

MATHEMATICAL
ELEMENTS

OF THE

Natural Philosophy,

Confirm'd by EXPERIMENTS:

OR, AN

INTRODUCTION

WJ TO *Hutchinson*

SIR ISAAC NEWTON'S Philosophy.

VOL. II.

Written in LATIN

By WILLIAM-JAMES'S GRAVESANDE,
Doctor of Laws and Philosophy, Professor of Mathematicks
and Astronomy at *Leyden*, and FELLOW of the ROYAL
SOCIETY of LONDON.

Translated into ENGLISH

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VOL. II

Written in Latin

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Doctor of Laws and Philosophy, Professor of Mathematics
and Astronomy at London, and Fellow of the Royal
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Translated into English

By JOHN GREGORY, Esq. of the Royal Society
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TO THE

Most Noble and Right Honourable

THOMAS

Lord PARKER,

BARON of Macclesfield,

Lord HIGH-CHANCELLOR of
GREAT BRITAIN, &c.

My LORD,

IF whilst your Lordship's Hours
are taken up with an Employ-
ment of the greatest Fatigue
as well as the highest Honour,
there can be any Time allow'd
for Recreation, your Lordship makes that
your Diversion, which few can attain to

DEDICATION.

without painful Application and laborious Study. To become a skilful Mathematician and a sound Philosopher, and at the same time shine in other Parts of Learning, requires a great and extensive Genius; but to lead a Life of Business, and be eminent in the Law, where Reputation is only got by constant Practice, as well as Brightness of Parts; and yet in those few Minutes of Leisure that are allow'd to breathe in, and are as it were stolen from Sleep, to play with all the Intricacies of Lines and Numbers, to view and understand the System of the World, the Proportion, Symmetry, and Harmony of its several Parts; to be acquainted with all the Experiments of Consequence that have been made, and be able to contrive new ones as useful as instructive, was only reserv'd to Men of uncommon Capacities, and to none in so eminent a Degree as to your Lordship.

DEDICATION.

THE Honour I have had of being admitted into your Lordship's Conversation, has given me an Opportunity to know which of the Mathematical Sciences are chiefly lik'd by you; and as ASTRONOMY and OPTICS seem to have the Preference, I thought this Translation of Dr. *GRAVESANDE*'s Second VOLUME would not be unacceptable.

HERE you have the Principles of the common OPTICS reduc'd into a small Compass, and confirm'd by new Experiments of the AUTHOR's own Invention; a fine Application of the Action of electric Bodies to discover the Nature of Fire; and Sir *ISAAC NEWTON*'s Doctrine of LIGHT and COLOURS prov'd by the most considerable of his

DEDICATION.

Experiments, which Dr. 's *Gravesande* performs with an Apparatus very ingeniously contriv'd, and nicely express'd by curious Figures. The last Part of this Volume not only leads a Beginner gradually on from the most simple to understand the most difficult *Phænomena* of ASTRONOMY, but gives such a physical Account of the celestial Motions as must be fully satisfactory to the best Geometricians; there your Lordship will see with Pleasure, that there are Professors abroad who understand the *PRINCIPIA*, and have so just a Value for that excellent Book, as to take Pains to propagate the wonderful Truths which it demonstrates, so as to make them plain to such Philosophers as are not yet able to go thro' all the difficult Propositions from whence they are deduc'd. But I will detain your Lord-

2 ship

DEDICATION.

ship no longer, either from the important Business of your Station, or the pleasing Truths of the AUTHOR, than while I beg leave to subscribe myself,

My LORD,

Your Lordship's

most obliged,

most obedient, and

most humble Servant,

J. T. DESAGULIERS.

DEDICATION

ship no longer, either from the impor-
tant Business of your Station, or the
pleasing Tasks of the Author, than
while I beg leave to subscribe myself

MY LORD,

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J. T. DESAIGUIERS



The AUTHOR's PREFACE.



Spoke of the Method of Reasoning to be used in natural Philosophy in the first Chapter of the first Volume, and in the Preface endeavour'd to vindicate the Goodness of the Method I have followed. There are several remarkable Specimens of this Method in the present Volume, which evidently shew

Sir ISAAC NEWTON's great Superiority of Genius above all other Philosophers.

Before him Naturalists were in the Dark in numberless Things relating to Light, and especially to Colours. For Instance, who ever suspected before that the Opacity of Bodies depended upon their Interstices? So that a Body becomes transparent when these Interstices are filled with a Medium of the same Density as the Particles of the Body itself.

His

The P R E F A C E.

His Account of the planetary System, and particularly of the Motion of the Moon, is not less worthy of eternal Praises, being likely to carry Astronomy to a greater Pitch of Perfection than the nicest Observations alone could possibly do; for if a Man is acquainted with the Laws that govern the System of the World, he will be able to make a better Use of his Observations, and to compute the Motions of the heavenly Bodies more exactly, than if he had nothing but Observations to direct him.

It was my Design in these Two Books to give my Reader a general Notion of the chief Things discover'd by Sir ISAAC NEWTON in natural Philosophy, and thereby to encourage him to the Study of the more abstruse, and at the same time more sublime, Parts of Mathematics, after he has learned the first Principles of Geometry, to fit him for the reading of these plain Elements; he will, as it were, go to the Fountain-Head when he reads the Writings of our great Philosopher, which will reveal such Things to him as were unknown to the profoundest Philosophers before him; and which, tho' publish'd, are still a Secret to all but thorough Mathematicians.

I have only a few Words more to say to the English Reader concerning the two English Translations of this Work.

As tedious and distasteful as an Author's Complaints generally prove, they cannot however be disallow'd, when they are grounded upon such an Injury done to the Author, as it is his Reader's Interest to be inform'd of.

Soon after the Publication of the first Volume of these ELEMENTS, it was translated into English, and printed in London, with the Name of a celebrated Professor of Mathematics, eminent for his Writings, inscrib'd on the Title-Page, as if he had look'd over and corrected this Translation;

The P R E F A C E.

Translation; but whoever examines any one Page in the Book, will immediately discover the Wrong done to that learned Gentleman, and the Abuse made of his Name, since he will every where perceive manifest Signs of the Translator's Ignorance of the very Principles of Physics and Mathematics, not to mention his Negligence with regard to the correcting of the Additions at the End of the Book, to the Numbers in the Margin, and the Distinction of the Propositions.

Dr. DESAGULIERS, whose Knowledge of Philosophy and Skill in making Experiments are so well known, began to translate the same Work about the same Time as the other, or rather before; but this obliging him to make more than ordinary Haste, he could not himself wholly avoid the usual Consequences of too much Precipitation, &c.



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celestial Motions.

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Mathematical ELEMENTS

OF

Natural Philosophy,

Confirm'd by

EXPERIMENTS.

VOL. II. BOOK III.

PART I. Concerning FIRE.

CHAP. I.

Of the Properties of Fire in general.



HO' we know several Properties of Fire, yet we are ignorant of a great many things relating to it.

I shall not invent Hypotheses, but reason from Experiments, and leave untouched what is not fully known.

Fire easily penetrates thro' all Bodies, however dense and hard they are. For we have never yet known

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any

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any Body, that by the Application of Fire has not been heated in all its Points; that is, in every Part.

545 *Fire moves very swiftly*, as appears from Astronomical Observations.

546 *Fire unites itself to Bodies*; for when they are brought to the Fire, they grow hot, as we said

547 before; *and in that Case they expand or swell*:

Which Expansion is also observed in such Bodies,

548 whose Parts do not cohere, in which Case *they also acquire a great Degree of Elasticity*, as is observed in Air and Vapours.

549 That *Fire is attracted by Bodies at a certain Distance from them*, will be shewn in the following

• 611 Part of this Book.*

If any Bodies are violently moved against one another, they will grow hot by such a Friction, and that to a great Degree; which shews that

550 *all Bodies contain Fire in them*: for, by rubbing, Fire may be put in Motion, and separated from Body, but can by no means be generated that way.

Having laid down these general Heads, we must examine several things more particularly.

C H A P. II.

That Fire adheres to Bodies, and is contain'd in them; where we shall also speak of Electricity.

AS we have already said, one may prove that Fire is contain'd in all Bodies, because there are no Bodies but what may be heated by Attrition; • 550 and that it coheres firmly with the Parts of Bodies, appears in Smoke and Vapours: for Smoke and Vapours are made up of Parts separated from Bodies, and agitated (sometimes

times very violently) by the Fire that is join'd with them.

There are, besides, several remarkable Phenomena arising from Fire, contained in Bodies, some of which we shall here mention; amongst which, there are such as relate very much to Electricity, for which Reason we must also treat of the Phenomena of Electricity.

DEFINITION.

Electricity is that Property of Bodies, by which (when they are heated by Attrition) they attract, and repel lighter Bodies at a sensible Distance.

Experiment 1.] Take two Pieces of Rock Crystal and rub them together, and immediately they will appear luminous all over, tho' they do not acquire any sensible Heat by that Attrition. Light (as well as Heat) is a Proof that there is Fire in a Body. The greatest Light is in those Points, where the Bodies touch one another.

Experiment 2.] Take a Glass Tube 15 or 18 Inches long, and one Inch in Diameter, and rub it with a Linnen or Woollen Cloth, and it will emit Light in the Dark.

Experiment 3. Plate I. Fig. 1.] This Tube, heated by rubbing, has a very sensible Electricity; for if light Bodies, such as Pieces of leaf Gold, and Soot, be laid upon a Plane, and the Tube be brought near them, they will be put in Motion, attracted, repelled, and driven several Ways by the Tube. The Tube acts at different Distances, according to the different State of the Air; sometimes at the Distance of one Foot; when the Air is full of Vapours the Effect is diminished.

555 There is one Thing remarkable, and very hard to explain in this Experiment, concerning the Direction of the Attrition; when you rub the Tube, one End of it is held in one Hand, whilst it is rubb'd with the other; which, if it be done from the Hand that holds towards the other End of the Tube, the Effect will not be sensible; but if you rub from the free End of the Tube towards the End held in the Hand, the contrary will happen. And this happens indifferently, whether you hold the open or the shut End of the Tube in your Hand.

556 In the following Experiments (*Plate I. Fig. 2.*) glass Globes are swiftly whirl'd about; to perform which, there must be a cylindric Neck at each End of every Globe; but only one of these Necks is to be open, and both are to have Brass Ferrels, such as are represented at G; a Cock E must be screw'd on to the Ferrel at the open Neck, and the opposite Brass Ferrel must have a little Wheel *r* of about an Inch and a half Diameter, join'd to it, with a small Brass Axis standing out: There is just such another Piece of Brass screw'd on to the Cock, so that it may be taken on and off at pleasure. These Axes go about a Quarter of an Inch into the Pillars SS, that support the Globe, and are Centers for it to whirl upon when it turns about its Axis.

The Pillars SS stand upon a strong horizontal Board of about an Inch and a half thick, framed into three other Boards, as may be seen in the Figure: In that which stands foreright, there is an Hole *f* that you may come at the lower Part of the Pillar, to make it fast with a Nut or Screw. The other Pillar S is likewise fasten'd by a Screw applied under the horizontal Board, and moved forwards and backwards in a Slit of 4 or 5 Inches long, before it be made fast, in order to take the

Glas

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Glass Globes in and out, and to take in such as are larger or smaller than others, according to the different Experiments to be made.

There is a great Wheel R which is turn'd round by means of the Handle M, and thereby gives a very swift whirling Motion to the Globe G.

In the Side of the upper Board, there is a Slit, along which the Pully *t* may be moved, by means of the Screw *c*, in order to keep stretch'd the Rope that goes round the Wheels R and *r*.

Experiment 4 & 5. Plate II. Fig. 1.] Apply a 557
Glass Globe of about 8 or 9 Inches Diameter, to the Machine above-mention'd, and let it be briskly whirl'd in a dark Place, the Hand all the while being held against it, to give it Attrition.

If the Globe be exhausted of its Air, it will appear all luminous within, but mostly so where the Hand touches the Glass.

But if the Globe has Air in it, and being 558 whirl'd in the same manner, the Hand be applied to it, no Light appears, either in the inner or outer Surface of the Glass; but Bodies at a small Distance from the Glass (as for Example, at a Quarter of an Inch, or nearer) become luminous; and so only those Parts of the Hand held against the Glass, which terminate, or rather environ the Parts that immediately touch the Globe, are luminous.

Experiment 6. Plate I. Fig. 2.] Take the Globe 559
made use of in the foregoing Experiments, and put it in between the Pillars, to whirl it as before: Then take a Brass Wire *a b d*, circularly bent in the upper Part, and fix it so that its curve Part may be about 4 Inches off of the Globe, with small Threads hanging from it, which being extended towards the Center of the Globe

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come within a quarter of an Inch. of the Surface of the Globe.

Whirl the Globe and apply the Hand, and immediately the Threads will be moved irregularly by the Agitation of the Air; but when the Glas is heated by the Attrition, all the Threads are directed towards the Center of the Globe, as may be seen in the Figure: And if the Hand be applied a little on one Side, or nearer the Pole of the Globe, the Threads will be directed towards that Point of the Axis which is under the Hand.

560 If the Air be drawn out of the Globe, this whole Effect ceases.

561 *Experiment 7. Plate I. Fig. 3.]* Take another Globe like the former, only differing in this, that the open'd Neck must have a larger Opening than that of the Globe G, so that you may put into it a round flat Piece of Wood o, that has a Brass Wire for its Axis. In order to fix this Piece of Wood in the Middle of the Globe, its Axis must be firmly screw'd to the Middle of the Cover that is join'd to the open Neck of the Globe at b: and the Cock E is also join'd to the Middle of the Cover on the Outside.

To the wooden Circle small Threads are fasten'd, which, being extended, almost touch the inner Surface of the Globe.

Turn the great Wheel so as to whirl the Globe, and rub it till it becomes warm, as was done in the former Experiments; if you cease to whirl the Globe, and the Hand be taken off, the Threads will immediately stretch themselves out like Radii from the Center towards the Surface of the Globe, yet they hardly remain one Moment at rest; for tho' the Globe be exactly shut, these Threads will be put in Motion, as appears

appears by blowing against the Globe, tho' you stand two Foot off, or farther, from the Globe. If you bring your Finger towards the Globe, tho' you do not touch it, the Threads next to it will be attracted by the Finger, and directed towards it; nay, and sometimes they fly from it. If you apply the whole Hand to the Globe, the Threads will be mov'd violently and irregularly.

And if all the Air be drawn out, as in the foregoing Experiment, the whole Effect ceases; and the Threads (both before and after the Friction) only hang down by their Gravity.

If we attend to all the foregoing Experiments, the following Conclusions seem to be naturally deduced from them, which we do not give out as certain, but very probable; for we must always distinguish Certainty from Probability.

*That Glass contains in it, and has, about its Surface, a certain Atmosphere, which is excited by Friction, * and put into a vibratory Motion;* for it attracts and repels light Bodies; * the smallest Parts of the Glass are agitated by the Attrition, and by reason of their Elasticity, their Motion is vibratory, which is communicated to the Atmosphere above-mention'd; and therefore that Atmosphere exerts its Action the farther, the greater Agitation the Parts of the Glass receive when a greater Attrition is given to the Glass.

The Fire, contained in the Glass, is expell'd by the Action of this Atmosphere; at least it is mov'd with it. For when light Bodies are put in Motion at a Distance from the Glass the Bodies also become lucid at a Distance. *

It is plain, that *this Atmosphere and Fire is more easily mov'd in a Place void of Air:* For if the Globe be exhausted of its Air, there can be perceiv'd no Light, nor any Effect of Electricity on the Outside of the Globe, * but the Inside of the Globe.

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Globe appears very luminous, and this Fire is perceiv'd to be in greater Quantity in this Experiment, than in that immediately after mention'd.

But the Action of the Electricity does also cease on the Inside when the Air is drawn out, which seems to overthrow what we said of the more easy Motion of the Atmosphere of Glass in *Vacuo*: But yet it is not probable that it shou'd move no whither in this Case. On the contrary, it seems to go the same way as the Fire, and to move that way where there is the least Resistance; and that the ceasing of the Action of Electricity is to be attributed to the Want of Air, by means of which, *the Threads are moved by the Atmosphere*; as we shall hereafter shew; and the Fire, which freely penetrates all Bodies, acts upon them violently *by means of the Air or Vapour*.

But laying aside Conjectures, tho' they have a great many Experiments for their Foundation, let us return to other Things relating to Fire; as several Experiments are to be made in *Vacuo*, we must describe the following *Machine* contriv'd to give Attrition to Bodies in *Vacuo*.

Plate II. Fig. 2.] Let M be the Air-Pump describ'd before, L L the Brass Plate of the Pump on which the Glass Receivers are set; on each Side of the Plate there is a wooden Pillar, A D, A D, which Pillars stand on the Board that carries the Air-Pump Plate, having their Base upon it, and a Part below the Base, and going thro' the Board with an Hole in it, to receive two cross Pieces of Wood or Wedges, that make them fast during the Experiment, but so that the said Pillars may be taken away afterwards.

The Glass Receiver, in which the Experiments are to be made, is about 9 Inches high, and 6 in

in Diameter, with a Cover * that has annex'd to it a Box or Collar full of oil'd Leathers thro' which a Brass Axis passes; and left in the whirling round of the Axis the lower End shou'd move, Part of the Box is made square, so as exactly to fit a square Hole in the Horizontal Board E E that presses on the Cover, and is made fast by bringing down the Nuts or Screws B B at each End, without bearing upon the Shoulders of the Pillars.

Towards the upper Part of the Pillars, there is a narrower horizontal Board F F press'd down also and fastened with Nuts, which has fix'd to it, on the under Side, a Piece of Brass g, with an Hole in it for the upper End of the above-mentioned Axis to turn in, whilst the biggest Part of this Axis, being in the middle of the Glass, has an outside Screw upon it, with the two Wing-Nuts *d, d*, moveable upon it, in order to fasten several Bodies upon the Axis.

The lower End of the Axis turns in the Hole *c*. The Brass Spring *ff* is joined to the Piece *c*, which screws it down to the Air-Pump Plate at the Hole where the Air is drawn out; but there must always be made a small Hole or Passage in the Piece *c*, for the Air to be drawn out at.

Having exhausted the Receiver, the Axis above-mentioned (by reason of the oiled Leathers in the Collar P) may be turned round without admitting any Air; but to give it a quick Motion, there is a little Brass Wheel *r* of about 2 Inches Diameter, with Points in its Groove, that the Rope, that turns it round, may not slip.

The great Wheel R, of about 3 Foot Diameter with its Frame, is brought up close to the Air-Pump, and made fast to its Foot by a Screw.

The Rope, that goes about the lesser Wheel *r*, is brought down over Pullies obliquely placed in

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in the upper Part of the Pillars C, C, and being over them, comes down to the great Wheel which it goes round; so that by turning the vertical Wheel R, the Brass Axis, above described, is carried round very swiftly, whereby a Motion is communicated to Bodies in *Vacuo* for several Experiments.

- 568 *Experiment 8.]* Take a Glass Globe of three Inches Diameter, or of two Inches and a half Diameter, with an Hole on each Side, where it may also have cylindric Necks. The Axis above-mention'd must go thro' those Holes, in order to give the Globe a whirling Motion; Pieces of Cork must be put on each Side of the Globe to cover its Holes, and made fast with the Nuts *d, d*, in the manner that they appear in the Figure, to hold fast together the Plates or little Wheels, thro' which the Axis goes.

This Globe, thus fix'd, will be swiftly mov'd in *Vacuo* with its Axis, by turning the Wheel R: To cause Attrition, a Piece of Woollen Cloth must be tied on to each Side of the Brass Spring *f, f*, which by its Elasticity presses the Globe hard on each Side.

Making the Experiment in the Dark, the Globe will appear luminous; and if the Motion be continued till the Globe grows hot by the Attrition, the Light will indeed be increased, but will appear fix'd in the Places where the Attrition is made.

- 569 It follows from that Experiment, that *the Fire, contained in Glass, does not want Air to make it visible*; for it grows hot, and shines when both the internal and external Air are taken out.

- 570 *Experiment 9.]* Take a round Piece of Wood of 2, or $2\frac{1}{2}$ Inches Diameter, and about $\frac{1}{2}$ an Inch

Inch thick, and let it have several Hollows cut in its Edge, that it may be encompass'd round with Beads of Plaister, which may be made fast to it with Threads going thro' them. Let this Piece of Wood, thus prepar'd, be made fast upon the Axis above-mention'd, in the same manner as the little Globe had been fasten'd; then give Attrition in *Vacuo*, as in that Experiment, and Light will be thereby produced in the Dark.

Experiment 10.] That Quicksilver contains Fire, 571
is plain from Experiments made upon it in *Vacuo*. For if Mercury well clean'd be shak'd about in an exhausted Glass, it will appear luminous.

If you put Mercury into a Glass Globe, the 572
Globe may be whirl'd round, as in the former Experiments, which will be delightful to see, if the Glass be mov'd slowly. If Mercury has no Tin mix'd with it, it may be clean'd with boiling hot Vinegar.

The Globe that has the Mercury in it, may be 573
exhausted by screwing on to it a Pipe about two Foot long, the lower End of which must be screw'd on to the Hole in the Middle of the Plate of the Air-Pump;

Then, if you cover both that Hole and the Hole thro' which the Air is drawn out with one of the Receivers above-mention'd, the Air will be easily drawn out of the Globe.

Plate I. Fig. 4.] Take a Plate O, and having 574
screw'd on to it the Pipe E E, whose other End is screw'd to the Air-Pump, put the Glass Receiver R upon it, and pump out the Air from it. The Pipe, through which the Air is drawn out, must stand up beyond the Plate in the Receiver about 4 or 5 Inches, be bent, and have a small Hole in it; which must be ta-

ken care of, for fear any Mercury shou'd get into the Air-Pump.

The middle of this Plate has a Brass Pipe B; whose lower End, that is without an Hole, comes down almost to the Bottom of the Vessel V, that contains very clean Mercury: There is an Hole in the Side of the Pipe, exactly shut by the Pin A. The upper End of this Pipe has a very small Hole in it, and comes up through the Plate into the Receiver.

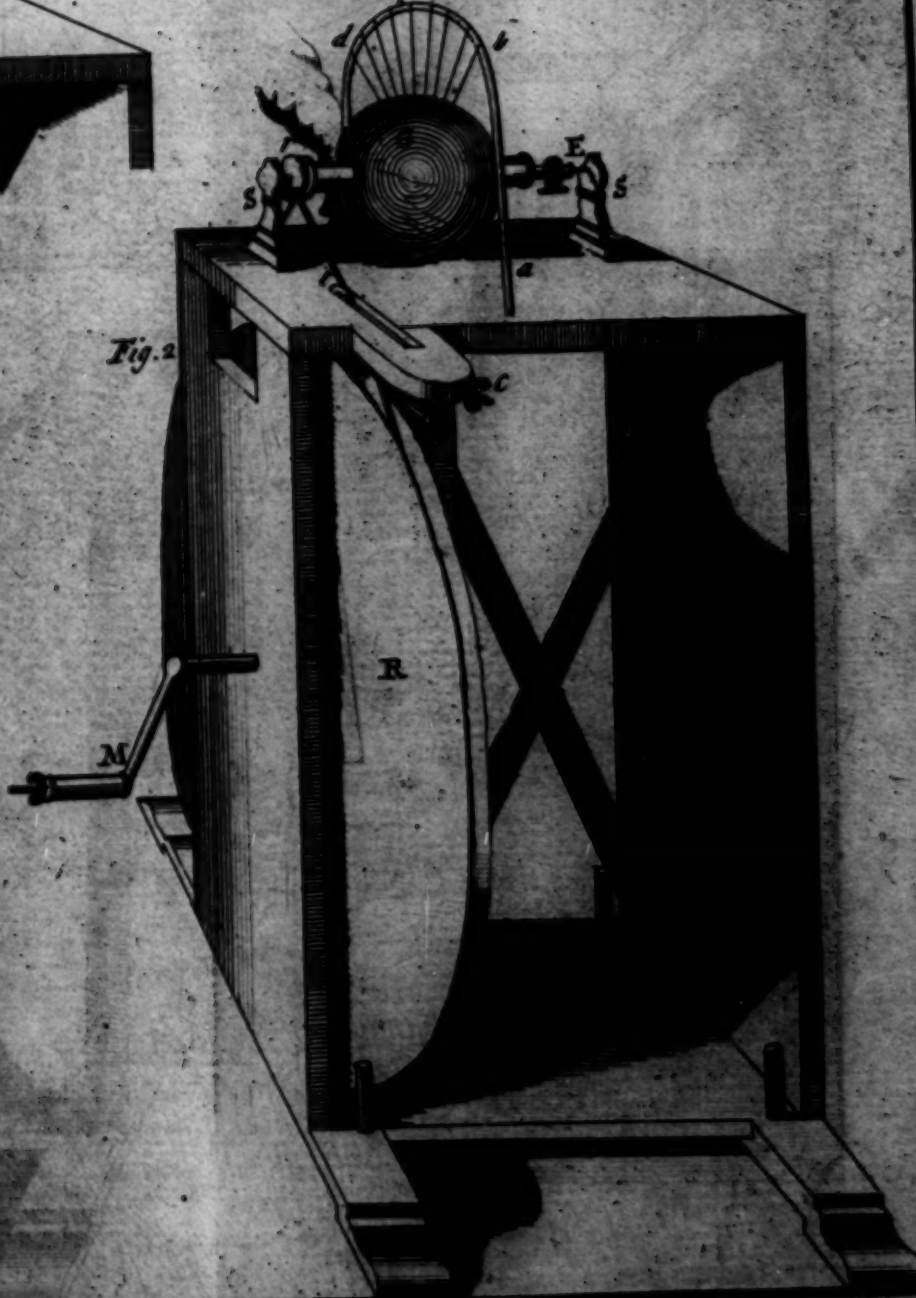
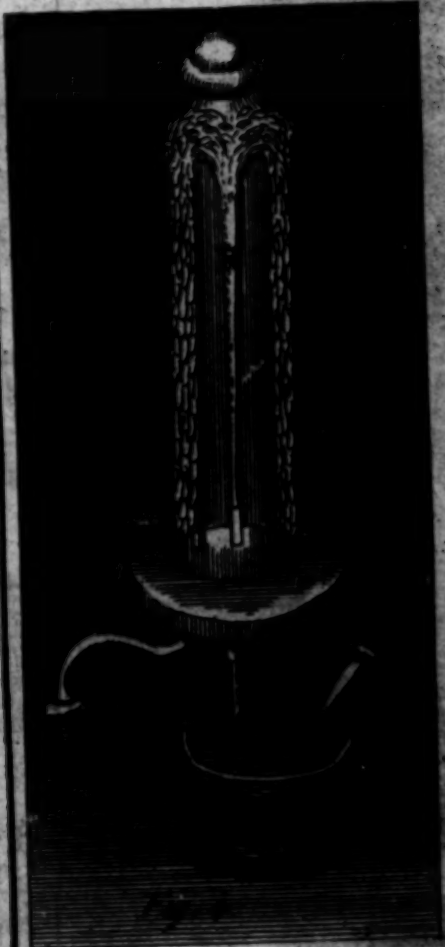
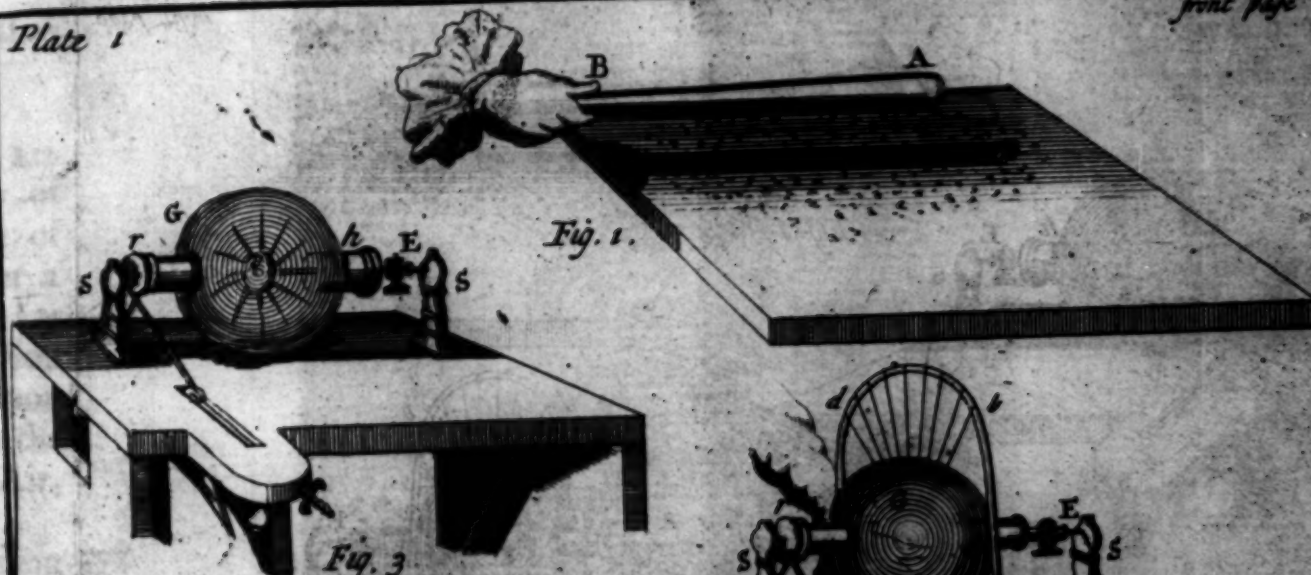
The Height of the Receiver R is about 16 Inches, and its Diameter 4 Inches; pump the Air out of it, and then open the Hole in the Side of the Pipe B, and the Mercury by the Pressure of the external Air will go up to the Tube, and will spout very strongly into the Receiver, striking against the Top of it.

The Experiment must be made in a dark Place, and the Mercury will appear luminous.

575 *Experiment 11.*] Chymists make a certain Preparation of Urine, which they call *Phosphorus of Urine*; which must be kept in Water. If it be roll'd up into small cylindrical Pieces like a Pencil, and you write with it upon a Paper, when you hold the Paper in a dark Place, you will see Letters of Fire. The *Phosphorus* itself, when you take it out of the Water, will grow hot and smoke; all which shews that the *Phosphorus* contains a great deal of Fire.

576 In this Experiment, we may observe that Water will upon the Fire contain'd in the *Phosphorus*; for it keeps it in, so that it cannot get out so long as the *Phosphorus* is cover'd with Water; but as soon as the Water is taken away, the Heat and Smoke immediately shew that the Fire issues out of the *Phosphorus*. The Air also does, in some measure, keep in the Fire contain'd in hot Water;

that





that is, hinders it from going out of it so fast as it will do in *Vacuo*.

Experiment 12.] Take two Vessels that are equal and alike, and put an equal Quantity of boiling Water into each of them. Let one of them be set upon the Air-Pump Plate under a Receiver, and pump out the Air; when you are pumping, the Water boils violently by the Action of the Fire that goes out of it; and then soon becomes only lukewarm, whilst the Water in the other Vessel, which remain'd in the open Air, has scarce lost any of its Heat.

One may observe something like this in shining Wood; for some Wood that is rotted in the Ground, shines when it is taken out; the Earth that encompass'd the Wood kept in the Fire, which goes out when that is remov'd, and the Wood continues to shine for some Days. In an exhausted Receiver, the Light is soon destroy'd, and is not restor'd by the Re-admission of the Air.

But it cannot be easily determin'd how Fire is confined in a Body by the ambient Bodies, nor is it easy to find out what Action produces this Effect; it is hardly probable that Pressure is here much concerned, since Fire does easily penetrate all Bodies by its Subtilty.

C H A P. III.

Concerning the Motion of Fire, where we shall speak of Heat and Light.

WE have said that Fire moves very swiftly,* and this Motion, in different Circumstances, produces different Effects. Heat and Light

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Light are to be attributed to the different Moti-
 579 ons of Fire. *Heat, in a hot Body, is the Agitation*
of the Parts of the Body and the Fire contained in it ;
 by which Agitation; a Motion is produced in
 our Bodies, which excites the Idea of Heat in
 580 our Mind. *Heat, in respect of us, is nothing but*
that Idea, and in the hot Body is nothing but Motion.
 Here we must call to mind what has been said of
 the Sensations. in general (N. 502.) which also
 may be referr'd to Light.

581 *When Fire enters our Eyes in right Lines, by the*
Motion that it communicates to the Fibres in the bot-
tom of the Eye, it excites the Idea of Light; of
 which Motion of the Fibres we shall speak more
 particularly hereafter.* *A rectilinear Motion is*

716
 582 the Motion of Light, as it appears from its be-
 ing easily stopped by an Obstacle. On the con-
 trary, such Motion is not requir'd in Heat: and
 that an irregular Motion is more for it, may be
 proved, because the Rays, that come directly
 from the Sun to the Top of a Mountain, pro-
 duce no sensible Heat; whilst in the Valley,
 where the Rays are agitated with an irregular
 Motion by several Reflections, there is often pro-
 duced a very intense Heat.

DEFINITION.

A Body is said to be Lucid, when it emits Light,
 that is, when it gives Fire a Motion in right
 Lines.

583 That there is not always Light, where there is
 Fire, is beyond all doubt; for we daily see hot
 Bodies that do not shine.

584 But whether there be any Body without Heat,
 cannot be determin'd. Heat, in all Bodies, is a
 Motion that may be infinitely diminish'd; and
 there may be such a Motion, tho' it be not sen-
 sible to us, because often we cannot discover
 any

any thing of Heat; for certainly there are a great many lucid Bodies, in which we can perceive no sensible Heat.* Concerning which it * 571 may be observ'd, that no Heat is sensible to us, unless the Body, that acts upon our Organs of Sense, has a greater Degree of Heat than that of our Organs. Which shews us, that the Judgment of our Senses, concerning Heat, is wholly uncertain.

When the smallest Parts of which any Body is made up are agitated, either by Attrition, or the Action of Fire applied externally to it, or any other Way; the Fire is separated from the small Particles, and agitated in the Body; then also the Fire and the Particles of the Body act upon one another by their Attraction, as is prov'd by the Experiments which we shall hereafter mention.* By this Action some Parts are * 611 separated from the Body, and carried off from it by the Motion of the Fire. And this is the Cause why *hard Bodies are often set on Fire by a* 585 *violent Attrition.*

Hence we deduce, that *the burning of Bodies is* 586 *a Separation of their Parts by the mutual Action of the Fire and those Parts on each other; some of those Parts, carried off by the Motion of the Fire,* 587 *make Flame and Smoke.*

We see besides, that a Body, that is burned by the Application of Fire, is not only dissolv'd by the Action of the external Fire, but also by the Action of the Fire contained in the Body itself; and that the Heat is increased by the Application of new Fire, and the Augmentation of the Agitation of the Fire which the Body itself contains; and therefore that *the Heat is not in* 588 *proportion to the Quantity of Fire.*

As to the Motion of Light, it is plain, as we said,* that it is perform'd in right Lines; but * 581 whether

whether it be successive or instantaneous, is disputed; that is, whether at the same Moment that a Body begins to shine, the Light is sensible at any Distance, or whether the Light goes on successively to Places more and more distant.

589 It seems clearly to follow from several Astronomical Observations, that that Motion is successive, and Philosophers did not long doubt of it; but by some later Observations, the Conclusions, drawn from the former, are weaken'd, and we are obliged to confess that the Motion of Light has something unknown to us.

To say that a Motion from one Place to another is not successive, implies a Contradiction, and it can scarce be doubted, that *Light moves from*
590 *one Place to another.*

For we observe that Fire is carried off in Vapours and Smoke; in which Case, Fire carries with it the Bodies to which it adheres, and yet is often mov'd very swift: If the Subtlety of Fire be consider'd, it will easily be found that it is immensely retarded by the Bodies to which it adheres; and, that, as soon as it is freed from them, it is mov'd with a very great Velocity.

There are several other Things very well worth observing, in respect to Light and Heat, but a great many of them are hard to explain. In natural Philosophy, when we are ignorant of the Causes, we must only mention the Effects.

591 We see several heated Bodies that become lucid, if their Heat be increased; such are Metals: They emit Fire by the Agitation of their Parts, but if the Motion of the Parts be increased, Part of the Fire is mov'd in right Lines, and the Body shines.

Thus

Thus if Smoke be made hotter by applying 592
Flame to it, it is chang'd into Flame; that
is, it becomes a lucid Body.

Experiment 1. Plate II. Fig. 3.] Having blown
out the Candle C, which sends out the Smoke
F; let another lighted Candle be brought to
it, and the Smoke of the first will be chang'd
into Flame, and that successively quite to the
Candle C, which is thus lighted by the Can-
dle A, tho' it be 7 or 8 Inches from it.

We have said that Air acts upon Fire*; one * 577
of its Effects upon it, that in many Cases is not
to be overlook'd, shews itself in respect of Light.
For *the Presence of the Air is often necessary for the* 593
Production of Light or Preservation of Fire, which
may be observ'd in the burning of all those Bo-
dies that go out when the Air is taken away
from them.

Experiment 2.] Put a lighted Candle under
the Receiver of the Air-Pump; exhaust the Re-
ceiver, and the Candle will immediately go out.

Plate II. Fig. 2. Experiment 3.] Take a round
Plate of Steel of about 3 Inches Diameter, with
an Hole in the Middle to receive the Axis which
is to be mov'd in *Vacuo**, and let it be press'd * 567
and fix'd between two round Pieces of Wood
by the Help of the Screws *d, d*; let Pieces of
Flint be fasten'd to the Spring *f, f*, so that by
the Motion of the Axis the Steel Plate may rub
nimblely against the Flints. As long as there is
any Air in the Receiver the Attrition will pro-
duce Sparks of Fire; when the Air is drawn
out, tho' you continue the Attrition you can
perceive no Light; but upon re-admitting the
Air, it becomes again sensible.

594 We observe also on the contrary, that *very oft-*
en the Absence of Air is necessary for Light, as
 might be observ'd in the Experiments already
 • 557 mention'd*. And lastly, That taking away the
 571 *Air, the same Light, which may be seen in the open*
 595 *Air, is sometimes increased.*

• 575 *Experiment 4.] With the Phosphorus* * above-
 mention'd write or draw Figures upon Paper,
 and they will shine in the dark, as we said be-
 fore; but the Paper will become much brighter
 • 577 *in Vacuo.*

C H A P. IV.

Of the Dilatation occasion'd by Heat.

596 **A** L L Bodies are dilated by the Action of
 • 547 Fire*; but that Dilatation changes as the
 Heat changes; so that it seems to depend rather
 upon the Motion than the Quantity of the Fire; for
 Bodies are expanded as well by rubbing as by
 applying Fire to them externally.

Experiment 1. Plate III. Fig. 1.] Take a square
 Iron Bar about 3 Foot long and $\frac{1}{2}$ of an Inch
 thick, and having laid it upon the Board between
 two firm Obstacles or Standards Q, Q, be-
 tween one of the Obstacles and one End of the
 Bar, thrust in an Iron Ruler about 4 or 5 Inches
 long (made Wedge-Fashion, being $\frac{1}{2}$ of an Inch
 broader at R than at r, and divided into equal
 Parts) as far as it will go; and observe the
 Division, that is against the End of the Bar,
 which must be a little oblique, that it may the
 better appy it self to the Ruler.

Then let the Bar be heated, either by Attri-
 tion or Fire; then let it again be put between

the

the Standards, and the Ruler thrust in between : As the Ruler will not go in so far as the Division that was observ'd before, it is plain that the Bar is lengthen'd by Heat.

Experiment 2. Plate II. Fig. 4.] That Liquids, as well as Solids, are dilated by Heat may be prov'd 597
in the following Manner. Take a Glass Ball G that has a small Tube E D for its Neck, and fill it with some Liquor up to what Height you please in the Tube, and the Liquor will ascend in the Tube, when you heat the Ball ; a small Degree of Heat will produce this Effect, even tho you use Mercury, the densest of all Fluids. The Experiment will be made in the same Manner, but better, if instead of a Ball you use an hollow Cylinder C with the long Tube B A join'd to it ; for the whole Liquor will be sooner heated in a Cylinder than in a Ball.

If the Tube E D or B A be divided into equal 598
Parts ; or the Tube with its Ball or Cylinder be fitted to a Frame on which equal Divisions are mark'd, the Heat may in some Sort be measured by this Machine : For the Liquor rises or subsides in the Tube, as the Heat of the neighbouring Bodies is increas'd or diminish'd. Such Instruments are call'd *Thermometers*, and are of common Use. They do indeed shew a Change 599
in the Heat, but it is uncertain whether they shew the Degree of Heat ; that is, it is not known what Relation there is between the Change of Expansion and the Change of Heat, so as to enable us to compare together the Degrees of Heat, by comparing the Degrees of Dilatation.

If the Ball G or Cylinder C be suddenly heated, 600
the Liquor in the Tube will immediately descend, but immediately after rise. By reason of the sud-

den Heat the Glass it self grows hot sooner than the Liquor contain'd in it; and therefore when the Glass is dilated by the Heat, the Cavity grows bigger, so that the Liquor descends; but the Heat being presently communicated to it, it rises again by its Expansion.

- From the Expansion of Bodies it is evident
 601 that *the Particles of which Bodies consist, from the Action of the Fire, acquire a repellent Force, by which they endeavour to fly from each other, and which acts contrary to that Force by which the Particles come to each other* *. As long as this last Force is stronger than the other, the Particles, cohere more or less, according to the different Degree of Heat. When the repellent Force is almost equal to the attractive, the Particles, which were before firmly join'd, scarcely cohere, yield to any Impression, and are easily mov'd one among another; whence we see that a So-
 602 lid Body is changed into a Liquid by Heat, which may be observ'd in all Bodies that are liquified by Heat, and return to their first State upon the Diminution of the Heat. It is a Question,
 603 *Whether all Fluidity is not owing to Heat?* which cannot be determin'd, because we know of no Body that is entirely without Fire in it; it is certain that Heat is not only the Cause of Fluidity in Metals, Wax, and such like Bodies, but that several Bodies, which are commonly reckon'd Fluid, are only so by reason of their Heat; thus
 604 *Water is melted Ice*; for when Part of the Heat of the Water is gone, it grows fix'd.
 605 Heat may be so increas'd that in some Bodies the attracting Force is overcome by the repelling Force; in which Case the Particles fly from each other; that is, acquire an elastic Force, as the Particles of Air have*, which
 431 Elasticity is increased even in the Air by Heat.

We

Plate 2.

Fig. 2

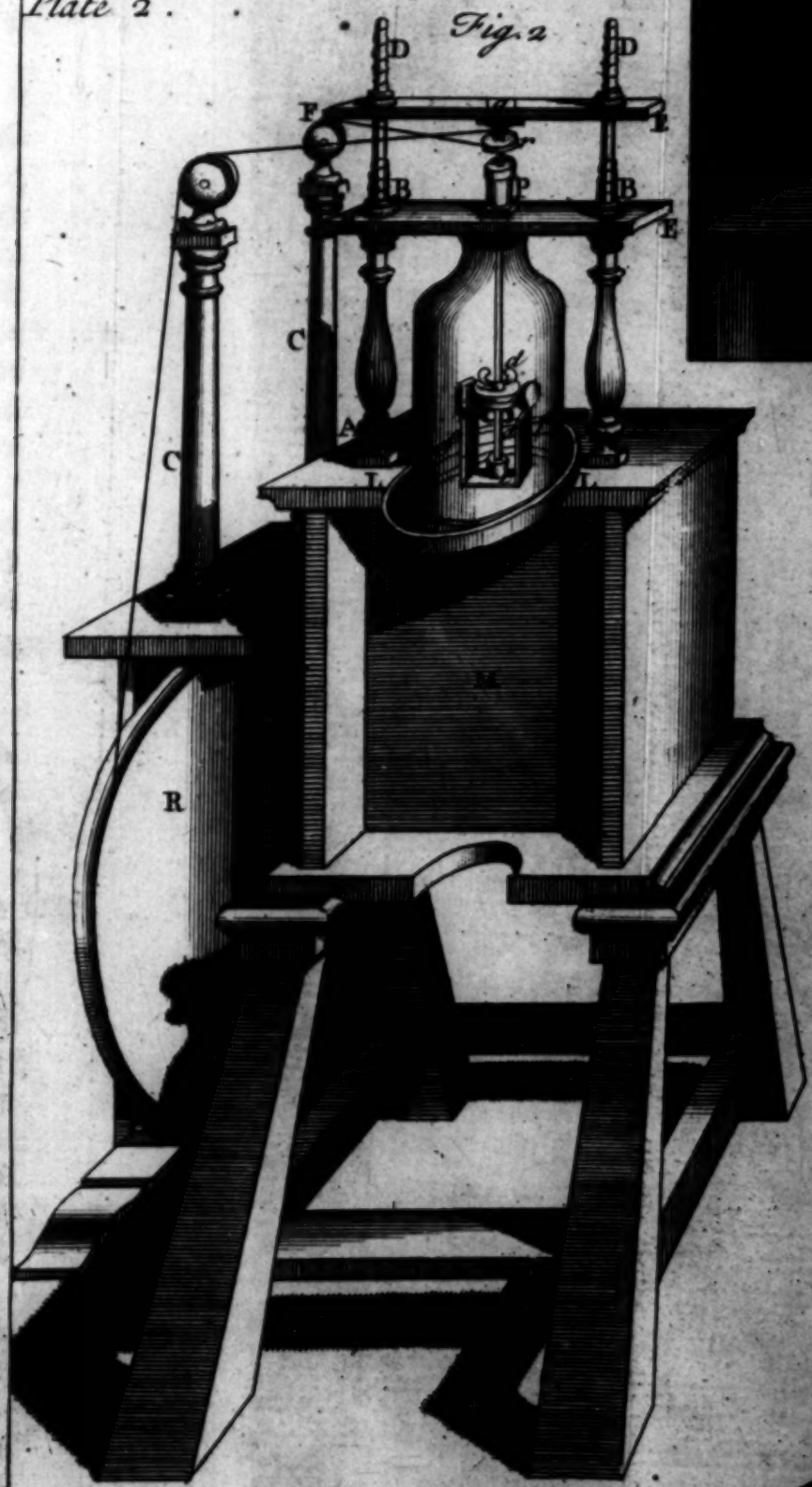


Fig. 4.

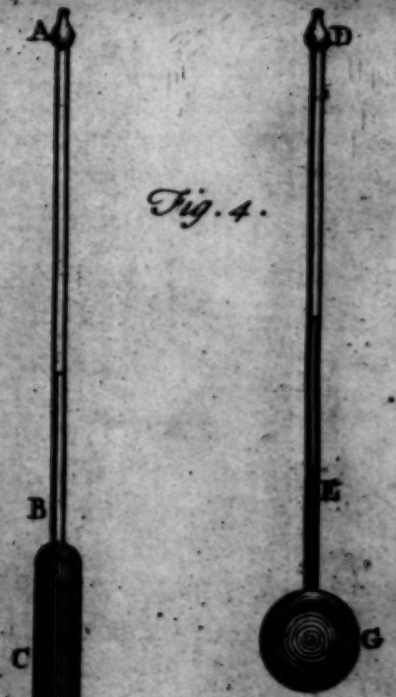


Fig. 3.



We may observe this Effect in Smoke and Vapours.

Experiment 3. Plate III. Fig. 2.] Take an hollow Brass Ball E, of about 4 Inches Diameter, with an Handle M; This Ball has a Pipe T with an Hole of hardly the twentieth Part of an Inch. Let this Ball be heated, and the Air in it will expand itself * and come out thro' the Pipe; * 597 then, immersing the Ball in cold Water, the Air will be again condens'd by the Cold, and the Water will go into the Ball, by the Pressure of the Atmosphere upon its Surface.

If this Ball, thus partly filled with Water, be laid upon the Fire, the Moment that the Water is chang'd into Vapours, those Vapours will go out thro' T; but if the Heat be increas'd, so as to make the Water boil violently, the Vapours, compress'd in the upper Part of the Ball, will by their Elasticity, endeavour to recede from each other every Way, and rush violently out of the Pipe. Such an Instrument is call'd *Æolipils*. 606

Experiment 4.] The following Experiment 607 shews a more sensible Effect of the Elasticity of Vapours.

Plate III. Fig. 2.] Take the Ball E, which is also of 4 Inches Diameter, made of Brass or Copper, but thicker than the former, and let it be placed upon a little light Cart, such as is represented in the Figure. In its upper Part, it has a square Pipe T. In the middle of this Pipe, there is a Separation, and the hind Part of the Pipe communicates with the Ball. There is an Hole of about $\frac{1}{4}$ of an Inch, in the middle of the Separation of the Pipe, which Pipe is open forwards. This Hole is shut up with a long Plate, that goes thro' two Holes in the
C 3 Sides

Sides of the Pipe, and applies itself to the above-mention'd Separation. As the Plate is made Wedge-wise, if you strike upon it with a Hammer, it will exactly shut the Hole.

Take this Ball off the Cart, and open the Hole, and, having heated the Ball, immerge it in Water, to let it be in Part fill'd, as in the last Experiment. Then, having shut the Hole, set the Ball again upon the Fire, till the Water boils violently; and then if you make it fast to the Cart, and open the Hole, the Vapour will fly out violently one Way, and the Cart be carried with the Ball the contrary Way.

608 The Vapour, being very strongly compress'd, endeavours to recede equally every Way, and therefore opposite Pressures destroy one another; but when the Hole is open'd, the Vapour, which goes out, does not press; therefore the Pressure one Way being taken off, that which acts in a contrary Direction prevails, and the Cart is mov'd along.

609 A Sky-Rocket rises up into the Air, because the Gun-powder, being set on Fire, acquires an Elasticity, and its Parts endeavour to recede every Way: As the Pipe, or Case of it, is open at one End, it is less press'd that Way, and consequently at the other End, the Pressure prevails, and carries up the Rocket.





BOOK III.

PART II.

Concerning the Inflexion, Refraction and Reflexion of Light.

CHAP. V.

Concerning the Inflexion of the Rays of Light.



HAVING premis'd, in the foregoing Part, what regards Fire in general; let us again farther examine the Properties of Light and the Phenomena arising from them.

The Things observ'd concerning Light are very wonderful, yet most of them are explain'd by a few Laws of Nature.

Light moves in right Lines*, can be inter- * 581
cepted by an Obstacle which wholly stops so much Light as comes upon it, but stops no more.

DEFINITION.

Any Light consider'd according to the Direction 610
of its Motion, if it be all carried in the same Direction, is call'd, a Ray of Light.

Such is a Beam of the Sun's Light going thro' an Hole.

Fire, as has been said, is attracted by Bodies * ; the notable Effects of which Attraction may be observed in the burning of Bodies ; they are also sensible in Light ; for *when Light passes near Bodies, it is turn'd out of the strait Way* ; which may be distinctly perceiv'd by the following Experiment.

Experiment 1. Plate III. Fig. 4.] Take a little Board T about 6 Inches long, and as high, with an Hollow *c c* in its Surface, in which two Plates of Steel slide, each of which has an Edge like a Knife ; the two Edges may be brought together in the middle of the Board, and be join'd together exactly. At the Place where they meet, there is a square Hole in the Board, of almost an Inch, that a Beam of Light, let into a dark Room thro' an Hole of a Quarter of an Inch in Diameter, may pass thro' the said square Hole, so as to come to the Plates.

If the Distance between the said Edges be of about the 10th Part of an Inch, and the Light be made to pass between them, the Board being set at the Distance of 3 Foot from the Window, if the Light falls upon the Paper A, at 5 Foot beyond the Board, there will appear, on each Side of the transmitted Beam, a Light like that of the Tail of a Comet ; which proves that the *Light is inflected* as it passes by the Edges of the Plates, as is plain from the Figure.

If you bring the Plates nearer (as for Example) within the hundredth Part of an Inch, instead of the Light above-mention'd, you will, on each Side of the Light of the Beam upon the Paper, see three colour'd Fringes, parallel to the Edges

Edges of the Plates; which will still appear more distinct, if the Hole in the Window be made less. I shall hereafter explain whence these Colours arise. Now it is enough to deduce from this Experiment, that Light is attracted by Bodies that inflect its Rays; for if there was not a Motion towards the Body, the whole Beam would have continu'd in its direct Motion.

But the Action of Bodies, by which they act upon the Light to attract it, exerts itself at a sensible Distance; as is plain from the Experiment. 612

Experiment 2. Plate III. Fig. 4.] Every thing else being as in the former Experiment, if the Plates be brought nearer together within $\frac{1}{4}$ Part of an Inch, no Light will appear upon the Paper between the Fringes above-mention'd; so that in this Case, all the Light, which passes between the Edges, is inflected on either Side, so as to produce the Fringes above-mention'd. Which plainly shews, that Steel acts upon Light at least at the Distance of $\frac{1}{4}$ Part of an Inch.

It is also proved, that That Action is increas'd as the Distance is diminish'd. 613

Experiment 3. Plate III. Fig. 4.] Things being still in the same Disposition, lessen the Distance between the Plates; and the Fringes will vanish successively, till the Plates being join'd together, no Light passes between them. The Fringes, which vanish first, are those that are produced by the Rays which are least inflected; and those vanish last which are produced by the Rays that are most inflected; that is, when the Edges come towards one another, the Shadow between the Fringes, made by each Edge, is continually increas'd, till, at last, the whole Light on each Side vanishes. Whence it plainly follows, that the

the Rays are more inflected, the nearer they pass by the Edges; that is, the Attraction is increased, as the Distance is diminish'd.

- 614 But this Attraction has something particular, for the Attraction of one Edge is increased, as the other is brought near it. Which is plainly seen in this Experiment; for as the Edges are brought towards one another, the Inflection of the Rays continually becomes greater.

C H A P. VI.

Concerning the Refraction of Light and its Laws.

- 615 **A** L L that gives Passage to Light, is call'd a Medium.

Glass, Water, and a Vacuum itself are Mediums.

When a Ray of Light goes out of one Medium into another, it is often turn'd out of the right Line.

DEFINITION II.

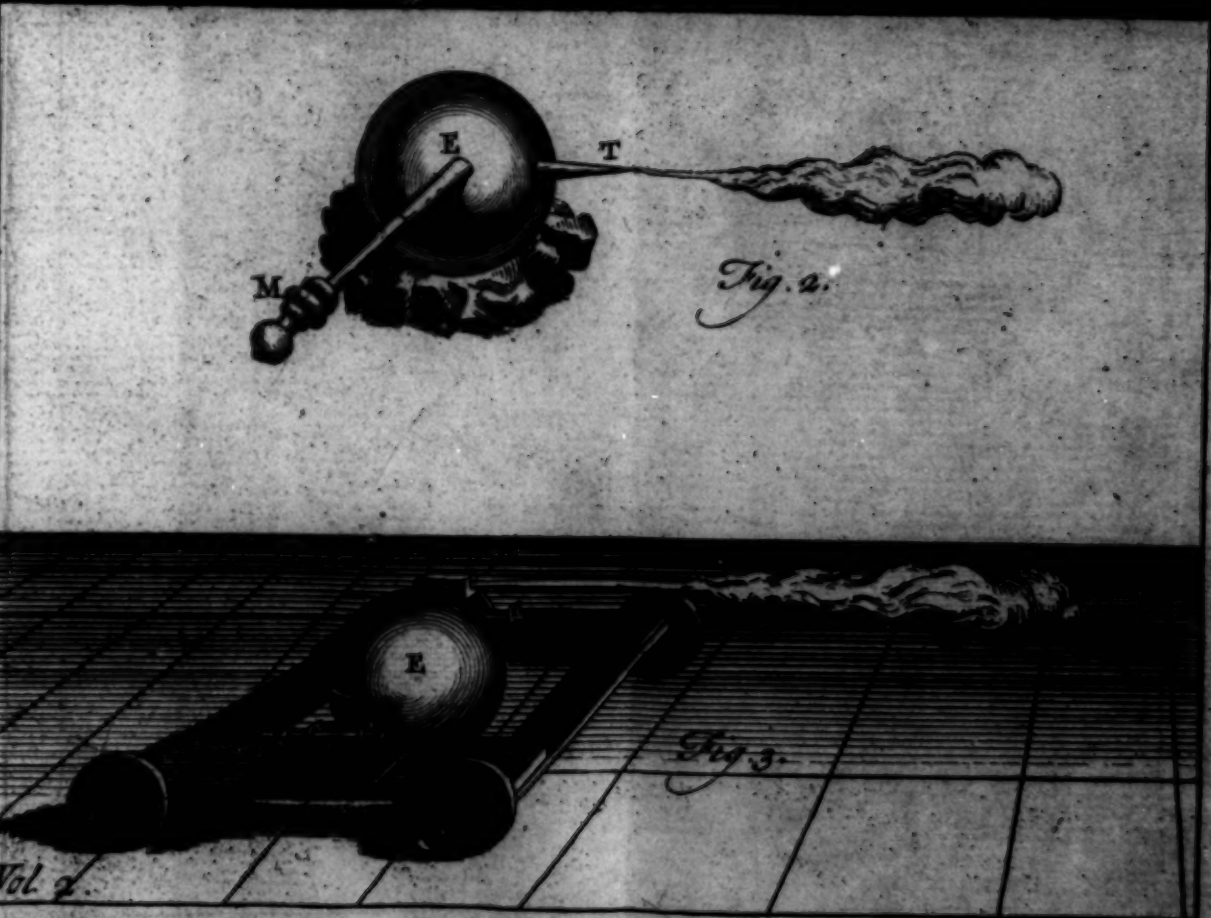
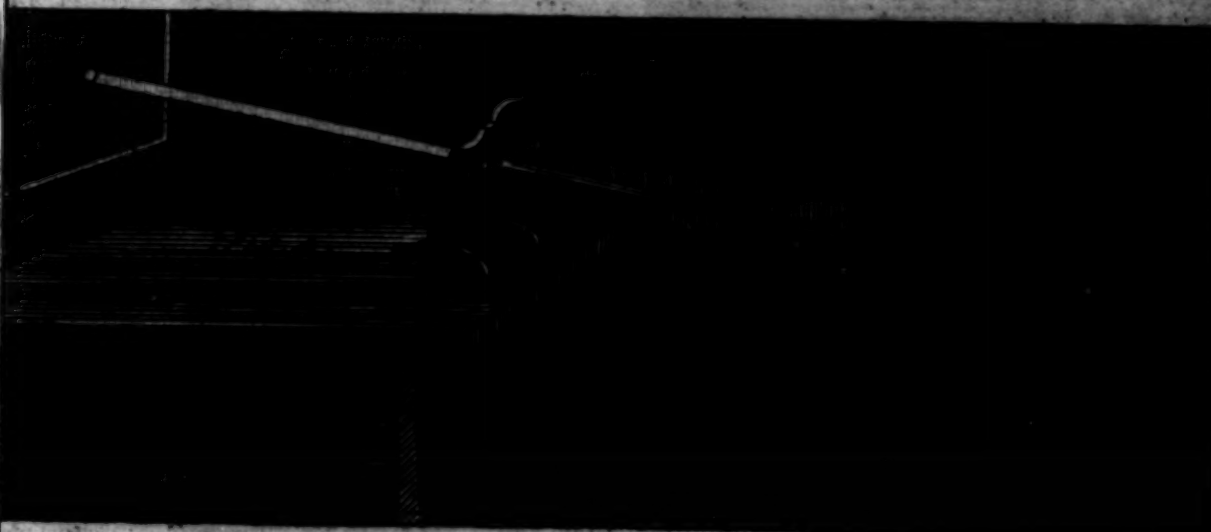
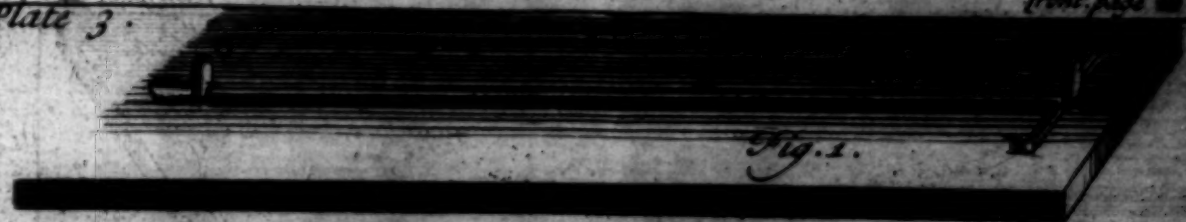
This Inflection is call'd *Refraction*.

- 616 In order to produce Refraction, the Mediums must be of different Densities, and the Ray must make an oblique Angle with the Surface that separates the Mediums.

- 618 Refraction arises from this, that the Rays are more attracted by a dense, than by a rare Medium; from which Attraction, that we have prov'd in the foregoing Chapter, all that relates to Refraction may be deduced.

- 619 Let EF be the Separation of the Mediums [Plate IV. Fig. 1.] and let X be in the denser, and Z in the rarer Medium. All the Particles have an attractive Force *, and this Force also obtains in Light. Let the Distance, at * which the Particles exert their Action, be that which is comprehended between the Lines EF and GH.

Therefore,





Therefore, the Light, which comes between those Lines, will be attracted by the Medium X.

At the Distance of the Line *GH*, only the extreme Parts of the Medium X act upon the Light; at a less Distance, both they and other Parts act so as to increase the attracting Force, when the Distance is diminish'd, as has been before observed*. In the denser Medium X, let the Line *IL* be supposed at the same Distance from *EF* as *GH* is in the Medium Z. Let the Light enter into the Medium X, and it will, on all Sides, be attracted by the Particles of the Medium, whose Distances from the Light are less than the Distance between *EF* and *GH*; for the Light is suppos'd to be attracted at that Distance by the Particles of the Medium X. • 613

As long as the Light is between the Lines *EF* and *IL*, the attracting Force is the strongest towards *IL*, because there are more Particles that draw that Way; but as the Number of Particles, that act the contrary Way, increases, that is, as the Distance from *EF* increases, the Force towards *IL* is diminish'd, till, in the very Line *IL* the Light be equally attracted every Way; which also obtains every where in the Medium X, beyond *IL*.

Suppose a Ray of Light to come in the Line *Aa*, and fall obliquely on the Surface, which divides the Mediums, or rather on the Surface *GH*, where the Action begins, by which the Light is driven towards the Medium X: When the Ray comes to *a*, it is turn'd out of the right Line, by the Force by which it is attracted by the Medium X; that is, by which it is driven towards it in a Direction perpendicular to its Surface. And, indeed, the Ray of Light is bent out of the right Line in every Point, as long as it is between the Lines *GH* and *IL*, between which



Therefore, the Light, which comes between those Lines, will be attracted by the Medium X.

At the Distance of the Line G H, only the extreme Parts of the Medium X act upon the Light ; at a less Distance, both they and other Parts act so as to increase the attracting Force, when the Distance is diminish'd, as has been before observed*. In the denser Medium X, let the Line I L be supposed at the same Distance from E F as G H is in the Medium Z. Let the Light enter into the Medium X, and it will, on all Sides, be attracted by the Particles of the Medium, whose Distances from the Light are less than the Distance between E F and G H ; for the Light is suppos'd to be attracted at that Distance by the Particles of the Medium X. • 613

As long as the Light is between the Lines E F and I L, the attracting Force is the strongest towards I L, because there are more Particles that draw that Way ; but as the Number of Particles, that act the contrary Way, increases, that is, as the Distance from E F increases, the Force towards I L is diminish'd, till, in the very Line I L the Light be equally attracted every Way ; which also obtains every where in the Medium X, beyond I L.

Suppose a Ray of Light to come in the Line A a, and fall obliquely on the Surface, which divides the Mediums, or rather on the Surface G H, where the Action begins, by which the Light is driven towards the Medium X: When the Ray comes to a, it is turn'd out of the right Line, by the Force by which it is attracted by the Medium X ; that is, by which it is driven towards it in a Direction perpendicular to its Surface. And, indeed, the Ray of Light is bent out of the right Line in every Point, as long as it is between the Lines G H and I L, between which

which the said Attraction acts; and therefore between those Lines it describes the Curve $a b$,
 6208 in the same Manner that Projectiles move*. Beyond the Line $I L$, the Action which bends the Ray ceases; therefore it goes on then in a right Line, according to the Direction of the Curve in the Point b .

The Distance, between the Line $G H$ and $I L$, is very small; therefore, when we consider Refraction, we take no notice of the bended Part of the Ray, which we look upon as if it was made up of two strait Lines $A C$, $C B$, meeting at C , namely, on the Surface which divides the Mediums.

Thro' C draw $N C M$, perpendicular to the Surface of $E F$.

DEFINITION III.

620 The Part $A C$ of the said Ray is call'd the incident Ray.

DEFINITION IV.

621 The Angle $A C N$ is call'd the Angle of Incidence.

DEFINITION V.

622 The Part $C B$ is call'd the Refracted Ray.

DEFINITION VI.

623 The Angle $B C M$ is call'd the Angle of Refraction.

In this Case, When Light goes out of a rare into
 624 a dense Medium, the Angle of Refraction is less than the Angle of Incidence, by reason of the Inflexion of the Ray; for these Angles wou'd be equal, if the Ray $A C$ continu'd to move strait on in the Line $C D$. The Ray $C B$ does also come nearer to the
 Per-

Perpendicular C M ; and therefore *the Refraction is said to be made towards the Perpendicular.*

On the contrary, *if the Ray goes out of a denser 625 into a rarer Medium, it will recede from the Perpendicular, because the denser Medium attracts the Ray in the same Manner, whether it goes out of a rarer into a denser, or out of a denser into a rarer Medium. Therefore if B C be the Ray of Incidence, CA will be the refracted Ray ; that is, Let the Ray come from either Side, and it will move 626 in the same Lines. Therefore, if there be two Rays, 627 and one comes out of a denser Medium into a rarer, and the other out of a rarer into a denser, and the Angle of Refraction of the one be equal to the Angle of Incidence of the other, the two remaining Angles of Incidence and Refraction will be equal to each other.*

Whence it follows, that *The Direction of the 628 Ray is not chang'd, if it moves thro' a Medium terminated by two Surfaces parallel to each other ; for as much as it is turn'd towards any Side at its Entrance, so much exactly is it turn'd the other Way as it goes out of the said Medium.*

If a Ray falls perpendicularly on the Surface that 629 separates two Mediums, it will not be turn'd out of the right Line by the Attraction of the denser Medium ; because in that Case it acts in the Direction of the Ray.

*Plate IV. Fig. 2.] To confirm what we have 630 said by Experiments, take a Trough or open Box P, about a Foot long, 4 Inches wide, and of the same Depth. Its two Sides *a b c d*, *a b c d*, are Planes of Glass parallel to each other, and this Box must be fill'd with Water two Inches and half high.*

The Experiments must be made in a dark Place ; the Light must be let in thro' a Slit in the moveable round Plate O, which is fix'd to the

Win-

Window-shut, and is easily turn'd round, that the Slit may be inclin'd at Pleasure: The Slit is 4 Inches long and $\frac{1}{2}$ Inch wide.

The Light, that is let in, strikes upon the Looking-Glass S, and the Slit is so inclin'd, and Speculum disposed, that the Beam, that comes thro' the Slit, may be horizontally reflected by the Looking-Glass, the Beam being in a vertical Position, that it may go thro' a vertical Slit in the Board T, of the same Dimensions as that in the Plate O. This Board is us'd to diminish the Breadth of the Beam, which is continually increasing, by reason of the Light that comes from the Sides of the Sun.

Experiment 1.] Things being dispos'd, as we have said in the Description of the Machine; let the Beam fall perpendicularly on the Surface $abcd$, and it will pass perpendicularly thro' the Water and the upper Part of the Trough, and not be turn'd out of the Way at all, either going in or coming out; by which is confirm'd what has been said (N^o 629.)

Experiment 2.] Now let the Beam fall obliquely on the Surface $abcd$, at fg ; the upper Part of the Beam will continue its Motion strait along b ; but the lower Part of it, in the Water, will be bent towards i , coming towards the Perpendicular; which confirms (N^o. 624)

Experiment 3.] Every Thing being, as in the former Experiments, the Beam, which, at i , goes out of Water into Air, is turn'd out of its Way fromwards the Perpendicular, and in such Manner, as to move in the same Direction as the incident Ray that goes into the Water at g ; for it goes

Book III. of Natural Philosophy.

31.

goes on parallel to the Ray $f b$ continued: which confirms (N°. 625, 626, and 628.)

In what has been said hitherto, we have only consider'd the Attraction of the denser Medium, because it overpowers; but we must not overlook the Action of the rarer Medium, because it diminishes the Action of the denser, which Action becomes so much the less on the Rays of Light, as the Mediums differ less in Density. Therefore *there is no Refraction where the Densities of Mediums are equal; and the Refraction is greatest where the Densities of Mediums differ most.* 631

The Laws of Refraction are deduc'd from the Acceleration which the Attraction generates; therefore this Acceleration must be examin'd.

DEFINITION VII.

The Space, terminated by the Planes GH and IL , is called the Space of Attraction. 632

[Plate 4. Fig. 1.] The Attraction acts between the Planes represented by these Lines but no farther*. 619

The Direction of this Action is perpendicular to the Surface that separates the Mediums, and consequently to the Surface IL ; and its Form is unequal at different Distances from the Surface. But it is equal at equal Distances, because both Mediums are supposed homogeneous, and alike in all their Parts.* 633 619

The Motion of the Ray AC may be resolved into two other Motions, according to the Directions AO and OC^* , of which the first is parallel to the Surface EF , and the second perpendicular to that Surface: The Celerities of which Motion will be respectively proportional to those Lines AO and OC , whilst AC denotes the Celerity of the Ray itself*. 192 192

The

634 The Motion, according to the Direction AO , is not chang'd by the Attraction perpendicular to the Surface IL , only the Motion according to OC is accelerated.

635 Keeping the Line AC , that is the Celerity of the Ray, its Inclination may be chang'd, whereby also the Celerity in the Direction OC is also chang'd; which Celerity wholly vanishes, if the Angle AaG be very small. In which Case, if after Light has enter'd into a denser Medium, its Motion be resolv'd into two, so that the Direction of the one be perpendicular to the Surface IL , its Celerity must be wholly attributed to the Attraction so often mention'd. For, as it enters into the Space of Attraction, a Motion is generated in that Direction; and, as it goes thro' that Space, in which a new Action does every where act upon the Light in the same Direction, it is continually accelerated. Which Acceleration obtains in every Passage of Light thro' the Space of Attraction; but it is different, according to the different Celerity with which Light comes perpendicular to the Surface which parts the Medium.

If the Attraction was equable thro' the whole Space of Attraction, the Laws concerning the said Attraction might be determin'd (as was said of the Acceleration of heavy Bodies after N^o 130.) by Help of the rectangular Triangle PQR , (Plate IV. Fig. 3.) in which the Lines parallel to the Base represent the Celerities, whilst Portions of the Area of the Triangle represent the Spaces gone thro'.

But here we have always the same Space run thro', namely the Breadth of the Space of Attraction, because we only consider the Motion which is perpendicular to the Surface, which separates the Mediums; therefore that Space is always represented by equal Parts of the Area of the Triangle PQR . Let Pdc represent such a Part

Part when the Celerity is equal to o . The Light enters the Space of Attraction in the above-mention'd perpendicular Direction, that is; when an incident Ray makes a very small Angle with the Surface that separates the Mediums; dc in that Case will denote the Celerity acquir'd by the Attraction, and with which the Light goes out of the Space of Attraction.

But if the Light goes perpendicularly into the Space of Attraction with the Celerity that is express'd by fg , it will go out of that Space with the Celerity bi , supposing the Areas Pdc and $fgib$ equal to one another, as appears from what has been said. The Triangle Pdc , Pfg , Pbi are Similar, and therefore their Areas are to one another, as the Squares of the homologous Sides dc , fg , bi ; but the Sum of the Areas Pdc , Pfg , is equal to the Area Pbi ; (by reason of the equal Areas Pdc and $fgib$;) therefore also the Sum of the Squares of the Lines dc and fg is equal to the Square of the Line bi ; whence it follows, that with the three Lines above-mentioned one may form a rectangular Triangle, whose Hypotenuse will be bi .

Therefore, *In a rectangular Triangle, one Side of 636 which is the Celerity with which Light enters the Space of Attraction perpendicularly, and the other Side the Celerity acquir'd in going thro' that Space, when the Celerity with which the Light enters into it is equal to o ; the Hypotenuse of the Triangle will be the Celerity with which the Light goes perpendicularly out of the Space of Attraction on the other Side:* Which universally obtains, which way so ever the Attraction is chang'd, in the Space of Attraction, according to the different Distance of the Planes that terminate that Space. Which to prove,

Let us suppose the Space of Attraction to be divided into two Parts, whether equal or un-

equal, by a Plane parallel to the Surfaces with which it is terminated. Let us suppose besides, that the Attraction is not the same in those two Parts, but yet that it does not vary in one of them. These Parts are to be consider'd as two different Spaces of Attraction. Let A (*Plate IV. Fig. 4.*) be the Celerity which the Light acquires in going thro' the first Part of the Space, when it enters the Space with the Celerity O. Let B be the Celerity acquir'd in going thro' the second Part of the Space, when the same Light enters that Part with the Celerity O. It is to be observ'd that this Demonstration relates to the Motion, perpendicular to the Surface, which separates the Mediums.

Let the Light enter the first Part of the Space above-mention'd with the Celerity O, it will come to the second with the Celerity A; if therefore a right angled Triangle be form'd with the Sides A and B, the Hypothenufe ED will express the Celerity with which the Light will go out of

• 636 the Space of Attraction*.

If the Light enters the Space of Attraction with the Celerity FG, let the rectangular Triangle HFG be drawn with the Sides of FG and A; the Hypothenufe HG will be the Celerity, with which the Light goes out of the first Part of

• 636 the Space of Attraction*, and enters into the second; and if you draw the rectangular Triangle HGI, whose Perpendicular is equal to the Line B, you will have the Hypothenufe IG to denote the Celerity with which the Light goes out, and continues its Motion after it has run
* 636 thro' the whole Space of Attraction*.

Now we must demonstrate that the Celerity IG is also the Hypothenufe of the rectangular Triangle NML, whose Side ML is equal to the Celerity FG, with which the Light enters the Space

Space of Attraction; and whose other Side $L N$ is equal to the Line $E D$, which is the Celerity that the Light acquires in going thro' the whole Breadth of the Space of Attraction, when it has enter'd it with the Celerity O ; which being also demonstrated in that Case, in which two different Forces of Attraction act, it is plain that the Proposition of $N^{\circ} 636$ is thereby prov'd.

But it is plain from the Consideration of the rectangular Triangles, that the Lines $I G$, and $N M$, are equal. The Square of the Line $N M$ is equal to the Squares of the Lines $N L$ and $L M$, or $F G$: $N L$ is equal to the Line $E D$, whose Square is equal to the Squares of the Lines $E C$ and $C D$, or of the Lines A and B , which are equal to $F H$ and $H I$: therefore the Square of the Hypothenufe $N M$ is equal to the three Squares of the Lines $F G$, $F H$, and $H I$. To which three Squares, the Square of the Lines $G I$ is equal; as having been prov'd equal to the Squares of the Lines $H I$ and $H G$, which last is equal to the Squares of the Lines $H F$ and $F G$.

If the Space of Attraction be divided into ever so many Spaces by plain parallel Surfaces, which terminate that Space, and the different Parts have different Forces of Attraction, the same Demonstration will serve; and the Number of Divisions may be made any how *in infinitum*; which Case obtains in the Refraction which Light suffers going out of any Medium into another of different Density*: to which Refraction⁶³⁷ therefore the Rule of $N^{\circ} 636$ may be applied.⁶³⁶

Plate IV. Fig 5.] Let Z be the rarer, and X the denser Medium, and let them be separated by the Plane $E F$; let a Ray of Light $A C$ fall obliquely on the Surface $E F$; let $A C$ denote the Celerity of the Light in the Medium Z , and

let that Line AC be settled; that is, let it be always the same, whatever the Inclination of the Ray is. With the Center C and the Semidiameter CA , draw a Circle; and let NCM be drawn perpendicular to EF ; from A , draw AO perpendicular to NC , and AQ to EF .

Let us conceive the Motion along AC resolv'd into two others, the one along AO , and

- 192 the other along AQ or QC *; the Line QC will denote the Ray's Celerity perpendicular to the Surface EF , which Celerity alone is increased
- 634 by the Attraction of the Medium*.

- Let CP be the Celerity which Light acquires, in passing perpendicularly thro' the Space of Attraction of the Medium X ; supposing the Celerity of Light at its Entrance to be O , the Hypotenuse OP of the rectanglar Triangle PCO will be the Celerity of the Ray AC in the Medium X , according to a Direction perpendicular to the Surface EF *; the Celerity of the Light in the Direction AO or QC , parallel to
- 637 the Surface EF , is not chang'd*. Therefore let
- 634 CV be equal to AO or QC , and VB perpendicular to EF , equal to the Hypotenuse PO , and draw CB ; the Motion along CB is compounded of the Two, and this Line by its Situation determines the Direction, and by its Length
- 190 the Velocity of the Light in the Medium X *; which
- 638 Celerity is not chang'd by the different Inclination of the Ray AC . For the Square of the Line CB is equal to the Square of the Line BV or PO , the Square of the Line CV or AO ; but the Square of the Line PO is equal to the Squares of the Line PC and CO : Therefore the Square of the Line CB is equal to the three Squares of the Lines PC , CO and AO ; which two last, if join'd, will give us the Square of the Semidiameter AC or CN ; that is, CB is equal

to PN, whose Square is also equal to the Squares of the Lines PC and CN, and which undergoes no Change from the different Inclination of the Ray AC.

The Line CB does in T cut the Circle, which is describ'd with the Semidiameter CA; from the Points B and T, draw BS and TR perpendicular to CM; by reason of the Similar Triangles CBS, CTR, BC will be to TC or CA, as BS to TR; which Lines therefore, by reason that BC and CA are settled, will have the same Ratio to one another, whatever be the Angle of Incidence. TR is the Sine of the Angle of Refraction TCR, and BS which is equal to CV, that is equal to AO, is the Sine of the Angle of Incidence ACO.

Therefore, in every Inclination of the Incident Ray, there is a settled and constant Ratio between the Sines of the Angles of Incidence and Refraction. 639

Now since BC and CA, which are as the Sines above-mention'd, do also denote the Celerities of the Light in the Mediums X and Z, it follows that those Sines are inversely, as the Celerities in the Mediums.

If the Medium Z be Air, and X Water, the 640
aforesaid Sines are as 4 to 3; and the Celerity of the Light in Air to its Celerity in Water, as 3 to 4. But if Z being still taken for Air, X be Glass, the Sines will be as 17 to 11. One Experiment determines this for all Mediums.

The Ratio between the Sines of any Angles is 641
the inverse Ratio of the Secants of the Complements, as it appears in this Figure, supposing a Circle drawn with the Semidiameter CQ or CV; for then AC (which is equal to CT) and CB will be the Secants of the Angle ACQ, and BCV Complements to the Angles of Incidence and Refraction, and inversely, as BS (which is equal

equal to AO) and TR , which are the Sines of Incidence and Refraction in the greater Circle.

And this Proportion of the Secants gives us a Method of easily reducing to Experiment the Prop. of N^o. 639.

Experiment IV. Plate IV. Fig. 6.] In this Experiment the Light is to be let into the dark Chamber thro' a Slit in the same Manner as in the former Experiments, and by means of a Looking-Glass reflected thro' the vertical Slit in the Board T .

Take a Trough P , nearly of the same Size as that which was us'd in the former Experiments, but which has only one End of Glass, namely, the little Side $abcd$. And let it be half fill'd with Water.

The vertical Beam of Light being made to fall obliquely on the Glass End of the Trough, the Part of it, which is above the Water, and goes directly forwards, and at b falls upon one of the long Sides of the Trough; but that Part of the Beam, which is refracted in the Water, goes along gi , and strikes against the same Side of the Trough at i . Whatever the Angle be which the Beam makes with the End or Side $abcd$, the Lines fb and gi will always be to each other, as 3 to 4; as is very easily shewn in several Inclinations, if you have two Scales, each with small Divisions on them; which Divisions in the one must be to the Divisions in the other, as 3 to 4: For the greater Line gi will always contain as many of the great Divisions, as the lesser Line fb will contain of the small ones. The Angle, which fb makes with the Plane $abcd$, is the Complement of the Angle of Incidence to a right

a right Angle ; and the Angle made by the Line gi with the same Plane is the Complement of the Angle of Refraction to a right Angle ; therefore gb and gi are the Secants of the Complements of the Angles of Incidence and Refraction, which have a constant Ratio to one another, as was to be confirm'd by this Experiment.

We have hitherto consider'd a Ray of Light ⁶⁴² going out of a rarer into a denser Medium, but the same constant Proportion of the Sines mention'd, N^o 639, holds good also in the contrary Motion of the Rays ; the Angles ACN , MCB (*Plate IV. Fig. 5.*) are not chang'd, whether the Incident Ray be AC or BC *. In that Case, if ⁶²⁶ BC be the Celerity of the incident Ray, CA will be the Celerity of the refracted Ray ; for the Motion of the Ray, going out of X into Z , is retarded in the same Manner by the Attraction towards the Medium X , as it is accelerated in the contrary Motion.

CHAP VII.

Of the Refraction of Light, when Mediums are separated by a plane Surface.

DEFINITION I.

RAYS proceeding from the same Point, as a ⁶⁴³ Center, and continually receding from each other, are said to be divergent.

DEFINITION II.

Those diverge more, which make a greater ⁶⁴⁴ Angle with each other.

DEFINITION III.

- 645 The Point, from which the divergent Rays proceed, is call'd the Radiant Point.
- 646 The more the Rays diverge, supposing them at equal Distances from each other, the less is the
- 647 radiant Point distant from them; and so on the contrary, the Rays are often mov'd by Refraction, as if they came from a radiant Point, tho' they do not really proceed from such a Point; that is, if the Rays should be continued or produced back the Way from whence they come, they would meet in one Point. In that Case also, the Rays are said to be divergent.

DEFINITION IV, and V.

- 648 Rays which concur in one Point, or would concur if they were continued, are said to be convergent;
- 649 and those are more convergent, which make greater Angles between themselves.

DEFINITION VI.

- 650 The Point of the Concourse of converging Rays, is call'd the Focus.

DEFINITION VII.

- 651 The Point, in which converging Rays, and such as (being intercepted or turned out of the Way before their Concourse) would have concurred, being continued, is call'd the imaginary Focus. Which Name is also given to that Point from which those divergent Rays are conceived to flow
- 652 which do not proceed from the radiant Point*.
- 652 The more the Rays converge, supposing their Distance from each other to be the same, the less Distance from them is the Focus, whether real or imaginary.

III.

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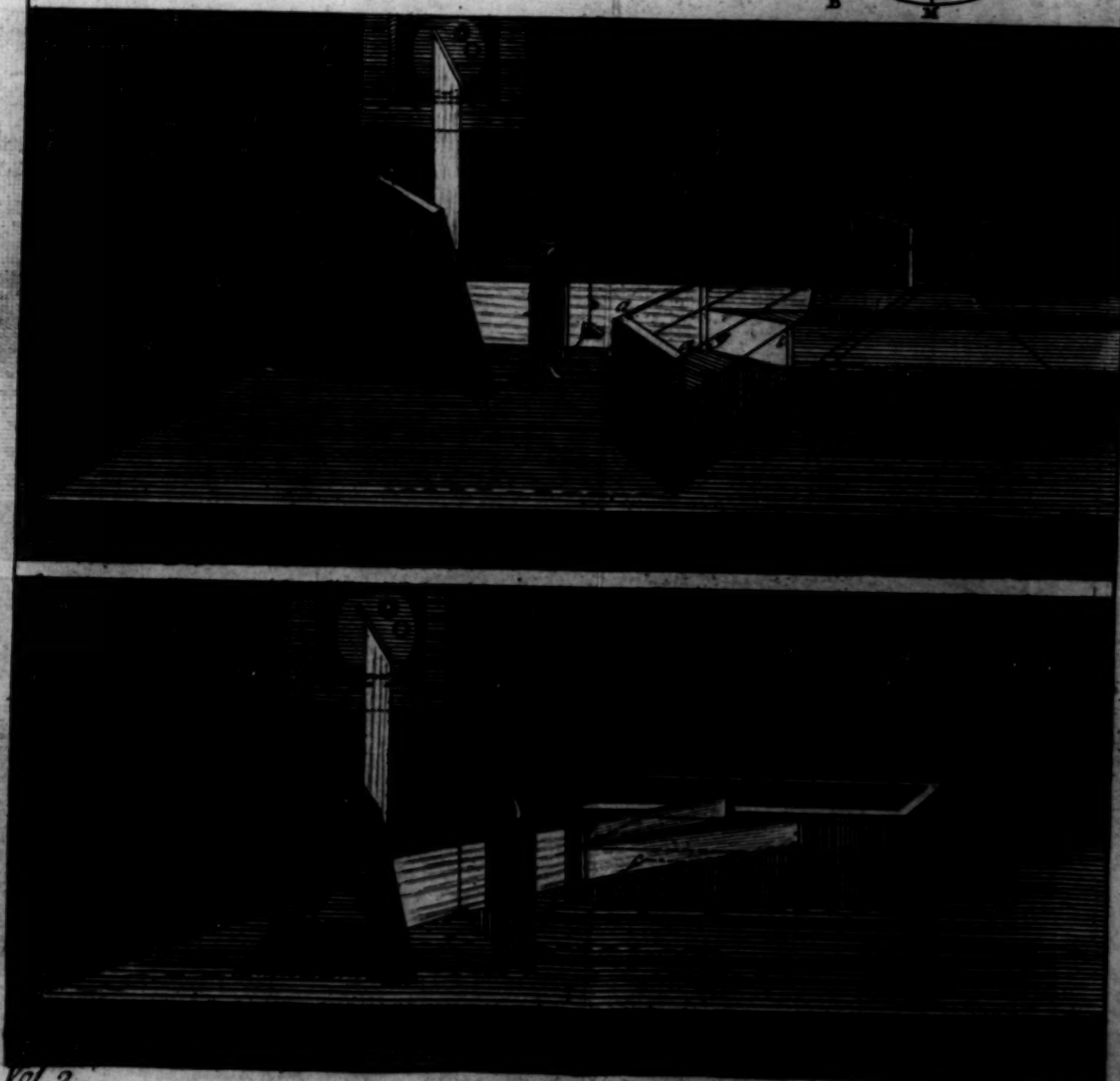
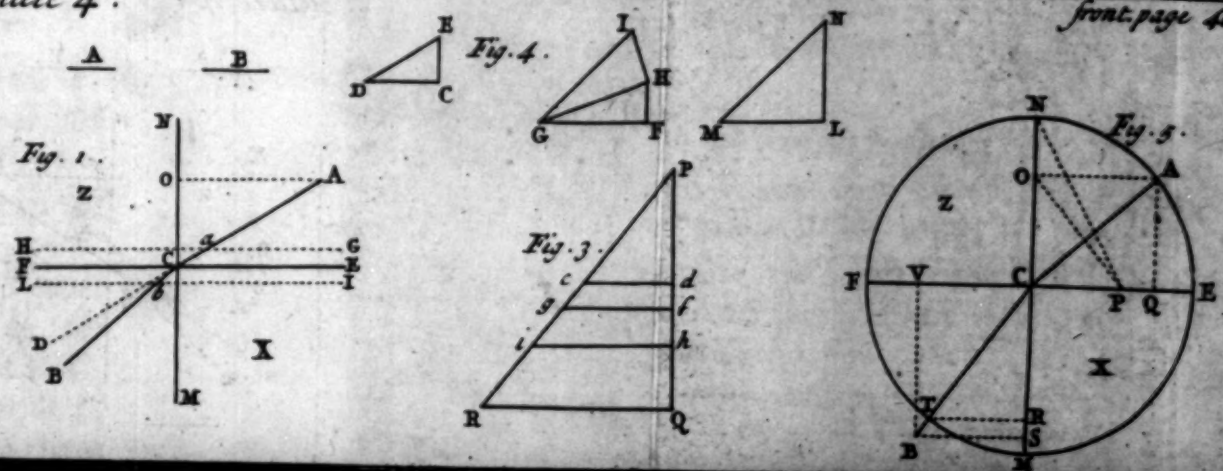
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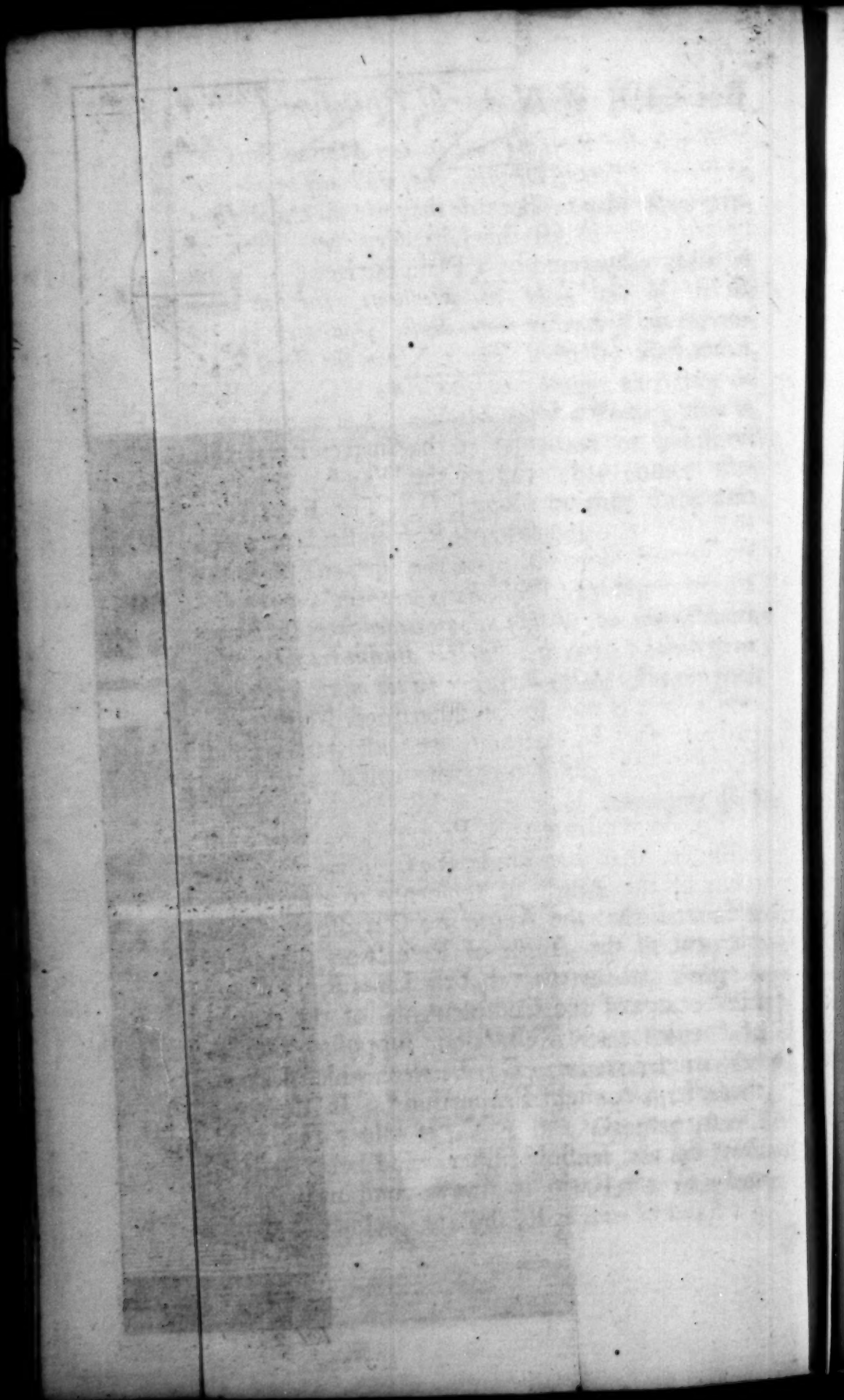
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Plate 4.

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If parallel Rays pass out of any Medium into another of different Density, they will also be parallel after Refraction: Because they are all equally inflected; for in all this Chapter, we speak of Mediums separated by a Plain Surface. 653

Let X and Z be two Mediums, the last more rare, and the other more dense, separated by the Plane ES , (Plate V. Fig. 1.) from the Point R C , let there proceed the divergent Rays P C , R o , R n , and enter into the denser Medium: Let one of them be R C , perpendicular to the Surface ES ; this last is not turn'd out of the Way*, but continues its Motion along CG . The Rays R o , R n , are refracted towards Perpendiculars, which are conceiv'd to fall upon the Surface ES in the Points o and n *, These Rays are mov'd in the denser Medium, as if they all proceeded from the imaginary Focus r , which is farther distant from the Surface than R , if the Rays are not too much scattered; which yet is not to be understood Mathematically; for, by a Point, we understand a small Space, such, as is otherwise call'd a Physical Point. • 629 • 624

To demonstrate this Proposition, we must consider, that the Angle R o C , is the Complement of the Angle of Incidence to a right Angle; and that the Angle r o C is also the Complement of the Angle of Refraction to a right Angle; and therefore that the Lines R o , r o , are the Secants of the Complements of the Angles of Incidence and Refraction, supposing the Semidiameter to be o C ; between which Secants, there is a constant Proportion*. In the small Divergence R o and R C , as also r o and r C , they do not sensibly differ, and between R C and r C , the Ratio is always constant; that is, r is fixed as well as R , tho' the Inclination of the Ray • 641 • 639

Ray be chang'd : And therefore Rn is refracted along nA , as if it had proceeded from r .

655 *If the Rays are too much dispers'd, this Demonstration will not serve, and the Place of Concurrence r cannot be taken for a Point: In this Case a little Circle must be imagin'd there, into which all the Rays concur, which will be the greater, the greater the Angle is that the divergent Rays make.*

656 *If some Rays, proceeding from R , are not too much dispersed, but fall very obliquely on the Surface ES , they will be refracted, as if they proceeded from a Point not very remote from the Point r : As is plain from what has been said.*

657 *The Rays, such as An , Bo , GC , which come converging from a denser Medium X into a rarer Z , concur sooner than they wou'd do, if they shou'd*

* 625 *continue their Motion in a denser Medium*, that is, become more convergent, and the real Focus is less distant than the imaginary one. In*

this Figure, the imaginary Focus is r , and the

* 625 *real Focus R^* . This Proposition therefore is properly the Inverse of the Proposition of N° 654.*

* 626 *and therefore * both are prov'd by the same Experiment.*

Experiment 1. Plate V. Fig. 2.] Thro' the Ball G , which is moveable in the Window-shut, and has an Hole going thro' the Middle of it, let a cylindric Beam of the Sun come into the dark Room, and be reflected horizontally by the Looking-Glass S ; let it then go through the Convex Lens of Glass that is fix'd in the Board or Stand T , and the Rays will meet together at R , and beyond R will move as if they proceeded from that Point; which therefore is the radiant Point.

Convex

Convex Lenses of Glafs are very common ; we shall hereafter mention their Properties*, it being needless to do it in this Place: Now we only want a radiant Point, and it is enough to shew how to make it. * 691
697

Take a Trough Box P, whose Side *a b c d* is of Glafs, and let it be fill'd with Water: The Rays, which diverge from the Point R, become less diverging when they go into the Water.

When convergent Rays as H D, I p, L q (Plate V. Fig 1.) having their imaginary Focus at f, go from the rarer Medium Z into a denser X, they become less convergent, and concur in the Focus F, which is more distant from the Surface E S *, as appears by applying here the Demonstration given in N^o. 654.* 658
* 624
* 652

*Rays proceeding from the Point F, and going out of a denser Medium into a rarer, become more diverging, and move as if they came from f, which Proposition is the Inverse of the foregoing, and is confirm'd by the same Experiment *.* 659
* 626

*Experiment 2. Plate V. Fig. 3.] Take the same Box B, as was used in the former Experiment; but here let in two Beams of the Sun into the dark Room thro' two Holes in the moveable Plate in the Window-shut; let them be both reflected horizontally, and transmitted thro' similar convex Lenses; thereby the Rays of the Beams will become convergent, having their Focus's at the same Distance; but if the converging Rays are made to run into the Water in the Box thro' the Side *a b c d*, they will be collected at a greater Distance; which will plainly appear by comparing together the Situation of the Points F in the Air, and f in the Water.*

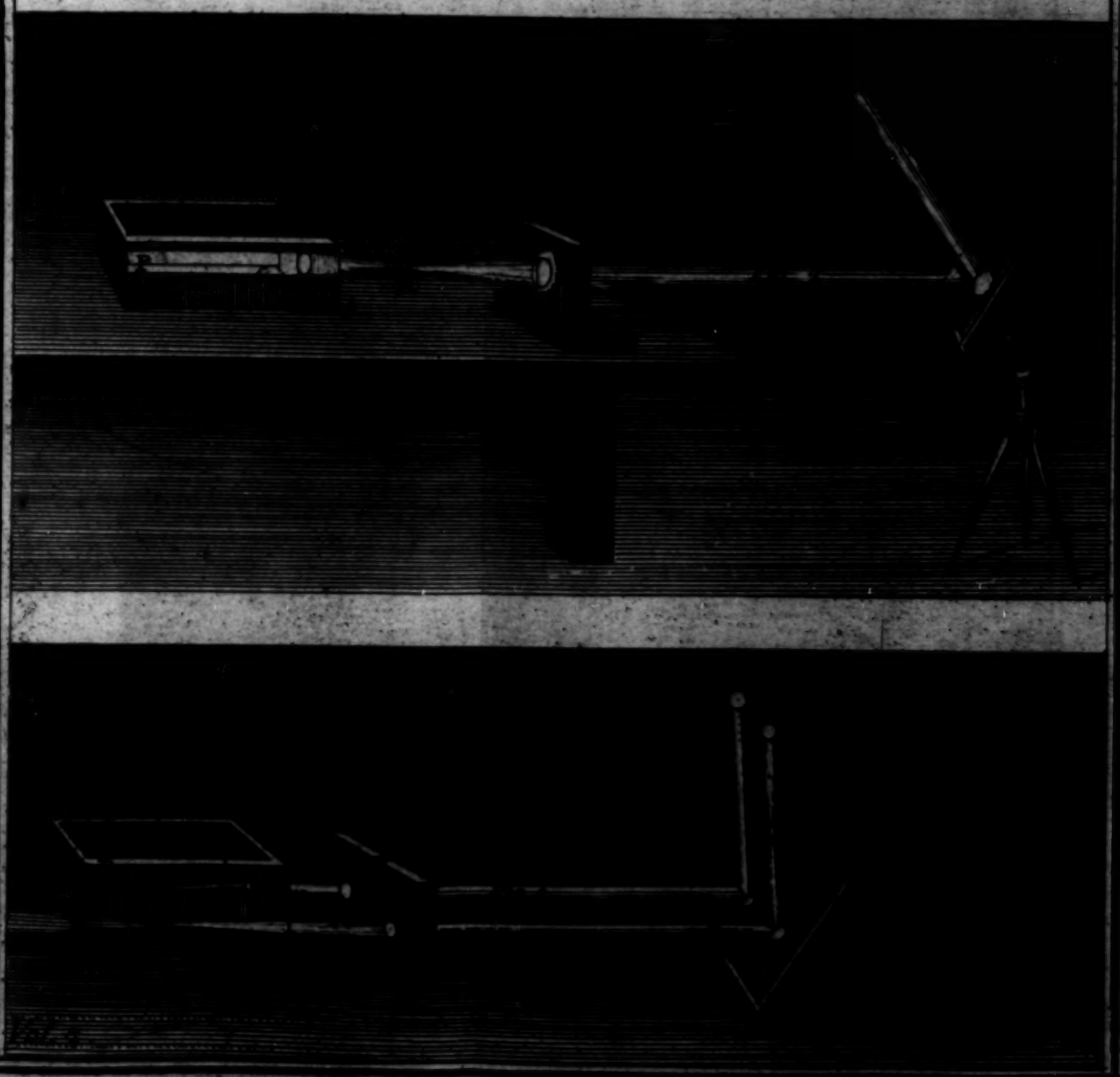
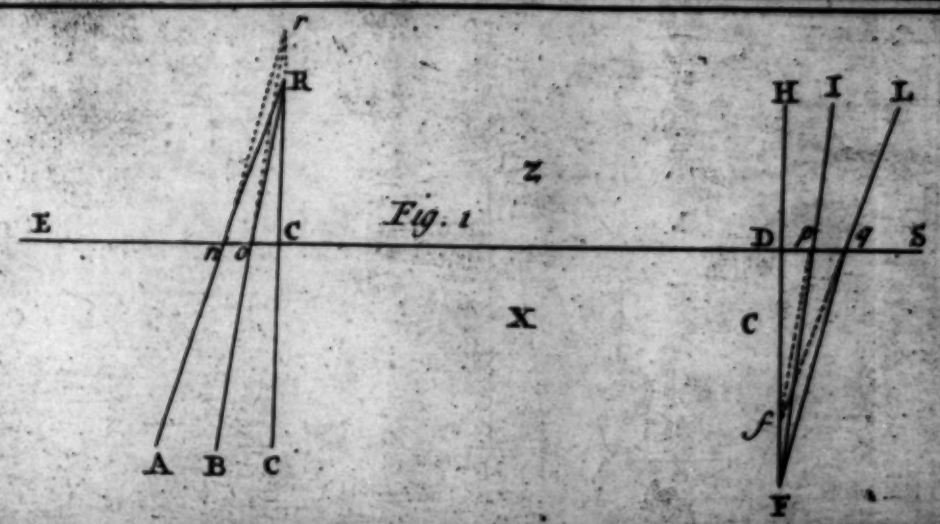
C H A P. VIII.

Concerning the Refraction of Light when a spherical Surface separates the Mediums.

Plate VI. Fig. 1.] **L** E T X and Z be Mediums differing in Density, the last the rarer, and the first the denser; let them be separated by the spherical Surface E S, whose Center is C, and whose Convexity is towards the rarer Medium.

- 660 To begin, by examining the most simple Case; Let us suppose parallel Rays, as B O and A n, going out of a rarer into a denser Medium, and falling upon a convex Surface, such as we have just describ'd; let one of them be B O, which, being continu'd, goes thro' the Center, and falls perpendicularly upon the Surface E S; and therefore is not turn'd out of the right Line*. All the Rays, which are not too distant from that Ray, come nearer to it by the Refraction of the denser Medium, and are collected into one Point F: As for Example; Let A n be a Ray which is refracted along n F; thro' the Point n draw to the Center C the Semidiameter C n, and let it be continu'd to p; as this Line is perpendicular to the Surface which separates the Mediums, the Angle of Incidence is A n p, which is equal to the Angle n C O; the Angle of Refraction is C n F. If the Arc n O be a very small one, these Angles are as their Sines, whose Ratio is constant*. Therefore these Angles n C O and
- 639 C n F are increas'd and diminish'd in the same Ratio, as long as their Difference, which the Angle n F O, which consequently follows the Proportion of the Arc n O, which is the Measure of the Angle n C O; as long as the Arc n O does

Plate 5





does not exceed 15 Degrees, the Angle $n F O$ is increased and diminished sensibly in the same Ratio, as the said Arc; and therefore all the Rays, between $A n$ and $B O$, do by their Refraction meet sensibly in the Point F .

Experiment 1. Plate VI. Fig. 2.] Let a cylindric Beam of the Sun, of an Inch in diameter, made up of innumerable parallel Rays (which are so on account of the Sun's immense distance) be let into the dark Room, as in the first Experiment of the foregoing Chapter, and be reflected horizontally by the Looking-glass S .

Fill with Water the Trough P , which is about 3 Inches high, and as wide, and one Foot long; let it have a Glass V made fast in one of its Sides; this Glass must be a Portion of a Sphere, thin, and every where of the same thickness, such as the Crystal of a Pocket Watch.

The convex Part of the Glass V must be outwards, that the Water next to it in the Trough may put on a spherical Surface; if the Rays above-mention'd go into the Trough thro' this Glass, because the Glass is thin, and has its Surfaces parallel, there is no sensible Change in the Motion of the Light by the Refraction of the Glass, and the Light enters into the Water in the same manner as if there was no Glass; let the Trough be so dispos'd, that one of the Rays may pass thro' the Center of the spherical Surface, and the others will come nearer and nearer to it, and at last concur with it at F .

Plate VI. Fig. 3.] Again, let X be the denser Medium, and Z the rarer, and let them be separated by the spherical Surface $E S$, whose Center is at C , and whose Convexity is towards the rarer Medium: From the radiant Point R let Rays proceed,

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ceed, and enter into the denser Medium thro' the Surface above-mention'd; so that of those Rays, that which is express'd by RO , being continu'd, may pass thro' the Center; this Ray is not refracted as it goes into the Water, and all the rest of the Rays come towards it by the Refraction, and when they are not too divergent are collected into one Point as F , in the same manner as was said of parallel Rays, with this difference, that the Focus F in that Case is more distant. The same Demonstration will also serve here as relates to parallel Rays, which is built upon this Foundation, that the Angle of the Incidence increases in the same Ratio as the Arc πO , which does also obtain here, when the said Arc does not exceed 15 Degrees. Let $R\pi$ be a Ray of Light, and from the Center C thro' π draw $C\pi p$; the Angle $R\pi p$ will be the Angle of Incidence; let it be divided into two Parts by the Line πq , parallel to the Line ROC ; the Part $p\pi q$ is equal to the Angle πCO , which is measur'd by the Arc πO , and which therefore follows the same Proportion as that Arc; and which also the Angle πRO (if it be very small) does follow, and is equal to the second Part of the Angle of Incidence, which also does wholly increase and diminish in the same Ratio as the Arc πO ; for the Ratio which holds in respect of every Part, taken singly, will also hold in respect of the whole.

662 The same Demonstration may be applied to any diverging or converging Rays, which in any Case are refracted in passing thro' a spherical Surface, and which (as appears by this Demonstration) when they diverge but little, have their Focus real, or imaginary, or run parallel. It is enough to have observ'd this in general.

The Focus F of the Rays that come from R, goes farther off when R is brought nearer; and so on the contrary. For the radiant Point being brought nearer, if the Point n remains the same, the Angle of Incidence is increased; and as it increases, so does also the Angle of Refraction $F n C$, and $n F$ intersects $R C$ at a greater distance.

Experiment 2. Plate VI. Fig. 4.] This Experiment differs from the foregoing only in this, that a cylindric Beam of the Sun reflected horizontally, must be transmitted thro' the convex Lens in the Board T, as was done in the Experiments of the foregoing Chapters, to form the radiant Point R, from which the Rays, going forward, diverging, are collected in the Water at a greater distance than if they had been parallel.

As you move the Board T, the Point R also changes its place; if this Point is farther off from the Surface that separates the Mediums, F falls nearer to it; on the contrary, if R be nearer, F is farther off.

The radiant Point may be brought so near to the Surface above-mention'd, that the Focus will recede to an infinite distance; that is, that the refracted Rays will run parallel.

Experiment 3. Plate VI. Fig. 5.] Things being dispos'd as in the former Experiment, by removing the Board T, let R be brought nearer to the Trough, and it may easily be so dispos'd as to make the refracted Rays become parallel.

Experiment 4. Plate VII. Fig. 1.] Now if the Experiment be repeated, bringing nearer to the Trough the radiant Point R, the refracted Rays will become divergent, but they will diverge less than
the

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- 666 the incident Rays. If the Rays, which out of a rarer enter into a denser Medium thro' a convex Surface, be converging and directed towards the Center of that spherical Surface, they will suffer no Refraction^{*}; but if they be directed towards another Point, since they are refracted towards the Perpendicular^{*}, they will be so infected, that the Focus of these converging Rays will always be between the Center of the Surface which separates the Mediums (to which all the Perpendiculars are directed) and the Point to which the incident Rays tend; that is, if the imaginary Focus of the incident Rays be at a less distance than the Center, the refracted Rays will be less converging; but if this imaginary Focus be beyond the Center, the refracted Rays will be more converging.

Experiment 5. Plate VII. Fig. 2.] Every thing being in the same manner as in the former Experiments, it is easy to confirm these Propositions by Experiments; for the Board T may be so dispos'd, that the convergent Rays shall enter the Water, so as to have their imaginary Focus at any distance in it.

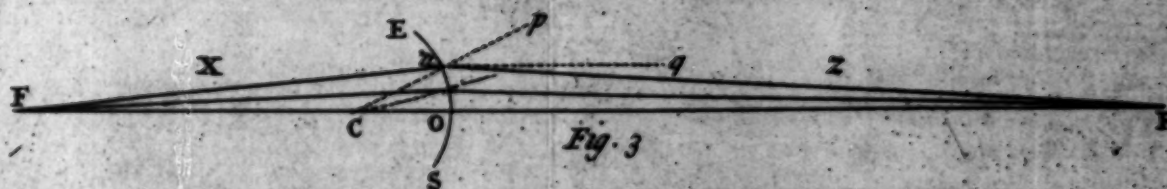
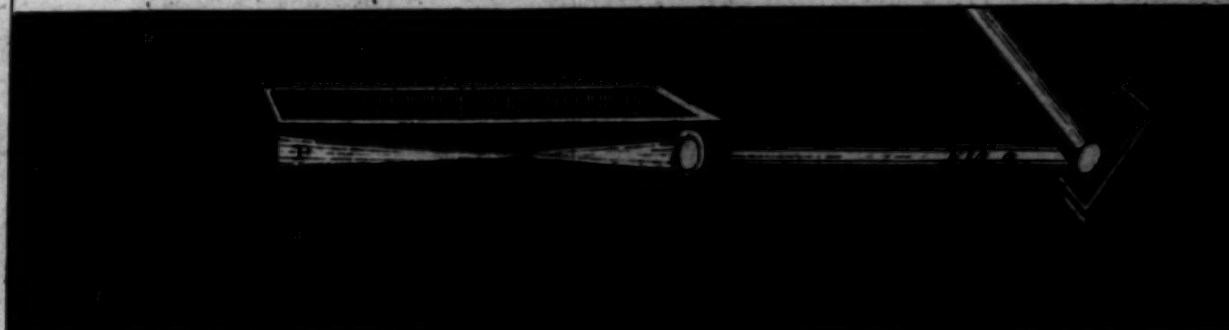
- 668 From what has been said hitherto it is easy to determine what happens in a contrary Motion of the Rays^{*}; that is, the Motion of the Rays from a denser into a rarer Medium, the convex Surface remaining towards the rarer Medium.

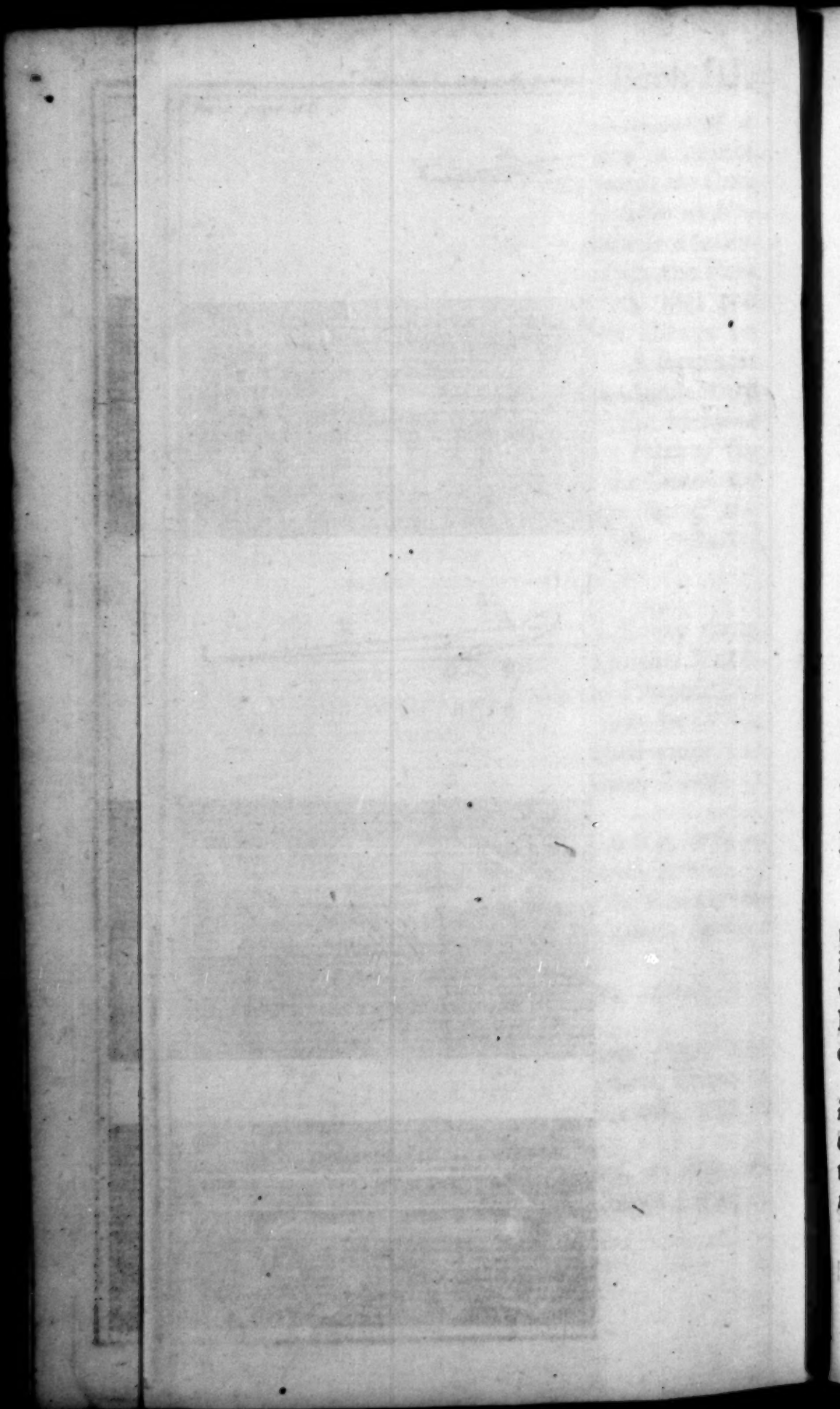
Parallel Rays; after Refraction, concur in a Focus^{*};

- 669 The Rays that come from a radiant Point meet also in a Point or Focus^{*}, and as that Point is brought near, the Focus goes farther off; and so the reverse on the contrary.
- 670 The radiant Point may be so placed, as that the Focus shall fly out to an infinite distance; that is, the Rays will become parallel^{*}.

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Plate 6.





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If the radiant Point be brought nearer, the re- 671
fracted Rays will diverge; they will diverge less
than the incident Rays, if the radiant Point be
more distant from the Surface than the Center*. * 667

But if the radiant Point be between the Surface 672
and the Center, the refracted Rays will be more
divergent*. * 667

If the Rays are convergent, they become more con- 673
vergent in every Case, which follows from the Re-
fraction being made fromwards the Perpendicu-
lar*, and which may be also deduc'd from N°. * 625
665.

Plate VII. Fig. 3.] Let us again suppose the Rays 674
to go out of the rarer Medium Z into the denser X,
and that the Hollow or Concave of the spherical
Surface E S, which separates the Mediums, is to-
wards the rarer Medium. If the Rays be parallel,
as B O, A n, B O, which goes thro' the Center
C of the Surface E S, will not be refracted; but
A n is refracted towards the Perpendicular C p
along n G*, and, being continued towards Z, * 624
intersects B C O at f, which is also true in respect
of the Rays between B O and A n*; those Rays * 662
therefore become divergent, having their imaginary
Focus f in the rarer Medium.

Experiment 6. Plate VII. Fig. 4.] This Experi-
ment differs from the first of this Chapter, only
in this, that the Glas V has its Concavity to-
wards the Air, the Trough P being in all other
Respects the same; in this Case the Rays of the
cylindric Beam diverge in the Water.

If Rays come from a radiant Point in C B, which 675
is beyond C (Plate VII. Fig. 3.) the Angle of In-
cidence A n C is diminish'd, and therefore also
the Angle of Refraction G n p grows less; that is,
the refracted Rays become more diverging, and the
V O L. II. E imagi-

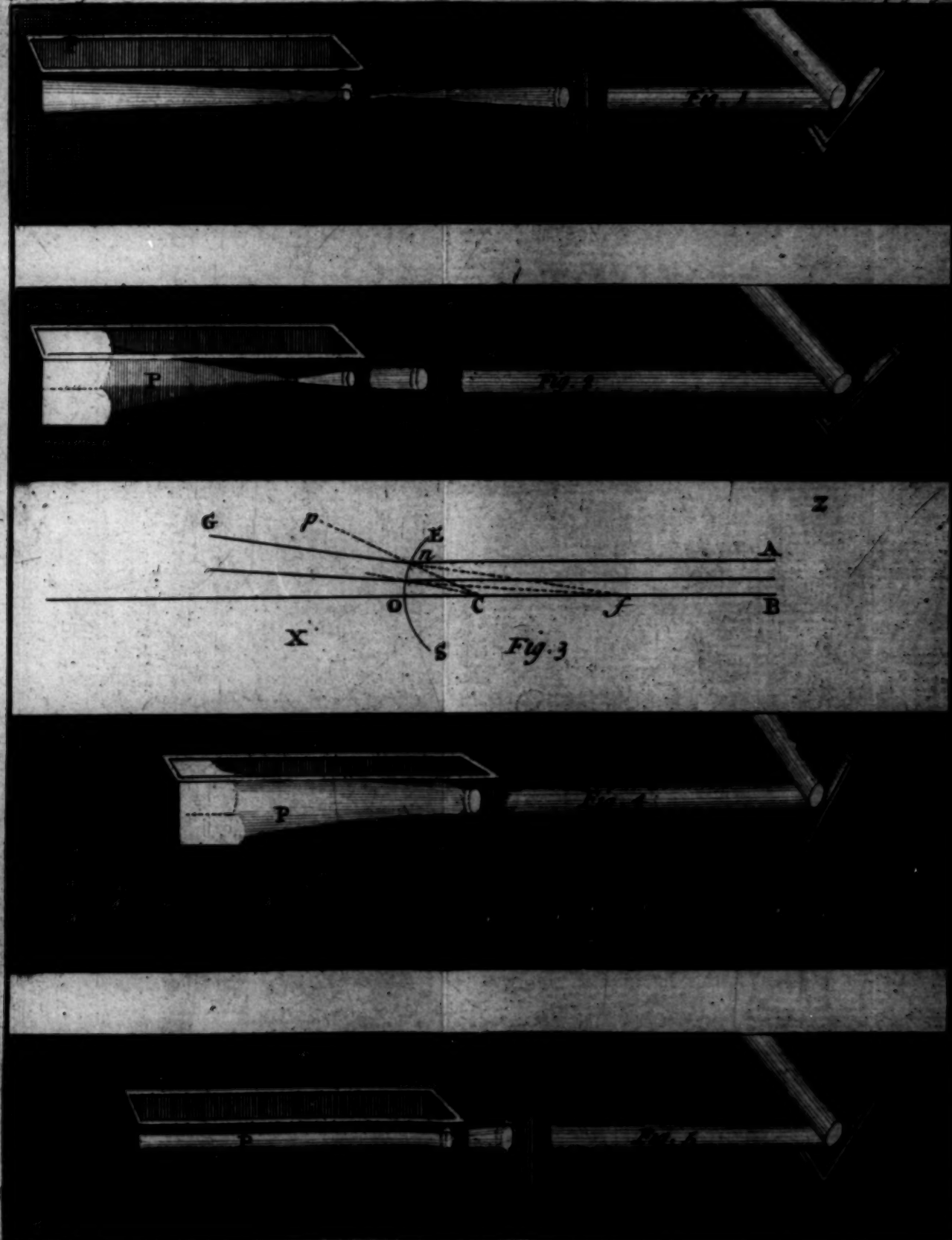
- 646 imaginary Focus f comes nearer to C^* ; till by the radiant Point coming nearer, at length it coincides with the imaginary Focus at C ; for in that Case the Rays undergo no Refraction*.
- 629 676 If the radiant Point comes nearer between C and O , the imaginary Focus is farther from O than the radiant Point, for it is always between that Point and C , by reason of the Angles of Refraction being less than those of Incidence*.

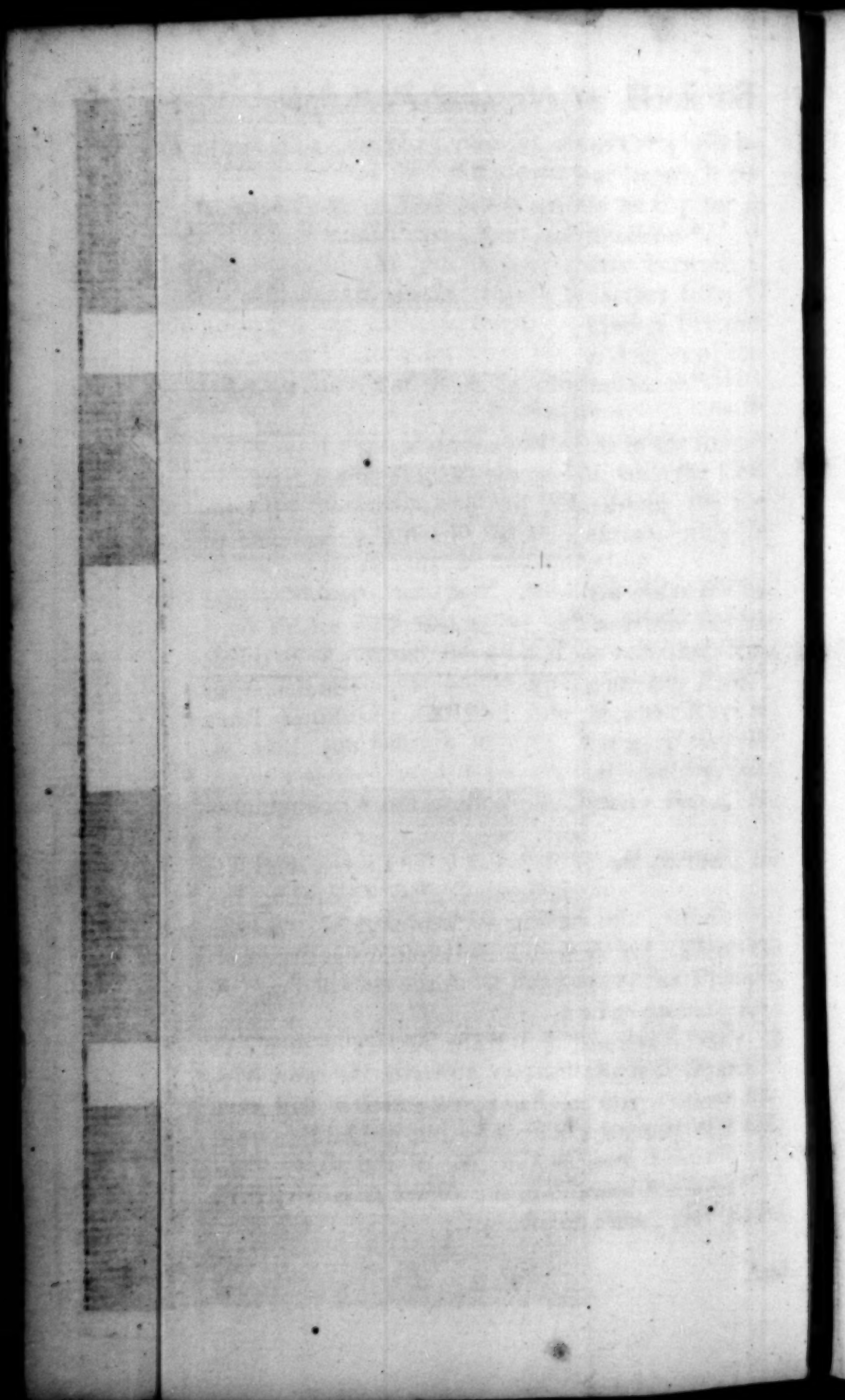
Experiment 7.] Things being, as in the former Experiment; if you use the Board, with the Convex Lens in it, to form a radiant Point, the Experiments proving these Propositions may be easily made.

- 677 *If the Rays are converging, and the Point of Concurrence be in the denser Mediums, near the Surface which separates the Medium, the refracted Rays will also converge, but less than the incident Rays.*
- 678 *If the imaginary Focus of the incident Rays recedes more and more from O , that is, if they converge less, the refracted Rays will also converge less; until, by the receding of the imaginary Focus, the Rays become parallel.*
- 679 *If the imaginary Focus recedes yet farther, the refracted Rays become divergent.*

Experiment 8. Plate VII. Fig. 5.] Here the Board must be so plac'd in respect of the Trough, that the Rays may enter the Water converging; and the Phenomena above-mention'd may be seen, according as you remove the said Board.

- 680 *Rays which from a denser go into a rarer Medium, the concave Surface being towards this last, are almost subject to the same Laws.*
- 678 *Parallel Rays by Refraction become divergent*.*
- 626 *If they proceed from a radiant Point, they become more divergent*.*
- 677





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And they diverge still more and more, as the radiant Point is brought nearer *. 682 • 678

Converging Rays, which tend to the Center of the Spherical Surface, undergo no Change *. 683 • 629

If they converge more or less, the imaginary Focus of the incident Rays is always between the Center of the Surface, which separates the Mediums, and the Focus of the refracted Rays, * which may recede in infinitum, so as to make the refracted Rays become parallel *. 684 • 675 • 676 • 674

We have hitherto consider'd such Rays as are but little inclin'd to the Surface which separates the Mediums; for we have mention'd such incident Rays as diverge but little, and one of which is perpendicular to the Surface that separates the Mediums. The same Propositions hold good in oblique Rays, yet in that Case all the Rays are inflected, which does not happen so in direct ones; for the Ray which is perpendicular to the Surface is not inflected. Oblique Rays also undergo a greater Refraction, that is, they are more inflected either to or from each other, than direct ones, supposing the Circumstances the same. 686 • 687

Plate VIII. Fig. 1.] Let Z be a rarer, and X a denser Medium, E S the Surface separating the Mediums, and having its Center at C; and the parallel Rays A n, B m will come together at F. The Rays that proceed from the radiant Point R will concur at F.

Plate VIII. Fig. 2.] If the Surface be turn'd so as to have its Concavity towards the rarer Medium, the parallel Rays A n and B m will have their imaginary Focus at f; but its Distance from the Surface E S, as also that of the above-mention'd Foci F and f in the Figure 1. is less, than if the Rays were direct.

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688 *All the Points of a lucid Body are radiant Points, and have each their particular Focus, which serves to explain the following Experiment, made to confirm what has been said of the oblique Rays.*

Experiment 9. Plate VIII. Fig. 3.] Take the Trough P mention'd in the first Experiment, and fill it with Water; let the Glass V be all cover'd, but a circular Part in the Middle of about half an Inch; have in the Trough a moveable white Plane T. If the Candle A be set at the Distance of 3 or 4 Foot from the Trough, let the Plane T be mov'd backward and forward in the Water, and when it is come to the Distance of the Foci of the Flame A, that Flame will appear exactly represented on that Plane, all the Foci forming a Picture. And this holds good, whether the Rays from the Candle fall obliquely or directly upon the Glass V, only that when the Rays are oblique (the Distance of the Candle A remaining the same) the Distance of the Plane T from V must be less. In this Case also the Candle and the Glass V will not be in the same right Line as the Representation of it, as it happens when the Rays are direct, by which the Proposition of N^o. 686. is confirm'd.

689 *The Candle is represented inverted, because the Rays, which proceed from different Points, intersect one another as they go thro' V, as may be plainly seen by looking at the first Figure. For which Reason, if there are two Candles, as A and B, the Representation of the last will be at b, and that of the first at a.*

690 *All the Changes that happen in Light, which we have taken Notice of in this Chapter, are so much the more sensible, as the Surface separating the Mediums is more curve; that is, a Part of a lesser Sphere.*

C H A P. IX.

Concerning the Motion of Light thro' a more dense Medium. Where we shall take notice of the Properties of Lenses.

THE Use of Glasses is common, they are more dense than Air, and the Rays out of Air pass thro' the Glass into Air again. According to the several Surfaces that terminate the Glass, Light undergoes different Changes as it moves in it; which to determine, the Glasses, or any Mediums encompass'd with a rarer Medium, and terminated with different Surfaces, must be examin'd. *If we consider only plane and spherical Surfaces, there are six Sorts.*

The Medium may be plane or flat on both Sides. 2. Plane on the one Side and convex on the other. 3. Convex on both Sides. 4. Plane on one Side and concave on the other. 5. Concave on both Sides. 6. *Lastly*, It may be terminated with a concave Surface on one Side, and a convex one on the other.

DEFINITION I.

If the Glass be made use of, and is not very thick, Glasses, whose Figure is mention'd in the last five Cases, are call'd *Lenses of Glass*.

In the second and third Case, a Lens is said to be convex; and if we distinguish those two Cases, in the second it is call'd *plano-convex*. And so in the fourth Case, it is said to be *plano-concave*; tho' both this Case, and the fifth, is generally referr'd to concave Lenses. But a *concarvo-convex* Lens is referr'd to the concave or convex Lenses, according as the one or the other Surface is predominant; and that is said

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to be predominant which is most curve; that is, which is a Portion of a less Sphere.

DEFINITION II.

693 *In every Lens, or Medium, terminated in any Manner above described, a right Line, which is perpendicular to the two Surfaces, is call'd the Axis. When both Surfaces are spherical, the Axes goes thro' both their Centers; but if one of them be plane, it falls perpendicularly upon that, and goes thro' the Center of the other.*

694 *In the Passage of Light thro' a Medium, terminated by two plane Surfaces, the Direction of*
 • 628 *the Rays is not changed**; which is the Case in *plane Glasses.*

695 *It is the Property of all Sorts of convex Lenses, that the Rays in their Passage thro' them are inflected towards one another; so much the more, as*

696 *the Convexity is greater: And so concave ones, that the Rays are deflected from one another, according as the Concavity is greater. For the Direction of*

• 694 *the Rays thro' a plane Glass is not changed;** but, by inflecting one or both Surfaces, another Direction is given to the Rays: There are more inflected towards the Axis of the Lens, by reason of the Convexity of the Surface of the Glass, and, by making the Surface concave, they are deflected from the Axis; as is plain in every Case, by comparing the Inflection in the plane Surface that is perpendicular to the Axis, with the Inflection in the spherical Surface at any Distance from the Axis. And the Difference of their Inflections, that is, the Change of the Direction of the Rays, increases, as their Distance from the Axes does; and it is to be observ'd in every Direction of the Rays, as well in oblique Rays, as in direct; but *the Changes are greater in oblique Rays, because the Angles of Incidence are greater; from which*
 697 *we deduce the following Properties of Lenses.*

That

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That *parallel Rays*, by passing thro' a *convex Lens*, concur in a *Focus*. 698

That *diverging Rays* either *diverge less*, or run *parallel*, or lastly, *converge*; in which Case, the *radiant Point* receding, the *Focus* comes nearer, and so on the contrary: But this is the Case when the *radiant Point* is farther distant from the *Lens* than the *Focus* of *parallel Rays*. 699

Lastly, that *converging Rays* converge more when the *Light* emerges out of the *Lens*. 700

The same things are observable in *oblique Rays*, concerning which it is to be noted, that the *Distances* of the *Foci* of the emerging *Rays* are less than in the direct, and the other Changes more sensible *. 701 * 697

All these same things may be deduc'd from examining the double Refraction in the Entrance and Emerision of Light; and this double Refraction is visible in every Case, by the following Experiments; by which the aforesaid Properties of *convex Lenses* are confirm'd.

Plate VIII. Fig. 4.] Take several Boxes like P, with Water in them, and thro' which Light is transmitted thro' the Glasses V and V, which are placed in the opposite sides of the Box, and are distant from one another about one Inch; these Glasses are thin. In the Box, which represents a *convex Lens*, on each side there is placed one, like that of the first Experiment of the former Chapter, which are so disposed as to have their *Convexities* without the Box; when a *plano-convex Lens* is to be represented, on one side there is a *plane Glas*s; a *concavo-convex Lens* is represented by two spherical Glasses, that are Portions of different Spheres, and the *Convexity* of the Portion of the greater Sphere must be turn'd towards the inside of the Box. 883

Whilst the *Light* passes thro' these Boxes, the Changes of the *Light* are visible to the Eye in

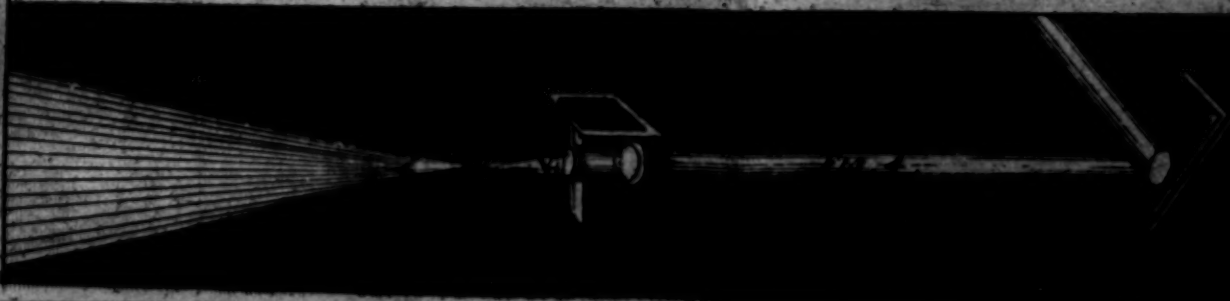
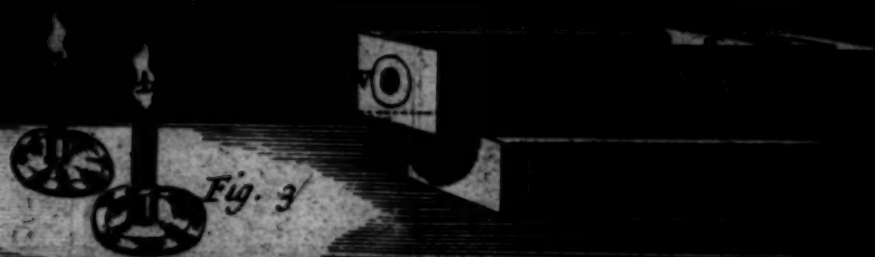
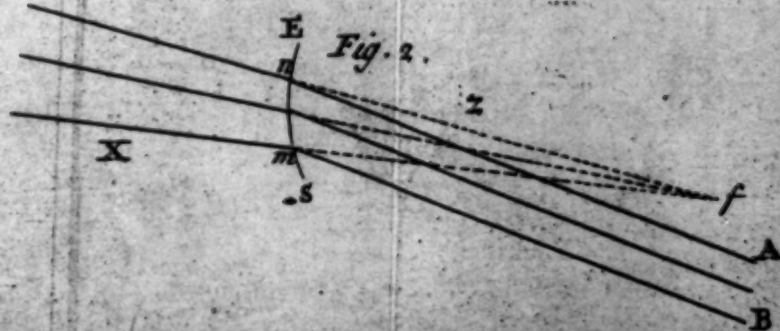
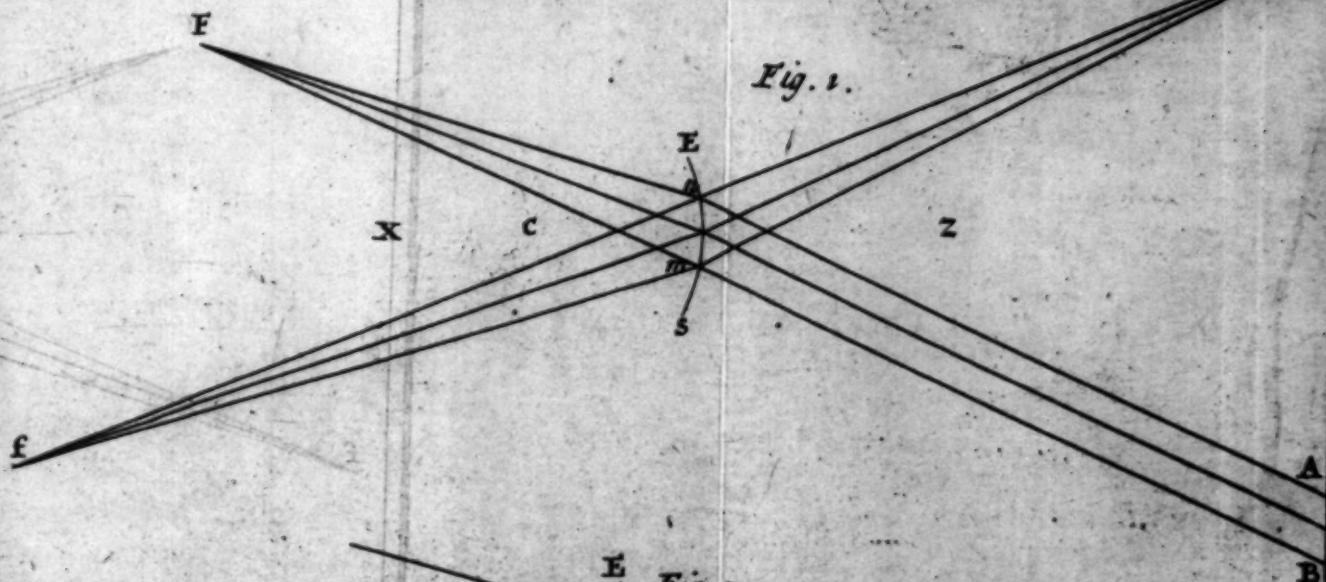
its Entrance into, and Emerfion out of a denser Medium, and by the Affiftance of thefe, all things relating to convex Lenses are clearly demonftrable.

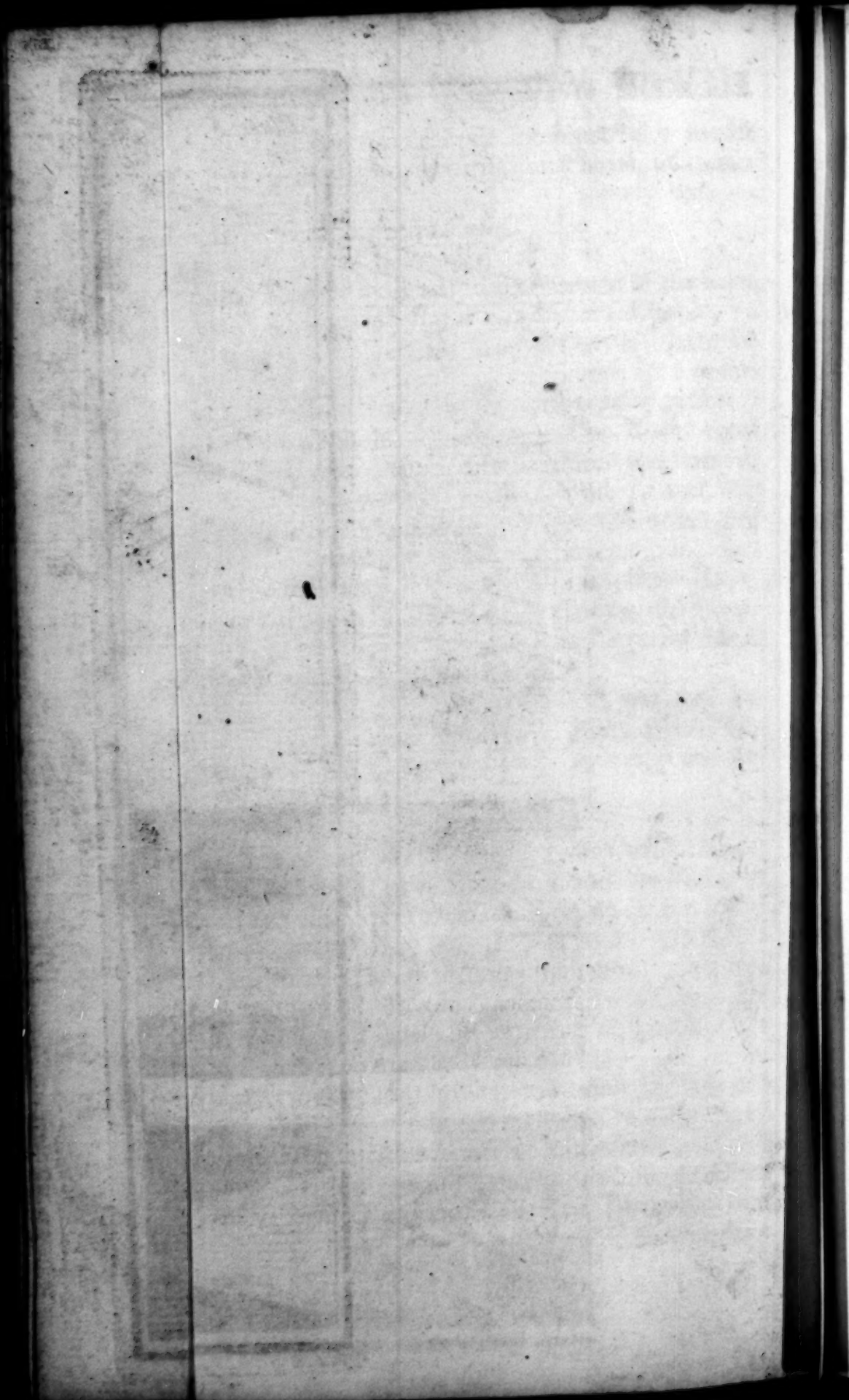
Experiment 1.] Ex gr. Let P be one of the foremention'd Boxes, with the fpherical Glaffes V, V, the Convexities being placed outwards; let it be fill'd with Water; in a dark Room let a cylindric Beam of the Sun be horizontally reflected from the Looking-glafs S; let this Beam enter the Box; the parallel Rays, of which it is form'd, will be inflected towards one another, and will converge; at their Emerfion on the other fide they will converge more, and concur in F. Experiments may be made of the incident Rays, which diverge or converge, by uſing the Board with the convex Lens, as in the Experiments of the former Chapter.

Every Point of a lucid Body, as was ſaid before, is a radiant Point, * and being placed at a due diſtance from a convex Lens, every one has
 • 688 its Focus. *

Experiment 2.] Let a lighted Candle be remov'd from a convex Lens beyond the Focus of parallel Rays; at the oppoſite Part of the Lens, upon a white Plane, by the Foci of the Points of the Flame, it will be represented; and this Representation will be inverted, by reaſon of the Interſection of the Rays in their Paſſage thro' the Glaſs.

703 *Convex Lenses are alſo burning Glaffes,* becauſe
 • 698 they collect the Rays of * the Sun, which, upon the account of the immense Diſtance of the Sun, are eſteem'd as parallel; but Rays united in a Focus (by reaſon that the Fire, that was before diſperſed, is now collected, and by reaſon
 ſon





son of the Motion of the Fire according to various Directions) do burn vehemently.

Experiment 3.] Take a convex Lens of any Magnitude, let it be so exposed to the Rays of the Sun, that the Axis of the Lens may be in the Direction of the Rays; if any combustible Body be placed in the Focus of the Sun's Rays, it will burn.

When, by reason of the Magnitude of the Lens, the Rays are not exactly enough collected, before they come to the Focus, they must be transmitted thro' another convex Lens that is less, by which they will be reduc'd into a smaller Compass, so as to burn more violently.

As for concave Lenses, and their Properties, they may be easily deduc'd from * what has been * 696 said.

Parallel Rays become diverging, by passing thro' a 704 concave Lens.

Diverging Rays diverge the more. 705

Converging Rays either converge the less, or become 706 parallel, or (as it happens in such as converge less) go out from the Lens diverging.

All which things happen to oblique as well as 707 direct Rays, but more sensibly in the first. * 697

Plate IX. Fig. 1.] Boxes, to represent the Ef- 708 fects of concave Lenses, are made in the same manner as those that represent the Effects of convex ones, * the difference is only in the Position of * 702 the Glasses; in the first Box the Concavities of the two spherical Glasses V, V, are outwards; in the second, instead of one of those Glasses, you have a plane Glas; in the third you have two spherical Glasses, but Sections of different Spheres; the Section of the greater Sphere has its Convexity outwards, and the other its Convexity inwards.

Expe-

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Experiment 4.] Parallel Rays of the Sun, reflected horizontally in a dark Chamber by the Looking-glass S, must be transmitted thro' the Box P, which is full of Water, and represents a Lens concave on both sides; as they go into the Box they will begin to diverge, and as they go out they will diverge more.

The remaining Experiments, relating to concave Lenses, are made in the same manner as has been said in respect of convex Lenses.

C H A P. X.

Of Vision; where we shall speak of the Make of the Eye.

THE Properties and Laws of the Refraction of Light, that we have explain'd, are of wonderful Use in representing Objects to our Mind.

By these Laws, the Objects are beautifully painted in their proper Colours in the bottom of the Eye; and this Picture, as I shall say hereafter, * is the occasion of the Ideas which are excited in our Minds concerning the Things that we see.

How this Picture is form'd in the Eye, cannot be explain'd, without examining a new Property of Light; namely, its Divisibility, which is past our Comprehension.

DEFINITION.

709 *A Body that is not lucid, and intercepts the Light, is said to be opaque.*

710 *Several among all the opaque Bodies, when exactly polish'd (except perfectly black Bodies, if there*

there are any such) have the Property of dividing Light, for they *reflect the Light, so that the Rays from every Point* being struck back, are divided, and *recede every way, so that all the single Points of the Body become as it were radiant Points*, from which Light goes every way.

Whence we deduce a Method of painting Objects upon a white Plane; for all the Points of the enlighten'd Body, from which the Rays come upon a convex Lens, have their Focus of the other side of the Lens.* The Foci of distant Objects, tho' not exactly, are sensibly at the same distance from the Lens; those Objects may by these Foci be represented in the same Place; which Representation is inverted, (by reason of the Interfection of the Rays as they go thro' the Glass) and sensible in a dark Place, in which Light comes in no way but thro' the Lens, and only that Light by which the Objects are represented. 711 * 699

This will do wherever the Lens is placed, and in respect of all the Points of Objects, enlighten'd by Rays of Light, from which right Lines without Interruption may be drawn to the Lens; in this manner the above-mention'd Divisibility in Light may be proved, and the Aptness that Bodies, that reflect Light, have to divide it.

Experiment 1. Plate IX. Fig. 2.] Make an Hole in a dark Place, over-against several Objects that are at least 50 Foot off or farther; let the Hole be V, and let it have a convex Lens in it that collects parallel Rays at the distance of about four or five Feet; if a white Plane be placed behind the Lens a little farther from it than that distance, all the Objects above-mention'd will be painted upon it in very beautiful Colours. It is to be observ'd, that the Lens must be placed in a Position

tion parallel to the Plane; and that by moving the Lens or the Plane, the distance must be found at which the Objects are represented most exactly.

This Representation of Objects has great Affinity with that by which the Objects that we see are represented in the bottom of the Eye, as will appear from the Make of the Eye.

712 The Figure of the Eye, when taken out of the Head, is nearly spherical; only the fore-part is something more convex than the rest.

The Section of the Eye is represented *Plate IX.*

Fig. 3.

The Part A A, which is most convex, is transparent, and call'd the *Tunica Cornea*.

The whole Covering of the Eye, except the *Cornea*, is call'd the *Sclerotica*, B A A B.

That Part of the *Sclerotica* which is next to the *Cornea*, is call'd the *Adnata*, or *White of the Eye*.

Behind the *Cornea*, on the inside, is a Coat call'd the *Uvea*, which has in its middle an Hole *pp*, call'd the *Pupil*.

The *Uvea* is made up of concentric circular Fibres, intersected at right Angles by strait Fibres; if the first are swell'd, the last are relax'd, and the Pupil is lessen'd or contracted; and a contrary Motion of the Fibres increases or widens it.

In the middle of the Eye, but nearer the fore-part, there is a transparent soft Body C C, like a convex Lens, whose hind-part is more convex than the fore-part; it is call'd the *Crystalline Humour*; its Axis coincides with the Axis of the Eye, that goes thro' the Centers of the Pupil and the whole Eye.

This crystalline Humour is sustain'd by small Fibres or Threads, which are fix'd to all the Points of its Circumference, and likewise to the
inside

inside of the Eye; they are inflected in the Form of an Arc, and every one of them is a Muscle; they are call'd the *Ligamenta Ciliaria*, and two of them are represented by *l C, l C*; they all cohere to one another, and together with the Crystalline, make a Separation in the Eye, and divide it into two Cavities, one forwards *p p*, and the other backwards *S S*.

The Cavity that is forwards is fill'd with a Liquor like Water, call'd the *aqueous Humour*.

The hind Cavity is fill'd with a transparent Humour, nearly of the same Density as the aqueous Humour, but not so fluid, call'd the *vitreous Humour*.

The hind Surface of the Eye within is lin'd with a Coat call'd the *Choroides*, which is again cover'd with a thin Membrane call'd the *Retina*.

At the back Part of the Bulb of the Eye, a little on one side, is the optic Nerve *N N*, so join'd to the Eye, that the Eye itself is as it were an Expansion of the optic Nerve, for the expanded Coats of the Nerve form the *Choroides* and *Sclerotica*, and the Fibres, which make up the *Retina*, concurring, make the Marrow of the Nerve.

The Eye is mov'd in the Head by several Muscles inserted in the *Sclerotica*, but we shall not treat of them here; as we only consider the Eye with respect to the Motion of Light, we purposely forbear to take notice of any thing else.

Rays that proceed from any Point, and enter the Eye thro' the Pupil, go out of a rarer into a denser Medium thro' a spherical Surface; and therefore if that Point be at a due distance from the Eye, the Rays after Refraction will converge; * in the same manner as in the Experiment of N°. 663. (Plate VI. Fig. 4.) in which the Glass *V* represents the transparent Cornea of the Eye, whilst the Water

731

661

in

in the Trough is instead of the aqueous Humour ;
 and therefore, *supposing only the Cornea and the*
 714 *aqueous Humour, there will be in the Eye an inverted*
 • 710 *Picture of the Objects.* *

711

Experiment 2. Plate IX. Fig. 4.] Let P be the
 Trough above-mention'd full of Water, and set
 in a dark Place, which has an Hole about half
 an Inch wide, so disposed towards enlighten'd
 Objects at a certain distance, that one may see
 them through the Hole ; let the Glass V of the
 Trough be applied to this Hole, and upon the
 white Plane T you will have an inverted Picture
 of the Objects ; by moving the Plane backwards
 and forwards, you will find the Place where the
 Picture is most distinct.

If the above-mention'd Picture, which we have
 imitated in this Experiment, was to be made in
 the Eye, it would be at too great a distance from
 the *Cornea*, and beyond the bottom of the Eye ;
 and therefore this distance is diminish'd by help of
 715 *the crystalline Humour*, which is denser, but en-
 compass'd with rarer Mediums ; for the Rays
 converging in the aqueous Humour, pass thro'
 the crystalline into the vitreous Humour ; that
 is, out of a rare Medium thro' a denser (which is
 terminated by two spherical convex Surfaces) in-
 to a rare Medium again ; by which Motion the
 • 700 Rays converge still more ; * and therefore they
 concur sooner, and the Picture above-mention'd
 falls within the Eye.

716 *The Objects which, as we have explain'd, are re-*
presented in the bottom of the Eye, are painted upon
the Retina ; and by the Motion of Light the small Fi-
bres, of which the Retina is made up, are agitated ;
by which Agitation, the Ideas of the Objects painted
in the Eye, are excited in the Mind. The Connexion
between the Ideas and the Motions by which
 they

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they are excited, is unknown to us, * as we said * 502
before: In determining the Causes of Sensations,
we can go no farther than the Agitation of the
Nerves.

*The more exact the Picture above described is, the 717
more distinct will the Objects appear. When the Rays 718
coming from the same Point are not exactly united
upon the Retina, its Picture is not a Point, but a
Spot, which is confounded with the Pictures of
the neighbouring Points; in which Case the Vi-
sion is confused.*

But when, according to the different distance of the 719
radiant Point, its Focus is brought nearer, or re-
mov'd farther off, * it is necessary that there should * 663
be a Change in the Eye, lest the Place, in which the
Picture is exact, should fall short of, or beyond
the Retina, and so the Vision should be confused.

But it is very difficult to determine what this
Change is, and Philosophers are divided in their
Opinions about it; I shall only observe in gene-
ral, that it is not very probable that the Figure of the
whole Eye is changed, in order to put back or bring 720
forward the Retina; and therefore we must expect
to find this Change within the Eye.

For if the Figure of the Eye was chang'd, as
this Change must be equally necessary in all Ani-
mals, the Eyes of all Animals would undergo the
same Changes; for the same natural Effects can-
not have different Causes. Now in the Whale the
Sclerotica is too hard to be subject to any Altera-
tion of Figure. Besides, if there was such a
Change in the whole Eye, it would arise from the
external Pressure of the Muscles, which would be
different in different Positions of the Eye, and
only regular in one Situation of it.

If now we examine the Eye within, it will ap-
pear necessary that there should be a Change in
the Crystalline, which by changing its Place or
Figure

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Figure in the Eye, will produce the desired Effect; for the Rays that fall upon the *Retina*, before they are united, will be made to unite just upon the *Retina*, if the Crystalline becomes more
 • 695 convex, * or if (its Figure remaining the same) it be brought forwards towards the *Cornea*.

721 That the Position of the crystalline Humour is easily changed, and that it is brought nearer to, or farther from the *Retina*, its Axis remaining the same, is plain, because the ciliary Ligaments are muscular; when these Muscles are swell'd, and become shorter, the Hollow which their Inflection makes at *Cl, Cl*, becomes less; by which means the vitreous Humour is compress'd, and therefore it presses upon the Crystalline, and pushes it forwards farther from the *Retina*; which is necessary when we look at near Ob-
 • 663 jects. *

699 From an Experiment, that we shall hereafter
 753 mention, it has been demonstrated, that there is another Change in the Eye that acts contrary to this; and we shall shew what is the occasion of it. The second Change is also to be referr'd to the
 622 Crystalline; which (when it is drawn by the ciliary Ligaments, to make it recede from the bottom of the Eye) becomes also flatter, and therefore it must recede farther than if its Figure was unchangeable; that is, the Change becomes more
 • 730 sensible; which we shall shew to be of Use. *

These Changes in the Eye have their Limits, for which reason also Objects appear only distinct
 724 within certain Limits, which are different Distances
 725 according to the difference of Peoples Eyes; and very often in the same Man, both Eyes have not the same Limits; which is almost of the same Use, as if the Limits of both Eyes were more distinct from
 one

one another : For one may see an Object distinctly enough with only one Eye. In some Persons also the nearest Limit of one Eye is farther off than the farthest of the other : In which Case near Objects and distant Objects are seen distinctly, but the intermediate ones appear confused.

The Picture in the Bottom of the Eye, as has been said, * is inverted ; whence a Question arises, why we see Objects erect ? To which we answer, by asking another Question ; Whether it is more easy to conceive the Connexion between an Idea in the Mind, and an erect Figure, than an inverted one ? We confess, that we have no Notion of that Connexion in either Case : But Experience teaches us, that there is a Connexion between an inverted Picture in the Eye, and the Idea of an erect Object ; and further than this we do not know.

If we look at the same Object with both Eyes, it will appear single ; but this happens only when the Object is painted in correspondent Points of each Retina ; which probably happens from the meeting of the Optic Nerves. For it is observed in all Animals, which see the same Object with both Eyes, that the Optic Nerves meet and separate again before they go to the Brain ; but in Animals which see different Objects with each Eye, the Optic Nerves go separately from the Eyes to the Brain.

Only one Point at a Time can be seen distinctly, namely, that which is represented in the Axis of the Eye ; if we look at one Point with both Eyes, we must so direct the Eyes, that their Axes continued shall meet in that Point ; which happens when we have our Eyes intent upon any Point.

By this Direction of the Axes we judge of the Distance of Objects; for the Situation of the Eyes alter according as the Axes make a different Angle, which Angle depends upon the Distance of the Object: Whence it happens, that without perceiving when we do it, by Use we get a Habit of judging of the Distance of
 729 Objects by the Direction of the Axes; which is sensible to us, because it depends upon the Motion of the Eye, that we feel. Therefore we may see the Use of having two Eyes placed at a certain Distance from one another; *as long as this Distance of the Eyes bears a sensible Proportion to the Distance of the Objects, we can judge of it pretty certainly.*

730 *We can also judge of lesser Distances with one Eye alone; because in the Variation of a small*
 721 *Distance, the Change in the Eye is sensible.**
 723

731 *In great Distances, if we look at known Objects, we judge from the apparent Magnitude and the Colour.*

732 *It is impossible to judge of very great Distances, except the same Objects be seen from different Places.*

733 *The apparent Magnitude of an Object depends upon the Bigness of the Picture in the Bottom of the Eye, which Picture depends upon the Angle under which an Object is seen, that is, the Angle which is form'd by Lines drawn from the Extremities of the Object to the Eye.*

734 *This apparent Magnitude is to be distinguish'd from the Magnitude which our Mind attributes to the Object that we see, which last is founded upon the Judgment, whose Foundation is not in the Appearance alone. Every Body knows that the Object appears the less according as it is more distant; whence it happens, that, according to the greater Distance of the Object, if it be known, the apparent Magnitude of the Object is increas'd*

in

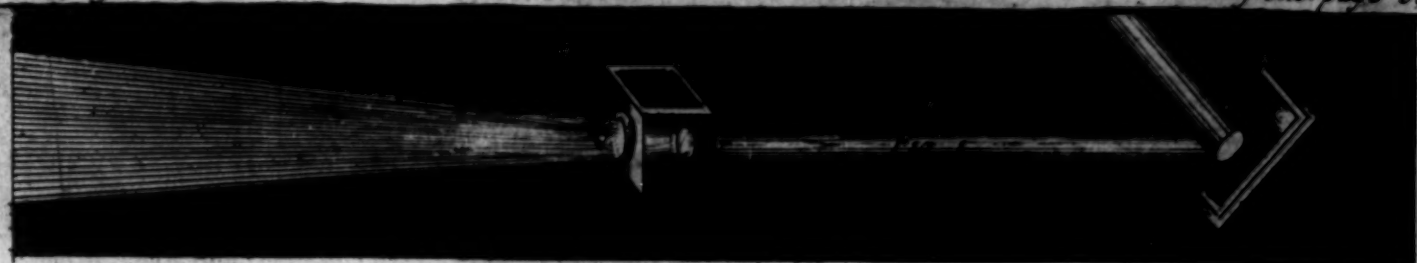


Fig. 2.

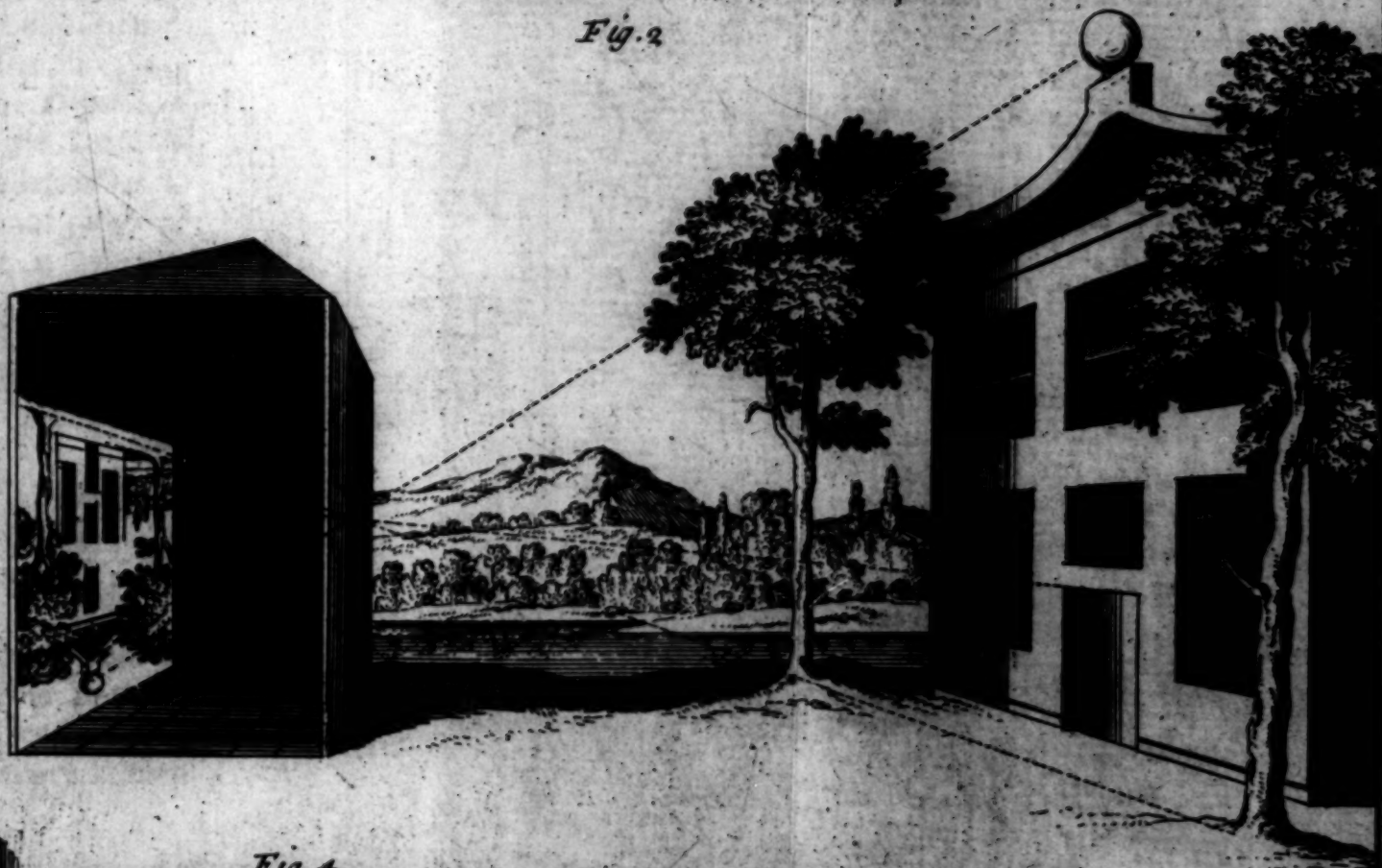


Fig. 4.



Fig. 3.

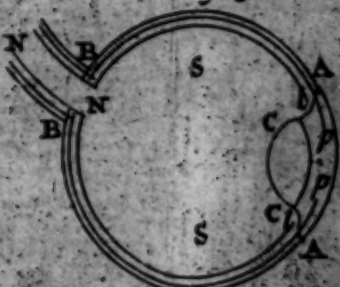
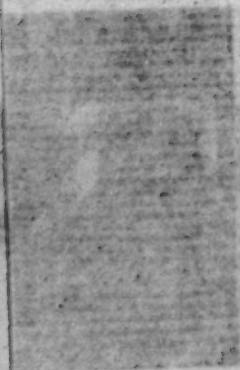


Fig. 5.





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in the Judgment that our Mind makes of it, which we do without any Attention to it. Therefore the same Object at the same Distance appears of a different Magnitude, if we judge differently of the Distance.

We have a remarkable Example of this in respect of the Sun and Moon, which appear greater when near the Horizon, than at a greater Height; tho', as is known to Astronomers, the Picture of the Sun in the Bottom of the Eye is the same in both Cases, and that of the Moon is less when it appears nearer the Horizon; we cannot judge of the Distance in either Case, * but it appears greater near the Horizon, by reason of the Interposition of the Fields and Part of the Heavens. If we see the Bodies above-mention'd thro' a Tube, this apparent Distance vanishes, as also the Magnitude which is deduced from it. From our Childhood upwards, and so continually, we join the Idea of Distance with the Increase of apparent Magnitude, (which is necessary for making a true Judgment concerning the Magnitude) whereby the Ideas are so closely join'd, that they cannot be separated, not even in those Cases, in which we know that they lead us into Error. Logicians teach us, how many Errors are to be attributed to Ideas so join'd.

CH A P. XI.

Of Vision thro' Glasses, and how to correct some Defect of the Eyes.

AN Object is visible, because all its Points are as it were radiant Points; * therefore a Point appears in that Place from whence diverging Rays are emitted.

737 *If Rays inflected any how enter th. Eye ... verging, the visible Point will be in the imaginary Focus of the Rays; for the Rays enter the Eye exactly in the same Manner that Rays would do that came directly from that Focus; and to have them unite upon the Retina, the same Situation of the Crystalline is requisite; so that, in respect to a Spectator, it is no Matter whether those Rays, after Refraction, or these directly enter the Eye; and there will be the same Motion in the Eye, when it fits itself for distinct Visions.**

*719,
730

738 *A Point appears the more enlightened, the greater Number of Rays coming from it enter the Eye.*

739 *When Objects are seen thro' a plain Glass terminated with parallel Surfaces, they appear to be nearer than when seen with the naked Eye. Let A (Plate X: Fig. 1.) be a visible Point; the Rays, going from it and entering the Eye, are between $A b$ and $A b$; these, being refracted in the Glass $V V$, move along $b c$, $b c$, and go out thro' $c d$, $c d$, which are parallel to the Lines $A b$, $A b$: * now because $b c$, $b c$, are refracted towards the Perpendicular, * $c d$, $c d$, fall between $b A$ and $b A$; that is, they intersect at a , which is nearer than A ; therefore the imaginary Focus of the Rays which enter the Eye is at a , in which the Point A appears to be.**

*737

740 *That Point also appears to be more enlightened, when seen thro' the Glass above-mentioned. For all the Rays between $A b$ and $A b$ enter the Pupil between d and d ; but as the Lines $A b$, $A b$, are parallel to the Lines $c d$, $c d$, and these are between those, $A b$ and $A b$ being continued would fall beyond d and d ; therefore if the Glass was taken away, the Rays, which now enter the Pupil, would take up a greater Space, and therefore would not all enter the Eye.*

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*Plate X. Fig. 2.] The apparent Magnitude of an Object is increased by the Interposition of a plane Glass; the Object A E is seen by the naked Eye under the Angle A d E; but if you use the Glass V V, by reason of the Refraction thro' A b c d and E b c d, it will be seen under the Angle c d c, which is greater than the last. But yet the Object is not greater in Proportion to the increased apparent Magnitude; * for it appears to be at a less Distance.**

The Increase of apparent Magnitude is so much the greater, as is the Difference of the Angles A d E and c d c; whose Difference increases as the Intersections of the Lines A d with b c, and E d with b c, come nearer to the Points b and b; which obtains as the Object is brought nearer to the Glass; and therefore it is the greatest of all, when the Object touches the Glass; which shews that Objects, even in the Glass itself, must appear magnified.

And in general, the Eye being placed in a rarer Medium, the Object that is seen in a denser Medium appears bigger, and is also brought nearer by the Refraction. This is every Day confirm'd by Experience, when we look at Objects in the Water.*

*Let there be a Point A seen thro' a convex Lens V V, (Plate X. Fig. 3.) and the Rays A b, A b, at c d, c d, will go out more diverging, as if they came from a; * therefore that Point appears to be at a greater Distance.* It appears also more en- lightened; for the Rays come nearer to each other as they go thro' the Glass, * and are also reduced into a less Space, wherefore a greater Number of them must enter the Pupil.*

The apparent Magnitude of an Object is also increased; that is, the Object seen thro' a convex Glass is seen under a great Angle, which ap-

pears from a Sight of the Figure; the Object A E, seen with the naked Eye, is seen under the Angle A d E, (*Plate X. Fig. 4, and 5.*) but now it is seen under the Angle c d c, which is greater; (in *Fig. 4.*) the Rays A b, E b, which are convergent, converge more as they go out of the
 *700 Lens; * or diverging (as in *Fig. 5.*) they come
 699 to the Eye converging. Therefore the Object appears magnified, both from the Appearance of
 *745 it being farther, * and its Magnitude being in-
 *734 creas'd; * and therefore the Magnitude, that we
 748 attribute to an Object, does not follow the same Proportion as the apparent Magnitude; for which Reason we shall not delay Time in Demonstrations about it: But we shall observe in general,

749 That the Angle, under which an Object is seen thro' a convex Glass, diminishes as the Eye recedes from the Glass; whilst the Object is not more distant from the Glass than the Point in which parallel Rays are collected: But if the Object is farther off, the apparent Magnitude is increased as the Eye recedes.

750 After the same Manner, if the Eye be between the Glass and the Focus of parallel Rays, the Angle above-mention'd is diminished as the Object is farther removed; the Eye being placed at a greater Distance, the same Angle is increased as the Object is farther removed; in which last Case the Object may be so far removed, as not to be visible beyond the
 754 Glass, as will be said anon.

Also in those Cases in which Objects are visible, they do not always appear distinctly.

751 For, that a Point may appear distinct, it is required that the Rays that proceed from a Point should
 *736 enter the Eye diverging, * and that the imaginary
 2 7 Focus of those Rays, in respect of the Spectator,
 724 should be within the Limits of distinct Vision.*

752 If the Object be removed beyond the Focus of parallel Rays, the Rays flowing from a Point
 of

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of the Object, enter the Eye converging; * which *699
 Case is impossible to the naked Eye: In this the
 Vision is always confused, and the Eye disposes
 itself so as to have the Vision the least confused
 that may be; from this Disposition we judge of
 the Distance, as we do in every Case in which
 we judge of it with only one Eye.* *730

But this Distance does not always appear to 753
 be the same; whence may be deduced what is
 said of the Change of the Figure of the Crystalline.* For if, supposing the Crystalline moveable, *722
 its Figure be unchangeable, in every Situation
 of the Object and the Eye, in which the Rays,
 coming converging from a Point, enter the Eye,
 there will be the least Confusion, if the Crystalline comes back towards the Retina as far as may
 be; so that in every Case there would be the
 same Disposition of the Eye, and the same Judgment concerning the Distance; which, as has
 been said, is contrary to Experience: But if it
 becomes flatter as it recedes from the Retina,
 there will be Changes in the Eye, which agree
 with the several Judgments made of the Distance in different Situations of the Object and
 the Eye.

If in the last Case, in which Rays, coming from a 754
Point converge, the Eye be so removed, that the Rays
unite before they come to the Eye, in all the Points
in which the Rays concur, there will be radiant
Points; which are the Foci of all the Points of
the Object, by which the Object is represented
*inverted upon a white Plane; * and which are *711*
visible Points in respect of the Eye, to which
the Rays can come after their Interfection. In *736*
that Case the Object appears inverted, because we
don't see the Object itself but its Representation
behind the Glass, which we have said to
be inverted. *711*

755 We have said that the Case is impossible to the naked Eye, in which the Rays, coming from a Point, enter the Eye converging; and therefore such Vision is always confused, because the Construction of the Eye cannot adapt itself to an impossible Case: Yet sometimes, but rarely, even in that Case, the Objects are seen distinctly; which, as it is occasioned by such a Defect of the Eye as takes away all distinct Vision from the naked Eye, I did not think proper to take Notice of such Exceptions of the general Rule.

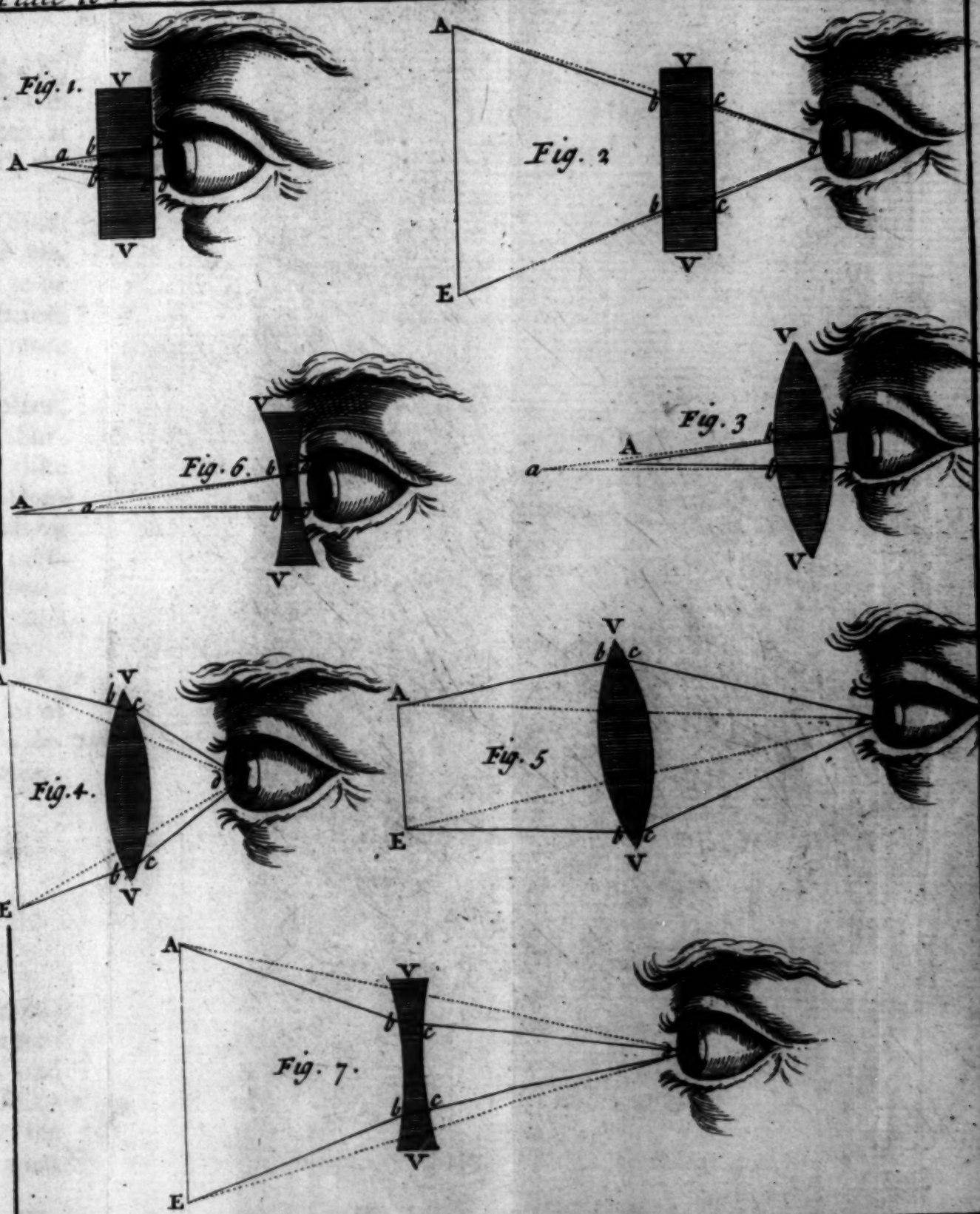
756 *The Fault of the Eyes of a great many old Men* is, that they can see none but distant Objects distinctly, those that are near appearing confused to them; which Defect is corrected by the Interposition of a convex Lens. The Rays, which flow from a Point which is near, concur beyond the Retina; passing thro' a convex Glass, they will diverge less as they enter the Eye, and so concur sooner in the Eye; that is, come to the Eye as if they flowed from a remote Point, such as is seen distinctly by an old Man.

757 *Thro' a concave Lens, Objects appear to be nearer, less enlighten'd, and less.*

Plate X. Fig. 6.] The Rays $A b$, $A b$, and all that are between, going thro' a concave Lens
 *705 become more diverging, * and enter the Eye as
 652 if they came from a Point a , which is less distant,
 737 where the Point A appears to be.

By making the Rays to diverge more, they are carried farther asunder; and therefore fewer of them enter the Eye, which diminishes the
 738 Illumination of the Point seen.

Plate X. Fig. 7.] The apparent Magnitude is also diminish'd, because the Rays $A b$, $E b$, by which we see the Extremities of the Object,
 *706 come to the Eye less converging, * and therefore the Angle $c d c$, under which the Object is seen
 beyond





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beyond the Lens, is less than the Angle $A d E$, under which it is seen by the naked Eye; therefore upon account of the Diminution, both of the Distance and the Angle above-mention'd, the Object appears diminish'd. *

*A concave Lens is of use to those who see no Objects distinctly but such as are near, such are call'd Myopes; thro' this Lens remote Points appear to be near, * and the Rays, which did concur before they came to the Retina, now enter the Eye more diverging, and meet upon the Retina.* 734 758 757

There are Glasses that have one Surface plane, and the other side is made of several plane Surfaces, that make Angles one with another (like a Diamond) thro' these the Rays that flow from one Point suffer different Refractions, and by every Surface are made to enter the Eye in a different Direction, as if they came from different Points; that is, the same Point forms several imaginary Foci, and therefore appears multiplied, for it is seen in several imaginary Foci *; which, as it happens in respect of every Point of the Object, thro' such a Lens, which is a Polyhedron (or multiplying Glass) the Object appears multiplied. 737 759

C H A P. XII.

Of Microscopes and Telescopes.

WE have shewn of what Use Glasses terminated with spherical Surfaces are for correcting the Defects of the Eyes of Old Men, and of the short-sighted *; how they serve for discovering the smallest Objects, and bringing the most distant (as it were) to the very Eye, is what we are now to consider. 756 758

We

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- We have said that the convex Glasses magnify
 • 747 the Objects*, which magnifying depends upon the Refraction of the Rays as they go thro' a convex Lens; whence it follows, that it is increased, if in the same Circumstances the Refraction be increased; which Effect may be produc'd, by augmenting the Convexity of the Lens, which will be the more convex, as the Surfaces that terminate it are Segments of a less Sphere, which can only be had in very small Glasses.

DEFINITION I.

- 760 Such small Lenses are call'd *Microscopes*.
 761 By a *Microscope* small Objects are vastly magnified, by which means, things which would be invisible to the naked Eye, are very distinctly seen.

DEFINITION II.

- 762 The Space seen thro' the *Microscope*, that is, the Circle in which Objects are visible thro' the *Microscope*, is called the Field of the *Microscope*.

- Experiment 1. Plate IX. Fig. 5.] If we look at the small Object A E thro' the *Microscope* V,
 • 745 it will appear magnified at ae^* .

- 747 There are also compounded *Microscopes*, made up of two or three Lenses; what Foundation they depend upon, will be sufficiently shewn by explaining one of those which is made up of two Lenses.

- Plate XI. Fig. 2.] Take a small Lens that is very convex, as V V, and let the Object A E be plac'd at such a distance from it, that all its
 • 710 Points shall have their Force beyond the Lens*;
 699 let the Object be brought so near, that the Foci
 • 699 may be remov'd to ae^* , and you will there have the Representation of the Object very much enlarg'd,

larg'd, which will be sensible if you receive it there upon a white Plane*.

* 711

Experiment 2. Plate XI. Fig. 1.] The Lens above-mention'd must be made last in the End of a Tube at V; and the other End of the Tube, which is wider, must be cover'd with a very thin Paper C C; the Object A E must be so plac'd, that the Foci of the Points of that Object shall be just at the distance of the Paper; if then the Object be well enlighten'd, you will have its Representation inverted, visible thro' the Paper at *a e*. By moving the Object, you will have the true Position that brings the Representation upon the Paper to be distinct.

Plate XI. Fig. 2.] All the Points of the Representation *a e* are radiant Points, and therefore visible*, which being seen thro' a large Micro- * 736 scope V V, shews the large Representation *a e* 754 at *a e**; that is, the Rays coming from the Ob- * 761 ject A E, after the Refractions thro' both the Lenses V V, V V, will enter the Eye, as if they came from an Object at *a e*.

Therefore thro' such a compounded Microscope 763 the Object appears inverted, and much more magnified than thro' a single Microscope.

DEFINITION III, and IV.

In this Microscope the smallest Lens, which is next 764 to one Object, is called the Object-Glass, and the other the Eye-Glass.

This last ought not to be too small; for the Points of the Representation *a b*, tho' they be radiant Points, do not emit Light every way; only the Rays which pass thro' the Object-Glass intersect one another in the several Points of the Representation *a b*; which Representation therefore will not be visible, unless the Rays that go thro' the Object-Glass do also go thro' the Eye-Glass.

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765 *Glass.* The Field therefore (or the Space that the Microscope can take in) depends upon the Magnitude of this Lens.

The Eye also must be so plac'd, that all the Rays that come to the Eye-Glass, going thro' it, shall come to the Eye; which is done by placing
766 the Eye at *d*, the Point in which all the Rays which come from the Center of the Object-Glass, and pass thro' the Eye-Glass, are collected.

Objects appear bright enough thro' Microscopes, because they are very near the Glass, and so the same Number of Rays pass thro' a small Lens as would not pass at a greater distance, unless thro' a great Hole; yet often, where Objects are the most magnified, they must be enlighten'd by
767 Rays collected thro' a convex Lens and thrown upon them.

The astronomical Telescope much resembles the compound Microscope.

DEFINITION III.

768 Instruments fitted to see distinct Objects, are called Telescopes.

769 That which we now treat of is call'd the astronomical Telescope, because it is not so fit for seeing Objects upon Earth, for it represents them inverted; but Astronomers do not much regard the Position of the Appearance of the Object.

This Telescope consists of two convex Lenses, the one an Object-Glass, which is plac'd next to the Object, and the other an Eye-Glass, plac'd next to the Eye; by help of the first, distant Objects are represented at a certain distance behind the Lens*, as near Objects are in the compound Microscope. If this Representation be observ'd thro' an Eye-Glass, it will appear enlarg'd and inverted, as we have said concerning the Microscope.

It

It is plain also, that in this Case, as well as in 770
the Microscope, the *Field depends upon the Breadth*
of the Eye-Glass; as also, that *the Place of the Eye* 771
must be determined in the same manner for the Tele-
*scope as for the Microscope**; for the astronomic * 766
Telescope differs from the compounded Micro-
scope only in this, that in the Microscope the
Lenses are more convex, and therefore less pro-
per for looking at distant Objects, especially in
respect of Object-Glasses. In the Microscope the
Object-Glass is more convex than the Eye-Glass;
but the contrary obtains in the Telescope.

Telescopes cannot be too long for observing
the Stars; but if they are above 20 Foot long,
they are of no use for seeing Objects upon Earth,
because of the constant trembling of the Air,
which is too sensible in Glasses that magnify very
much.

A short *astronomic Telescope will serve to see Ob-* 772
jects upon Earth, by adding to it two convex Lenses,
which are also call'd Eye-Glasses; the three Eye-
Glasses are alike, and less convex than in the
astronomic Telescope, the Object-Glass remain-
ing the same.

Plate XI. Fig. 3.] Take an Object-Glass V V, 773
which represents a distant Object at ea ; then
take besides three Eye-Glasses D D, D D, D D;
the first must be so plac'd, that the Rays coming
from the Point of the Representation ea , shall
become parallel when they have pass'd the Lens*; * 699
in that Case the Rays which come from the mid-
dle Point of the Object-Glass, will be collected
at G; the second Lens must be so plac'd, that
these Rays which are collected at G (where they
intersect one another, and move as if they came
from that Point) may go out parallel after they
have pass'd thro' it*; which being perform'd, * 669
the Rays coming from the Object-Glass to e , and
there .

there intersecting and forming that Point of the Representation of the Object, being refracted thro' the first Lens, pass by G parallel to one another; thro' the second Lens they are refracted in the Direction D e, and collected at e*, so as to make it the Point of a new Representation. In the same manner the Point of a new Representation corresponds to the Point a of the second Representation; which being also true concerning the intermediate Points, there will be found an erect Representation of the Object at a e.

Experiment 3. Plate XI. Fig. 4.] Let three little Boards D, D, D, with Eye-Glasses in them, that collect parallel Rays at the distance of about five Inches, be moveable upon a Plane between two Rulers, in such manner that the three Glasses may be in the same Line as the Hole V, thro' which alone the Light enters into the Room, and in which there is an Object-Glass, which is fix'd in a short Tube, in order to exclude all the side Light.

This Object Lens is such, as is able at the distance of three Feet from V, to represent distant Objects inverted at F; which Representation will be visible, if you let the Rays fall upon a white Plane in that Place*. Five Inches farther from F you must place the first Eye-Glass, and ten Inches from that you must place the second; at f, which is five Inches from it, you will have an erect Representation of the same Objects, which will also be visible, if receiv'd upon a white Plane.

774 Plate XI. Fig. 3.] If the Representation a e be seen thro' a third Eye-Glass, supposing the Eye at o, in which the parallel Rays a D and a E are collected, the Object appears magnified, brought near, and erect; for it is seen under the Angle D o D,

D o D, when with the naked Eye it would appear under a very small Angle ; it will also appear near, because, tho' it be seen beyond a e, yet the distance at which it appears has no sensible relation to the distance of a very distant Object.

Experiment 4. Plate XI. Fig. 4.] Supposing every thing as in the foregoing Experiment, let there be a third Eye-Glass plac'd ten Inches from the second, and 5 Inches from that a little Board, or Eye-Stop, with an Hole O ; if the Eye be plac'd at O, the Object, as has been said, will appear erect, magnified, and near. If the Board O be displac'd, that is, be brought nearer, or removed farther off, the Field of the Telescope is diminish'd ; because there is but one Situation of the Eye, in which all the Rays which pass thro' the Eye-Glasses can come to the Eye.

We are to take notice, that the Eye-Glasses made use of here, are not convex enough in respect of the Object-Glass V ; but they are best for making the third Experiment.

All the Points of the Object do also appear more enlighten'd ; for the Rays which, coming from any Point of the several Points of the Object-Glass, do intersect one another in a Point of the Representation, by reason of the small distance of the Eye-Glass from that Representation, are but little dispers'd before they come to the Eye ; so that they all go into it : and therefore the Illumination given by the Telescope, is to that which is given by the naked Eye, as the Surface of the Object-Glass, thro' which the Rays pass, to the Surface of the Pupil. • 738

One may also with two Lenses make Telescopes, which shall shew Objects erect, enlighten'd, and magnified ; these must be shorter, for if they be above a Foot long, they become almost useless, because

because their Field will be so small; that is, they will take in so little of an Object.

- Plate XI. Fig. 5.]* Let V V be an Object-glass, the inverted Representation of a distant Object
- 711 will be at $e a^*$; the Rays are so intercepted by the concave Lens D D, that such of them as come from the Center of the Lens V V, are inflected
 - 705 as if they proceeded from the Point f^* ; by the same Refraction in the Rays which concurr'd at
 - 706 a , become diverging*, having their imaginary Focus at a ; which also happens in respect to all the Points of the Representation $e a$, and instead of it, you have an imaginary Representation which is erect at $a e$; that is, the Rays enter the Eye as if they came from the Object plac'd at $a e$.

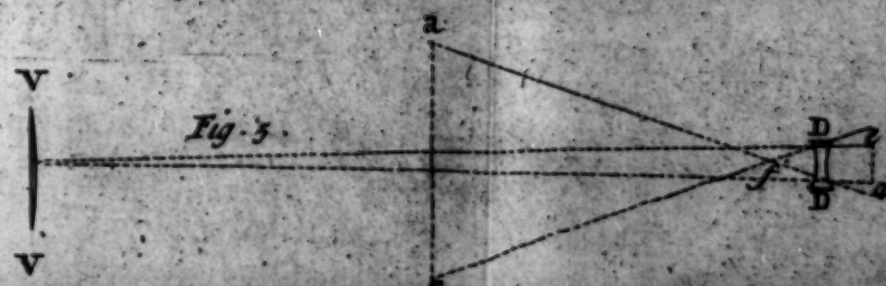
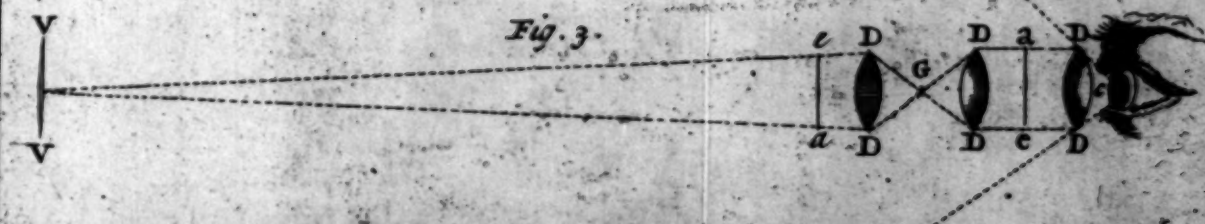
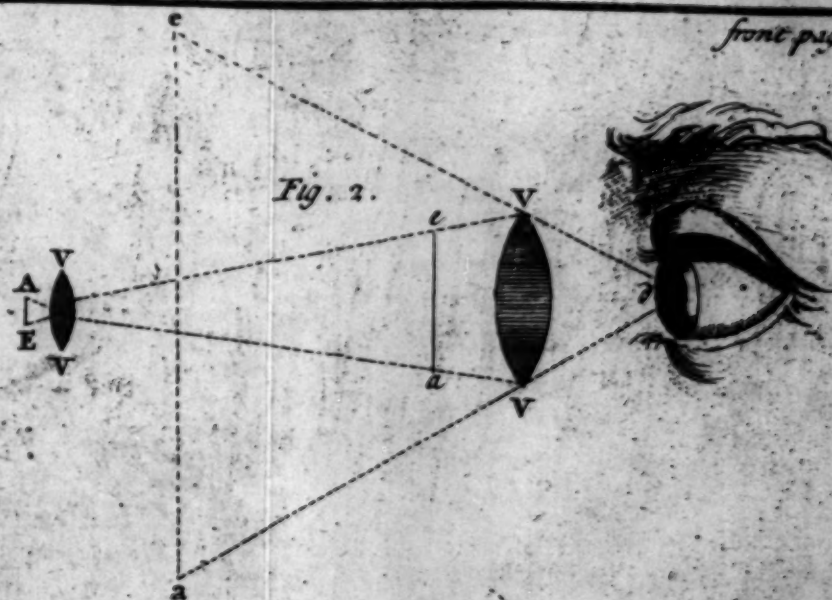
777 The Rays in all respects go out diverging from the Eye-Glass, and therefore *the Eye must be brought as near as possible to the Eye-Glass.*

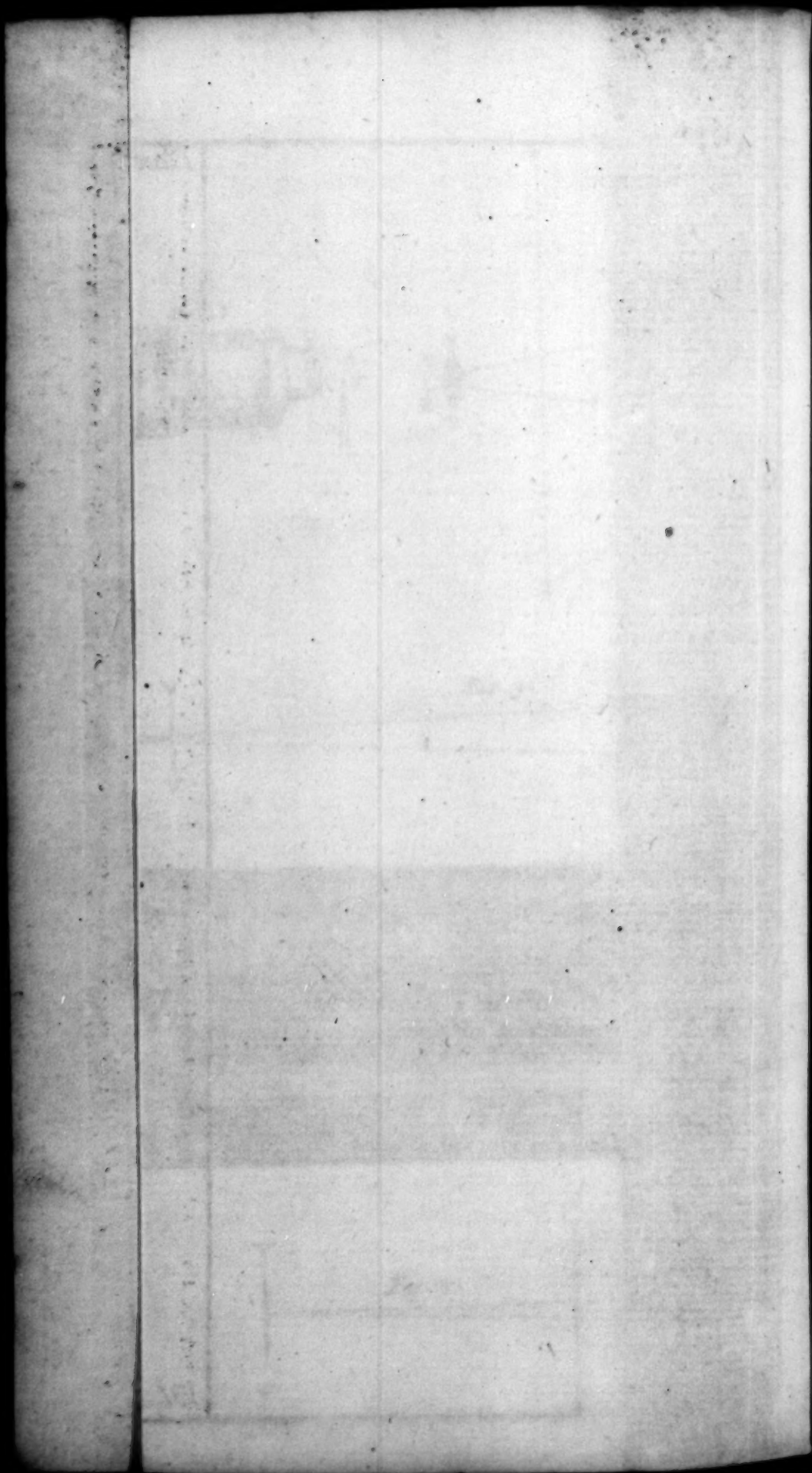
778 *In this Telescope the Field depends upon the Bigness of the Object-glass; for the Rays, which from a Point come obliquely to the Center of this Lens, very often do not enter the Eye; whilst other Rays from the same Point, which pass thro' the Lens near its Circumference, do come to the Eye.*

C H A P. XIII.

Of the Reflexion of Light.

- W**E have shewn that Light is reflected from opake Bodies, and that every Point reflects it every way*; the occasion of this is the Inequality of the Surfaces, which are made up of an innumerable Quantity of small Planes, which in all sensible Points are directed every





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every Way; which will be easily conceiv'd, if we imagine a Surface cover'd with an innumerable Quantity of small Hemispheres. That this is true, we deduce from the Reflexion of Light from a polish'd Surface; that is, from a Surface whose Inequalities are taken off; which in all its Points reflects the Light only one Way, which holds in Curve as well as in plane Surfaces: Nay, from Surfaces that are not at all polish'd, the Light is mostly reflected that Way, and it would be all reflected if they were polish'd, as daily Experience shews.

[Plate XII. Fig. 2.] Let AC be a Ray of Light coming obliquely upon a plane Surface; let CO be perpendicular to this Surface, and the Rays be reflected along CB.

DEFINITION I.

The Ray CB is called the *reflected Ray*.

780

DEFINITION II.

The Angle OCB is called the *Angle of Reflexion*.

781

The *reflected Ray*, together with the incident Ray, in the same Plane which is perpendicular to the reflecting Plane.

782

For the Action of this Plane, by which the Light is reflected, is directed perpendicularly to a Plane, which is supposed like in all Points.

783

The Angle of Reflexion is equal to the Angle of Incidence.

784

[Experiment 1. Plate XII. Fig. 1.] Take a plane Looking-glass S, which may be set in any Position by Means of a Ball and Socket join'd to the Foot that sustains it; thro' a Hole, in the Plate of Metal L, that is in the Window, let in a Sun Beam of about a Quarter of an Inch Diameter

ter into the Room ; the Glass must be so disposed, that the Beam may come thro' an oblique cylindric Cavity (of the same Bigness as the Beam) made in a little upright Board T : If you turn this Board Side for Side, the reflected Ray will go thro' the same Cavity. This holds good, whatever the Inclination of the Cavity be, as may be demonstrated by using different Boards.

By which Experiment N^o. 779. is prov'd, as well as N^o. 784.

785 *Plate XII. Fig. 2.] If the reflected Ray becomes the incident Ray; that is, if the Light comes along the Line BC, it will return in the Line CA, that is, the first that was the incident Ray will become the reflected Ray; as appears from the Equality of the Angles BCO, OCA.*

From this Equality of the Angles of Incidence and Reflexion, we farther deduce, that
786 *the Light, after it has fallen upon a Body, recedes from it with the same Force that it came upon it.*

Let the Motion along AC be resolv'd into two
192 *Motions along AO and OC*, supposing AO parallel to the reflecting Plane, and OC perpendicular to it. Let AO be continued; the Motion in that Direction is not alter'd from the Action of the Plane: Therefore let AO and OB be equal; if the Light recedes from the Plane with the same Force, with which it came upon it, the Motion occasion'd by the Repulsion is represented by CO, and in that Case*
190 *the reflected Ray goes thro' B*; that is, the Angle OCB is equal to the Angle OCA, which agrees with the Experiment.*

787 *As to the Reflexion of Light, it is to be observ'd, that Light does not run against the solid Part of Bodies, when it is reflected by them; but that it is reflected in those Places, where it could very freely pass.*

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pass. I shall prove this by several Experiments, by which many other wonderful Properties of Reflexion are discover'd.

It is a common Experiment observ'd by every 788 Body, that when Light is mov'd thro' any Medium, as for Example, Glass, Water or Air, it does not undergo a sensible and regular Reflexion; but that it is reflected there, where two Mediums of different Density are separated; so it is reflected in the Surface of Water or Glass.

Could Light in such Quantity strike against the Particles, just where the Mediums are separated, whereas it moves thro' both the Mediums for a great Space without striking against any such Particles; Are there more of those Particles near the Surface than elsewhere? *Light also is more* 789 *abundantly reflected in a denser Medium, when it comes against the Surface of a rarer; than when on the contrary, moving in a rarer Medium, it strikes against the Surface of a denser.*

Experiment 2. Plate XII. Fig. 3.] In a dark Place in which the Light enters thro' an Hole in the Plane L, let there be placed a triangular Prism of Glass AB; let the Light enter the Prism thro' one Side; if it comes to the next Side making an Angle of Incidence greater than 40 Degrees, it is wholly reflected, and does not at all penetrate into the Air; but Light moving in Air is never wholly reflected by the Glass.

But if the Reflexion be made by the striking of Light against the solid Parts of Bodies, there must be more such Parts in Air than in Glass; for if Light was reflected from the Glass itself into the Air, the Light would never come to the Separation of the Mediums; that the Light can also go out of Glass in the very Places, where it is reflected, is prov'd by following Experiments.

periments. Therefore in the Neighbourhood of the Glas there must be so many Parts in the Air, that there may be no Way for the Passage of the Light, to cause it to be wholly reflected into the Glas; but it is plain that there are no such Parts, because Light comes thro' the Air in all Directions quite to the Glas. Even in the same Place of the Surface which separates the Glas and Air, the Light which comes from one Side is reflected, whilst that which comes from the other Side is transmitted. Which clearly proves that the Light is reflected in the very Place where it can go thro'.

790 *Experiment 3. Plate XII. Fig. 3.]* Every Thing being as in the former Experiment, if the Obliquity of the Light be chang'd, Part of it will pass thro' into the Air.

Who wou'd conceive that Light, which passes from Glas into Air, and does not run against the solid Parts, shou'd all of it (by a little increasing the Obliquity) run against those Parts; when in each Medium, as has been already said, there are Pores and Passages in all Directions?

791 *Experiment 4. Plate XII. Fig. 3.]* Take a Glas triangular Prism AB, moveable about its Axis; which is made so by fixing brass Plates to its Ends, with brass Wires perpendicularly fix'd to them: The Prism must be so laid upon the Trough P, as that the said Wires may bear upon the Brims of it, which are made a little hollow to receive them, yet so as to let them turn, that the Prism may move freely about its Axis: Let it be so plac'd as to reflect the Light in the same Manner, as in the second Experiment. Let the Trough be fill'd with Water up to the Prism; then the Light, which, striking against the

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the Air, was wholly reflected, now running against the Water, does partly enter into it, and is only reflected in part.

Which Experiment does not at all agree with a Reflexion made by a Stroke upon the solid Parts.

In the third Part of this Book we shall also shew, that thin Plates, which reflect Light, will transmit it, if they become thicker*. ^{*894}

The fourth Experiment also proves that *the reflecting Power is so much greater, as the Mediums, which are separated by a reflecting Surface, differ more in Density*; for Glass and Air differ more in Density than Glass and Water. 792

In this Experiment we also see that *Reflection is made by the same Power by which the Rays are refracted, which produces different Effects in different Circumstances.* 793

A Ray, which goes out of a denser into a rarer Medium, by the Attraction of the former, is made to recede from the Perpendicular*; if ^{*625} the Obliquity of the incident Ray be increas'd, the Obliquity of the refracted Ray will also increase, till it comes at last to move in the very Surface which parts the Mediums. And this obtains, when the Sine of the Angle of Incidence is to the whole Sine, as the Sine of Incidence, in the denser Medium, is to the Sine of Refraction in the rarer; for in that Case the Angle of Refraction is a right one. If the Obliquity of the incident Ray be more increas'd, it is plain that the Ray cannot penetrate into the rarer Medium: This is the Case in which the Light is wholly reflected; which Reflexion depends upon that Attraction by which the Rays are refracted. For when the Ray is moved thro' the Space of Attraction, it is bent towards the denser Medium*; if it be in the denser Medi ^{*618}

um, and so bent, that, before it has gone thro' the whole Space of Attraction, the Tangent to the Curve be parallel to the Surface that separates the Mediums, the Curve, being continued, turns back again; and therefore the Ray is reflected by the Attraction of the denser Medium, and this Continuation of the Curve is similar and equal to the first Part, and makes the Angle of Reflexion equal to the Angle of Incidence; because the Light returns thro' the same Part of the Sphere of Attraction; and the same attracting Force acts upon the Light in correspondent Points of the two Parts of the Curve. Thus a Projectile, in its Ascent and Descent, describes similar Curves.

Yet that *all Reflexion does not depend upon that*
 794 *Attraction, in the same Manner*, is evident; for in that Case, in which the Refraction is made, Part of the Light is reflected; for the Light does not wholly penetrate out of the rarer into a denser Medium; for even in that Case, in which the Attraction is the most oppos'd to the Reflexion that is possible, yet some Rays are reflected.

Yet it cannot be doubted but that, *in every*
 795 *Case, Reflexion has relation to the refracting Power.*
 796 *Where Light passes without Refraction, there it is*
 617, *not reflected**; but where the Refraction is greatest,
 788 *there the Reflexion is also strongest**; which is true,
 631,
 792 not only when Light, moving in a denser Medium, strikes against a rarer, as in the fourth Experiment; but the same Thing is observ'd, when Light strikes against the denser Medium: Thus supposing the Light to move in Air, the Surface of Glass reflects more strongly than that of Water; and that of a Diamond yet more strongly. If Glass and a Diamond be immers'd in Water; the refracting Power is less in the Separation of those Bodies with Water, than where those

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those Bodies touch the Air*. These Bodies also ^{*631} reflect Light less strongly in Water than in Air. From this Relation of the reflecting and refracting Powers, we deduce, that *Light is driven* 797 *back at a certain Distance from the Bodies*, in the same Manner that the refracting Power does also act at some Distance from the Body: This Proposition is confirm'd from what has been demonstrated concerning Reflexion, which does not depend upon a Stroke made against the solid Parts of Bodies; and this is fully made out, if we consider, *that polish'd Bodies reflect the Light* 798 *regularly* (which we observe in Looking-Glasses) *tho' there be a great many Scratches in their Surfaces*: For as they are polish'd with the Powder of *Emery* and *Putty*, tho' their Parts are very small, yet they leave very great Scratches on the Surface in Respect of the Particles of Light; whence in the Surface itself the Reflexion must needs be irregular; but if we conceive the Reflexion to be made at some Distance from the Surface, the Irregularities are diminish'd, and almost wholly taken off, as is easily understood by any one that considers it with Attention.

C H A P. XIV.

Of Plane Mirrors.

Plate XII. Fig. 4.] **L** E T *b c* be the Surface 799 of a Plane Mirror or Looking-Glass; A a radiant Point. Let the Plane of the Glass be continued, and from the Radiant A let a Perpendicular A C fall upon it; if this Line be continued, and C a be made equal to C A, a will be the imaginary Focus of the reflected Rays that proceed from A. Let A b be the incident Ray; b f the reflected Ray; which continue beyond the Glass; because

- because the Angles of Incidence and Reflexion are
 784 equal to one another, their Complements also,
 which are the Angles $A b C, f b d$ are equal; to this is
 equal its opposite and vertical $a b C$: The Trian-
 gles $A b C, a b C$, which are rectangular, have the
 Side $C b$ common, and the Angles $C b a, C b A$,
 equal; therefore they agree in all Respects, and C
 A and $C a$ are equal to one another: Which De-
 monstration may be applied to all other Rays
 which flow from A , in whatever Plane, perpen-
 dicular to the Glas, they be conceiv'd to be.
 Therefore where-ever the Eye is, if the reflected
 Rays come to it, they will enter the Eye as if
 they came from a ; and the Point A will appear
 737 in that Place; but *the Appearance of that Point*
 800 *will have the same Position in respect of the Mirror,*
behind it, as a radiant Point has before it.
 801 If this be applied to all the Points of the Ob-
 ject, it will appear, that *the Object will appear*
behind the Glas in the same Position that it has
before the Glas.

C H A P. XV.

Of Spherical Mirrors.

- 802 **E**VERY spherical Surface may be confi-
 der'd, as made of innumerable small
 Planes; and a Plane, which touches the Sphere
 in any Point, is as it were a Continuation of such
 a small Plane.
 803 Spherical Mirrors are either *Concave* or *Con-*
vex.

The first are made of Part of an hollow
 Sphere polish'd.

The second are Parts of Spheres polish'd on
 the Outside.

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*A Ray coming upon any spherical Mirror, together 804 with its reflected Ray, is in a Plane, which being continued, goes thro' the Center of the Sphere *, for such * 782 a Plane is perpendicular to the Surface of the Sphere. A Line which is drawn thro' the Center of 805 the Sphere and Point of Incidence, being continued makes equal Angles with the incident and reflected Rays * ; for that Line is perpendicular to the * 784 Surface, and those Angles are Angles of Incidence and Reflexion ; therefore the Ray that goes thro' the 806 Center, or which being continued, would go thro' the Center, when reflected returns upon itself.*

Plate XII. Fig. 5.] Let bc be a Portion of the convex Mirror, A a radiant Point ; let Ab , Ad , Ac , be incident Rays, the reflected Rays will be bf , dg , cb ; if from the radiant Point A a Tan- 807 gent be drawn to the Mirror, the reflected Ray will be a Continuation of the Incident, or rather the Reflexion of the Rays terminated in the Point of Contact.

If bf , dg , cb , the Rays that are reflected from 808 the convex Mirror, be continued, with all the intermediate ones, by their Intersections they will form the Curve aa , which all these Rays touch, and the neighbouring Rays intersect in the Periphery of the Curve ; so that they always enter the Eye as if they came from a Point of the Periphery ; in which therefore the Point A does always appear, * 737 as long as the reflected Rays can come to the Eye, and the Eye is mov'd in a Plane which goes thro' the Center of the Sphere ; but when the Eye is mov'd out of that Plane, the Radiant appears in another Curve, because there are such Curves in every Plane, which may be conceiv'd to pass thro' A and C .*

Since all these Curves, and each of them whol- 809 ly are behind the Glass, all the Objects also appear behind the Surface of the Glass.

The

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810 *The Objects likewise appear erect.* For if the Point A be mov'd about the Mirror, the whole Curve $s s$ is carried with the same Motion; which proves (in respect to the erect inverse Situation) that the Points of the Representation have the same relation to each other as the Points of the Object itself.

As the Point A is farther removed from the Glass, the whole Curve does also recede by a contrary Motion; but supposing A at an infinite distance, that Point of the Curve which is the farthest remov'd from the Surface, will be distant from a quarter of the Diameter; whence it follows, that *the Objects appear diminish'd*, because all the Representations are compriz'd in small Limits.

812 *If the Eye be mov'd, the Appearance of the Object is also mov'd, and its Figure chang'd;* for all the Points are mov'd in their own Curves, and that unequally, according to the different Situation of the Eye in respect of each Curve; whence of necessity the Figure must be chang'd.

Experiment 1. Plate XII. Fig. 6.] If any one sees his Face in a spherical convex Mirror standing at A, he will see his Face at s erect, diminish'd and unlike; by the Motion of the Eye one may observe the other things mention'd in respect to fix'd Objects.

813 Let $b d$ (Plate XIII. Fig. 1.) be a concave Mirror, and a Portion of a Sphere whose Center is C; let parallel Rays fall upon the Surface of the Mirror, one of which, $C d$, is suppos'd to pass thro' the Center; this Ray by Reflexion returns upon itself, and the Rays next to it being reflected, become converging, and concur with it in the Focus F, which is the middle Point between d and C. Let $A b$ be a Ray very little distant from $C d$; draw the

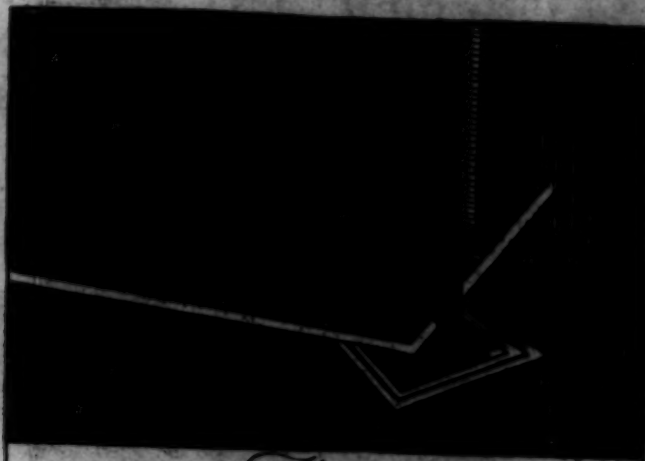


Fig. 1.

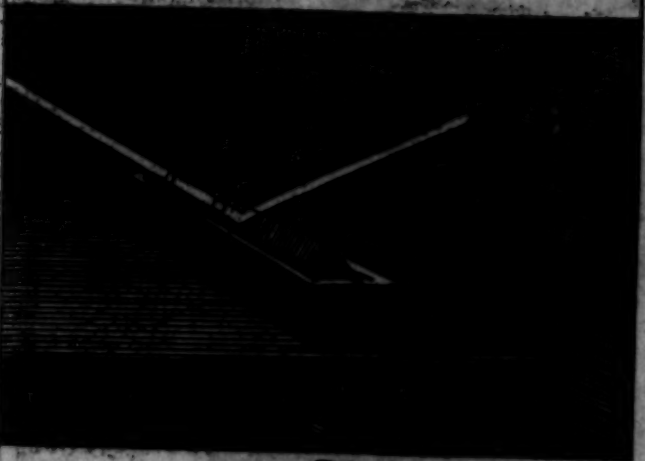


Fig. 3.



Fig. 6.

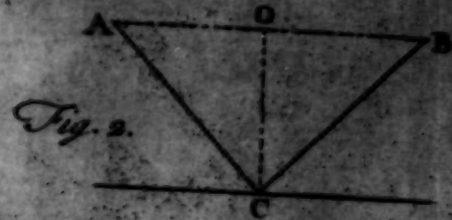


Fig. 2.

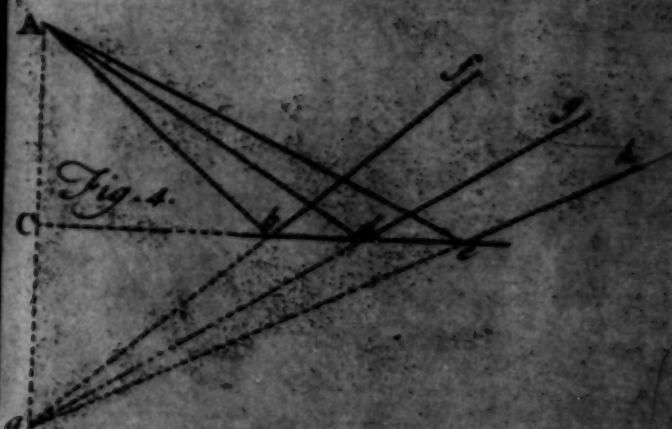
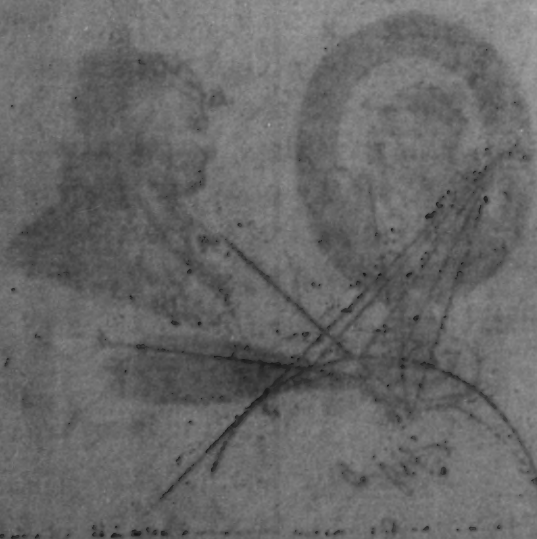
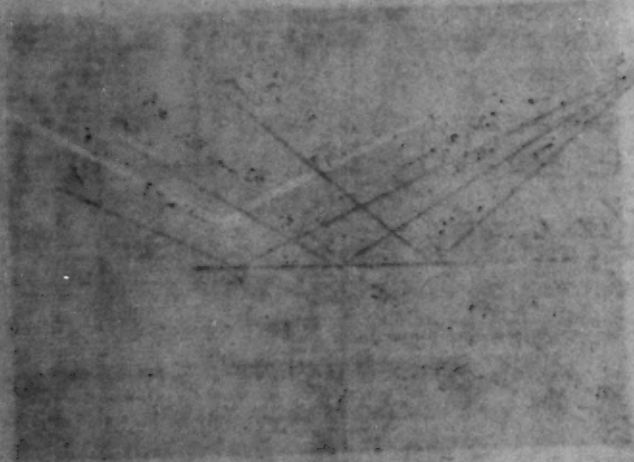


Fig. 4.



Fig. 5.



the Semidiameter Cb ; the Angle of Incidence will be $A b C$, to which the Angle of Reflexion $C b F^*$ is equal, as also the alternate Angle $b C F$; * 805 therefore $b C F$ is an Iſosceles Triangle, and the Sides $F C$ and $F b$ are equal; because $b d$ is very ſmall, $F d$ and $F b$ do not ſenſibly differ, therefore $F C$ and $F d$ are equal; which Demonſtration will ſerve for all the Rays that are but very little diſtant from $C d$.

If parallel Rays are farther diſtant from $C d$, they do not meet at F ; yet they will all come together into a little Circle, if the Diameter of the Mirror does not exceed the fifth or ſixth Part of the Diameter of the Sphere of which it is a Portion.

Burning Mirrors are made upon this Foundation, which collect the Sun's parallel Rays into a Focus. 814

Experiment 2. Plate XIII. Fig. 2.] Let there be a concave Mirror S , made of Metal, or of Glaſs quick-ſilver'd behind; let it ſtand upon the wooden Foot P , whoſe upper Part is bor'd ſo as to receive a Cylinder of Wood made faſt to a tranſverſe Piece $A A$, which ſerves for turning the Glaſs round with a horizontal Motion; and the Mirror itſelf muſt move upon two Ends of an Axis between the Pillars $A B$, $A B$, ſo as to be inclin'd in any Angle, and the Screws B, B , will make it faſt in any Poſition.

Having expoſ'd the Burning-Glaſs to the Sun Beams in ſuch a manner, that the Ray, which comes upon the middle of the Glaſs, is perpendicular to its Surface, ſince all the others are parallel to it, they are collected in a Focus, at a quarter of a Diameter's diſtance from the Glaſs, and there burn violently.

If

If the Diameter of the Surface of the Mirror (as it is in mine) be of about 15 Inches, and the Focus is 18 Inches distant from it, Wood will immediately be in a Flame, and thin Plates of Lead presently melt.

If we consider the Rays that are at some distance from Cd , and parallel to it, those of them that are nearest one another being reflected, will intersect before they come to Cd , and in that

815 Case, that is, *where parallel incident Rays fall obliquely on the Glass, being a little dispersed by Reflexion, they are collected in a Point.*

816 *If the Focus, in which parallel Rays are collected by a concave Mirror, becomes the radiant Point, the Rays, which are but little dispersed, are reflected*

• 813 *parallel to one another*.*

785 From these Properties of a concave Mirror we

817 deduce the Method of representing Objects in a dark Place, much like what was before shewn in

• 711 *respect of a convex Lens*.*

Plate XIII. Fig. 3.] Let there be an Hole F thro' the Wall; let $a b$ be a concave Mirror, so plac'd as to collect the parallel Rays that are perpendicular to the Wall at F ; the Rays coming
• 816 from F in that Direction, are reflected*; and such are the Rays, which being reflected from the external Objects, intersect one another at F .

Let $A F$ be Rays coming from a Point of a distant Object; these Rays are by the Mirror reflected perpendicularly to the Wall; and because Rays coming from a distant Point, and passing thro' a small Hole, may be look'd upon as parallel, these Rays will, after Reflexion, be collected into one Point at a , at the distance of
• 815 the Wall*, that is, in its Surface; where therefore the Point will be represented. In the same manner the Rays which come from a Point thro'

thro' B F, are collected at *b*; which, as it is true with regard to all the Points of an Object, will give the Representation of it upon the Wall; and if the Wall be white, and the Object enlighten'd by the Rays of the Sun, the Picture will appear in very lively Colours.

Experiment 3. Plate XIII. Fig. 4.] In a dark Place cover the backside of the Window-shut with white Paper, an Hole being made in the middle of it little more than half an Inch diameter, so as to answer to an Hole behind it in the Window-shut, over-against which, at a distance no less than of 50 Foot, there are several Objects enlighten'd by the Sun; let a concave Mirror, whose Surface is 15 Inches wide, and which collects parallel Rays at the distance of 18 Inches, be plac'd at that distance from the Window, in such manner, that a Line passing thro' the Center of the Hole, and the Center of the Surface of the Mirror, be perpendicular to the Plane of the Paper and the Surface of the Glass; then the Objects will be represented upon the Paper in a Circle concentric with the Hole, and whose Diameter is equal to the Diameter of the Mirror; you must join to the Hole, on the outside of the Room, an hollow truncated Cone, to exclude the Light which does not come from the Objects to be represented.

Let *b c* (Plate XIII. Fig. 5.) be a concave Mirror, C the Center of its Concavity, A a radiant Point farther distant than C the Center of the Glass; A *b*, A *d*, A *e*, incident Rays, whose reflected ones *b f*, *d g*, *e h*, with their intermediate ones, by their mutual Intersections form the Curve *a a*, which they touch; therefore the Point A appears in that Curve*; and if the Eye be mov'd in the Plane of the Curve, the Appearance will change Place

Place in that Curve. But in all the Planes which may be conceiv'd to pass thro' C A, there is such a Curve, and they all concur in the Line C A, namely, at the Point *a*; therefore in that Point *a*, the reflected Rays are the most abundantly collected, which therefore is call'd the Focus of the Rays coming from A. On the contrary, A will be the
 819 Focus, supposing the Radiant at *a*.*

In this Figure there is only part of the Curve drawn, which is produc'd by one Part of the Line A C; such another Part must be conceiv'd on the other side, and both join in the Focus of the radiant Point.

820 As the radiant Point recedes, the Curve comes nearer the Mirror.

821 As the Radiant comes forward, the Curve recedes from the Mirror, and moves towards the Radiant, till they concur at the Center C; in which, if the Radiant be plac'd, all the reflected Rays
 806 will coincide with the incident*, and the whole Curve will, as it were, be swallow'd up in the Center.

822 If the Radiant is yet brought nearer to the Mirror, so as to be between the Center and the Glass, the Curve will recede farther, and be then beyond the Center; and in the Curve, that Point will recede most of all, in which all the Curves concur which are conceiv'd in several Planes, that is, the Focus of the radiant Point; and that Focus

823 will be at an infinite distance, when the Radiant is distant from the Mirror just the fourth Part of the

813 Diameter of the Sphere*; then also the Curve is
 816 extended in infinitum, and the two Parts which

concur in the Focus of the Radiant are separated; this separated Part is seen at *a a*: (Plate XIII. Fig. 6.) If the Radiant be brought still nearer, the

824 curve Parts decline from one another, because the Rays, such as A *b*, and those nearer it, being reflected,

flected, do not touch the Curve, but become di-
 vergent; that is, those reflected Rays being conti-
 nu'd beyond the Glas, will intersect one another,
and form a new Curve behind the Glas, which has
two Legs, one of which is seen at *a a*; they con-
 cur in the Line *C A* continu'd, namely, at *a*, and
 receding from the Glas, are stretch'd out in *infi-*
nitum. And there is also on each side of the radi-
 ant Point, a Point in the Surface as *d*, which se-
 parates the Rays that form the Curves *a a* and
a a; and the Ray *A d*, being reflected in *d g*,
 touches neither of the Curves, if it be infinitely
 continu'd toward each Part *g, g*, tho' it is con-
 tinually coming nearer to each Curve. If the
 whole Sphere was compleated in respect of the
 opposite Part of the Sphere, the Radiant would
 be beyond the Center, and the reflected Rays
 would form the Curve which we have mention'd
 before, * by which the separated Legs, as *a a*, * 818
 would be join'd. These things thus premised, we
 proceed to explain the Phænomena of concave
 Mirrors.

If the Mirror be enlighten'd by a lucid Body, the 825
Rays which come from all the Points of the Object be-
ing reflected, will form Curves, but are chiefly col-
*lected in the Foci of these Points; * therefore if these* * 819
Foci are in the Surface of a white Plane, there will 826
be upon it a Representation of the lucid Body, as in
the second Experiment of Chap. IX. and that Re-
presentation will be inverted, for the Line which
joins the radiant Point with its Focus, goes thro'
*the Center of the Sphere; * in which therefore* * 819
all such Lines intersect one another; and this In-
tersection is between the radiant Point and the
*Focus, * in which the Point is represented.* * 818

But as the lucid Body is brought nearer to the Mir-
*ror, the Appearance recedes farther, * and in this* * 820
Case becomes bigger.

Experi-

Experiment 4.] Hold a lighted Candle between the Mirror and the Center of the Sphere of which it is a Portion, yet so that it may be more distant from the Mirror than that Center; if then there be a white Plane perpendicular to the Line that passes thro' the Candle and the Center of the Mirror, and this Plane be held beyond the Center, you will have upon it an inverted Representation of the Candle; the proper Place will be found by moving the Plane forwards and backwards; as likewise by this Proportion, viz. As the difference of the Distances of the Candle, from the Center of the Sphere and from the Mirror, is to the fourth Part of the Diameter of the Sphere, so is the Distance of the Candle from the Glass to the Distance required. As the Candle is brought nearer to the Speculum, the Plane must be mov'd farther off, and the Representation will grow bigger.

827 *Plate XIII. Fig. 5.*] *Objects placed beyond the Center, appear between the Glass and the Center;* for all the Points appear in a Curve, as at *a a*;
 818 * *the Images of the Objects will also be diminish'd and*
 828 *inverted,* for they are reduc'd into a narrow Space; and as the Point *A* moves downwards, its Representation will be carried upwards; for the Line *a a* keeps the same Situation in respect of *A C a*, as it is carried round the Center *C*.

Experiment 5. Plate XIII. Fig. 7.] Lest the Representation of the Object should be less vivid, the Mirror is to be included in a Box. If you have a Mirror, whose Surface is about eight
 818 Inches wide, and which is a Portion of a Sphere of a Foot and a half in diameter, shut it up in
 812 a Box *P*, in whose fore-part there is an Hole

of about 6 Inches Diameter, and from which the Glass is distant about 6 or 7 Inches; and let this Opening be turn'd from the Light. Now if any Person, as A, beholds himself at the Distance of about 2 Foot from the Glass; his Face will appear inverted in the Box towards the Hole; and if the Beholder comes nearer, he will see a Head coming out of the Hole.

○ The Representation of a Point, placed in the Center of a Sphere, coincides with the radiant Point itself, and is as it were swallowed up by it.

○ If the Eye be plac'd in that Center, no Object can be seen by it; for then only the Rays, that flow from the Eye, will be reflected to it.

○ If the Object be between the Center and the Point, in which parallel Rays are collected after Reflexion, the Object will also appear without the Glass, at a greater Distance from the Glass than the Object itself; the Representation is inverted, which is proved in the same Manner as in N^o. 828; and magnified, because it is farther remov'd from the Center, than the Object itself is distant from it; for the Representation recedes from the Center in infinitum; whilst the Object goes thro' the fourth Part of the Diameter of the Sphere.

If the Object be less distant from the Mirror than the fourth Part of the Diameter of the Sphere, according to the different Situation of the Eye, the Object appears either before or behind the Glass.

vd Plate XIII. Fig. 6.] If the Eye be so plac'd, that those Rays may come to it which form the Curve *a a*, as towards *f*, it will see the Appearance of the Objects towards the Glass magnified; because the Curves, as *a a*, which belong to several Points, are diverging.

○ If those Rays come to the Eye which form the Curve *a a*, the Object will appear without the Glass: And in both Cases the Representation is

erect: for as the Point A ascends or descends, the Curves as and aa , in which it is represented, are agitated with the same Motion.

834 If the Eye be in the Point, in which the reflected Rays, that belong to each Curve, intersect one another, as in O, the Appearance of the Object will be double.

It is plain in every Case of the Appearance, that the Points have not the same Relation to one another, as the Points of the Object; and therefore that a Concave Mirror never represents the Objects exactly. But the most irregular Representation of all is that which is in such Lines, as as .

836 There are also cylindric Mirrors convex and concave; these in one Respect are plane, and in another Respect spherical; and therefore the Representation of the Objects is very irregular: which Irregularity, since it depends upon a regular Figure, may be so determin'd, that Pictures may be drawn, which, tho' truly irregular, will in such a Glass appear regular by Reflection, in a determinate Situation of the Eye.

C H A P. XVI.

Of the Magic Lantern.

838 THERE are several Machines made by the Combination of Mirrors and Lenses, which afford useful and pleasant Appearances; whose Explanation may be easily deduced from what has been said.

Among many other I shall only chuse to explain one, in which Figures, that are painted upon small Pieces of Glass, are represented monstrously large upon a white Plane. This is a

Phano-



Fig. 7.

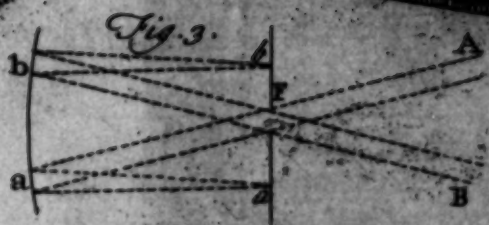


Fig. 3.

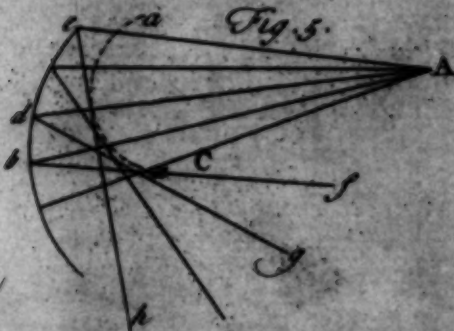


Fig. 5.

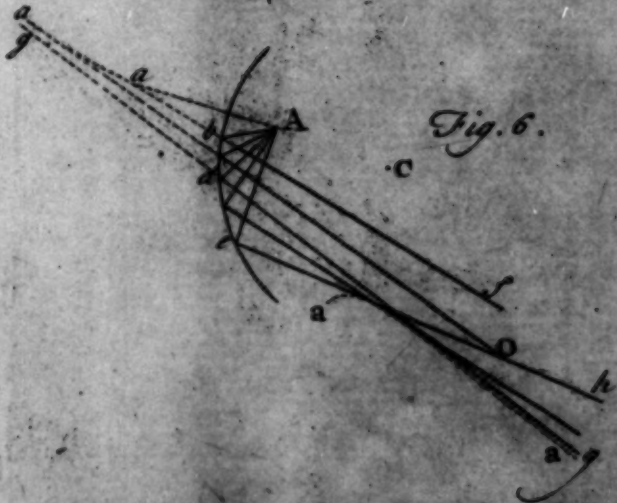


Fig. 6.



Fig. 2.



Fig. 4.



Page 10

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Phænomenon wonderful enough to deserve a particular Explanation; the Instrument that performs this is call'd a *magic Lantern*, which Optic Writers have not altogether pass'd by, but yet have not sufficiently explain'd.

Plate XIV. Fig. 1.] Let there be a wooden Box about a Foot and half long, 14 Inches high, and as wide; there must be a concave Mirror S of 8 Inches diameter, and a Portion of a Sphere of 18 Inches; this Mirror is fixed to a Foot which moves upon Rulers along the Length of the Box.

There is also in this Box a Lamp L, sustain'd by a wooden Foot, which is moveable long-wise between two Rulers, in the side of the Box; the Pipe of the Lamp stands forward, in such manner that the Center of the Flame is over-against the Center of the Surface of the Mirror; this Flame is made up of four little Flames, which by touching one another, make one square Flame two Inches wide.

In the top, or upper Plane of the Box, there is an oblong Hole, which has a Cover that slides to two Grooves, or between two Rulers or Ledges; thro' this Cover passes the Chimney C, which (as you see in the third Figure) stands up about half a Foot above the Box; the Chimney is moveable with the Cover, whilst the Opening remains shut, that the Chimney may be always over the Lamp.

In one of the little Sides of the Box, which is over-against the Mirror, there must be a round Hole about five Inches wide, which must have in it a convex Lens of Glass of the same bigness V, convex on both Sides, which are Portions of a Sphere of one Foot diameter; the Axis of this Glass being continu'd to the Surface of the Mirror, will be perpendicular to it, and fall upon

its
H²

its Center, as likewise to the Plane of the Flame, thro' whose middle Point it also passes.

This Hole is shut and open'd by a Plane moveable in a Groove, which is mov'd by a Cylinder that stands out of the Box at E.

To this Hole without the Box answers the Tube T, whose Length and Diameter is of about six Inches, at the end of which there is a Ring, in which the second Tube *t* moves, of about four Inches diameter, and five or six Inches long.

In the lesser Tube there are two Lenses, the first in that End which is thrust into the Tube T, and it is of the same Convexity as the Glass V, and three Inches and a half diameter; the second Lens is three Inches from the first, and flatter, being terminated by Portions of a Sphere of four Foot diameter; between these Lenses, at the distance of about an Inch from the second, there is plac'd a wooden circular Stop or Aperture, which shuts up all the Tube, except an Hole of an Inch and a quarter diameter in the middle of the Wood.

The Objects that are to be represented, are to be painted upon a flat thin Piece of Glass, which must be mov'd without the Box over-against the Glass V, between it and the Tube T, the Picture being in an inverted Position; if these Pictures are round, they may be of five Inches diameter; that they may be mov'd easily, they are put into flat Boards, three in a Board; the Picture also may be painted upon long Glasses, which may be successively made to slide before the Glass V.

Plate IV. Fig. 3.] This whole Box stands upon a Frame or Foot, made so that it may be fix'd at different Heights; there are flat Pieces of Wood fix'd to the Box at bottom, which slide in Grooves in the Frame; each of them has a Slit in it, so that

that the Box may be made fast at any desired Height, by the help of Screws join'd to the Frame, and moveable in the Slits.

The whole Machine is plac'd at the distance of 15, 20, or 30 Feet from a white Plane, which Distance must be different, according to the bigness of the Plane; for this Distance may be equal to the Length of the Plane; the Box must be just at such an Height, that the Tubes in the side of the Box may be exactly opposite to the middle of the Plane.

The Lamp being lighted, the Box must be shut, and the Figures which are painted upon the Glass will be represented upon a white Plane; by moving backwards and forwards the Tube that has the two Lenses in it, you will find the proper Position of the Glasses required to give a distinct Representation. As for the Disposition of the several Parts of the Machine, which immediately serve for exhibiting this Appearance, we shall here more particularly explain.

Plate XIV. Fig. 2.] The Parts in this Figure 838 are shewn separately; SS is the Mirror, *ll* the Flame which consists of four Flames in the Line *ll*; VV is the Glass V of the first Figure; OO is a Picture painted upon a flat thin Piece of Glass; *a a* the biggest Lens; *d d* the flattest Lens; *b b* the wooden Stop between the Lenses; *f* the Aperture or Hole in the middle of the wooden Circle.

These things being dispos'd as has been already explain'd, and as may be seen in the Figure, the Rays which proceed from a Point of the Picture OO, by going thro' the Lens *a a*, become less diverging, and fall upon the Lens *d d*, as if they came from a Point more remote*; from this * 669 Lens they go out more converging*, and are * 699 collected upon the Surface of the white Plane,

where they exhibit the Point of the Picture that
 • 711 is painted on the Glass *; this Picture is illuminated both by the Rays that proceed from the Flame *ll*, and by the Rays reflected by the Mirror *SS*.

839 For the Perfection of this Machine, it is required, 1st, that the Figure *OO* be enlighten'd as much as possible; 2^{dly}, that it be equally enlighten'd in all its Points; 3^{dly}, that all the Light by which every Point of the Picture is enlighten'd, go thro' the Lenses *aa* and *dd* to the white Plane, and serve to make the Representation; 4^{thly}, that no other Light but that go out of the Box, lest the Representation should be less lively by reason of extraneous Light.

The first Requisite depends upon the Bigness of the Flame, and of the Mirror, and of its Concavity; the more concave it is, the nearer it is to be brought to the Flame, and then the more Rays will be intercepted and reflected; yet Care must be taken that the Mirror (which may be made of very good Glass) be not too much heated.

When the Flame and Mirror are so contriv'd, that the Picture is the most enlighten'd that it can possibly be, and every where equally enlighten'd, the Flame and Mirror must be so plac'd, that the inverted Image of the Flame shall fall just
 • 826 upon the Picture *. Now as the Representation
 • 826 of the Flame can be increas'd and diminish'd *, the Mirror and Flame must be so disposed, that the Representation of the Flame shall cover the whole Picture upon the Glass, but so as not to exceed it; for then the Picture is as much enlighten'd by the reflected Light as it can be, and all its Points are equally illuminated; the direct Light also does pretty near equally fall upon all the Points of the Picture; this Light would indeed be increas'd by bringing the Flame nearer,
 but

but the reflected Light would be diminish'd, and the Diminution of this last would be greater than the Increase of the other.

The Glass *V V* serves to inflect the Light, by which the Picture *O O* is illuminated, before it comes to the Picture; by which Inflection all the Light comes to the Lens *a a*, and serves for the Representation on the white Plane.

All the Light that is of use for this Representation goes thro' the Hole *f*, and the Rays coming from different Points, intersect one another there; wherefore the Picture upon the Glass, which is plac'd inverted, is represented erect upon the white Plane; by the Ring *bb*, all the Rays which do not serve to form the Representation, are intercepted, lest they should enter the Room, and make the Picture less distinct; this Ring or Aperture also intercepts those Rays by which one Point is more enlighten'd than another, whereby the Light, which (from what has been said) is equally enough diffus'd, is yet made more equal; but unless the Stop or Aperture *bb* be just where the Rays intersect, it does a great deal of Mischief.





BOOK III.

PART III.

Of Opacity and Colours.

CHAP. XVII.

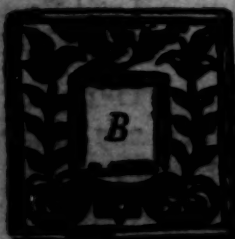
Of the Opacity of Bodies.

DEFINITION.

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ODIES that transmit the Light are called transparent; such are all Mediums*, except a Vacuum.

There is no Body, whose smallest Parts are not transparent; no one who is used to Microscopes will doubt of this. There are some Parts of Metals, which, tho' very small, do not transmit the Light; but if they be dissolv'd in Menstruums; that is, if they be divided into much less Parts, they become transparent. One may also prove by a very easy Experiment, that Light cannot go thro' several opaque Bodies.

Experi-

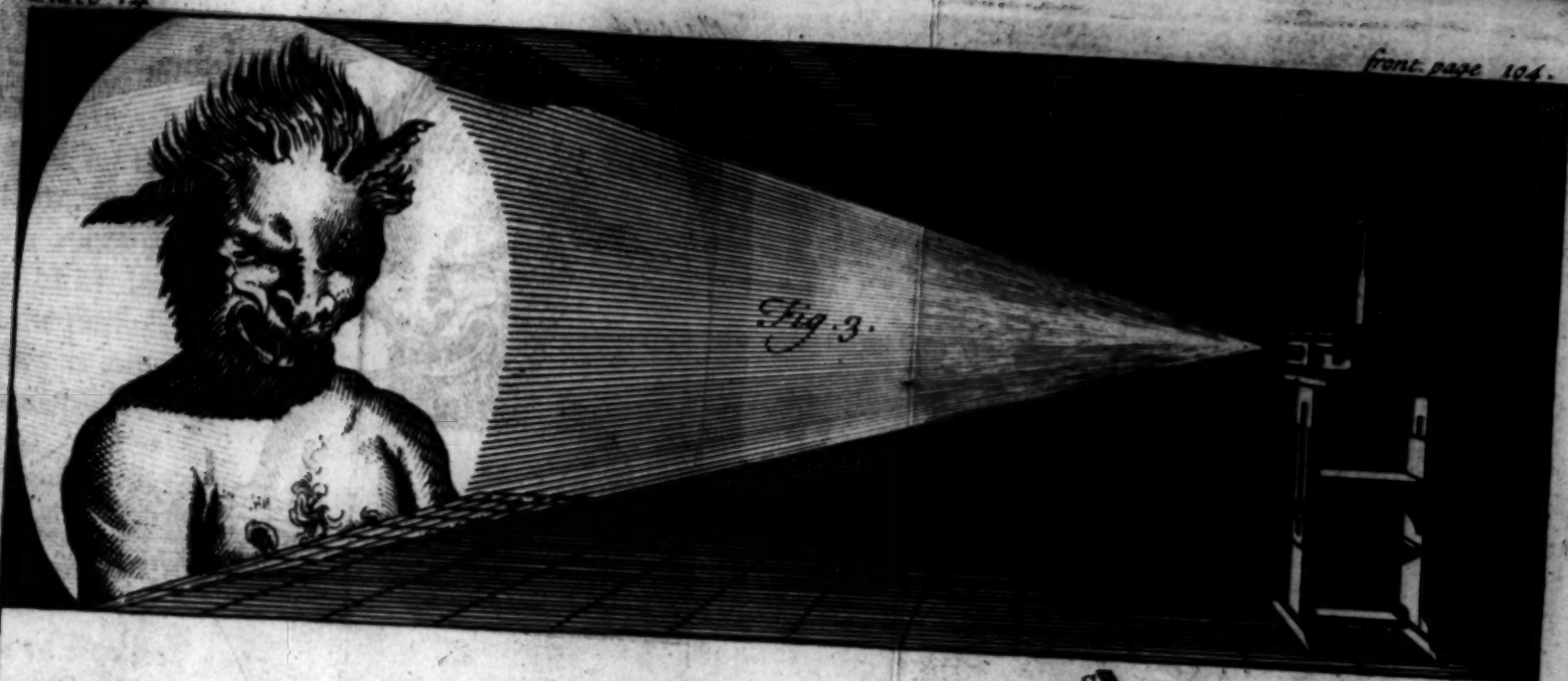


Fig. 2.

l
...
v

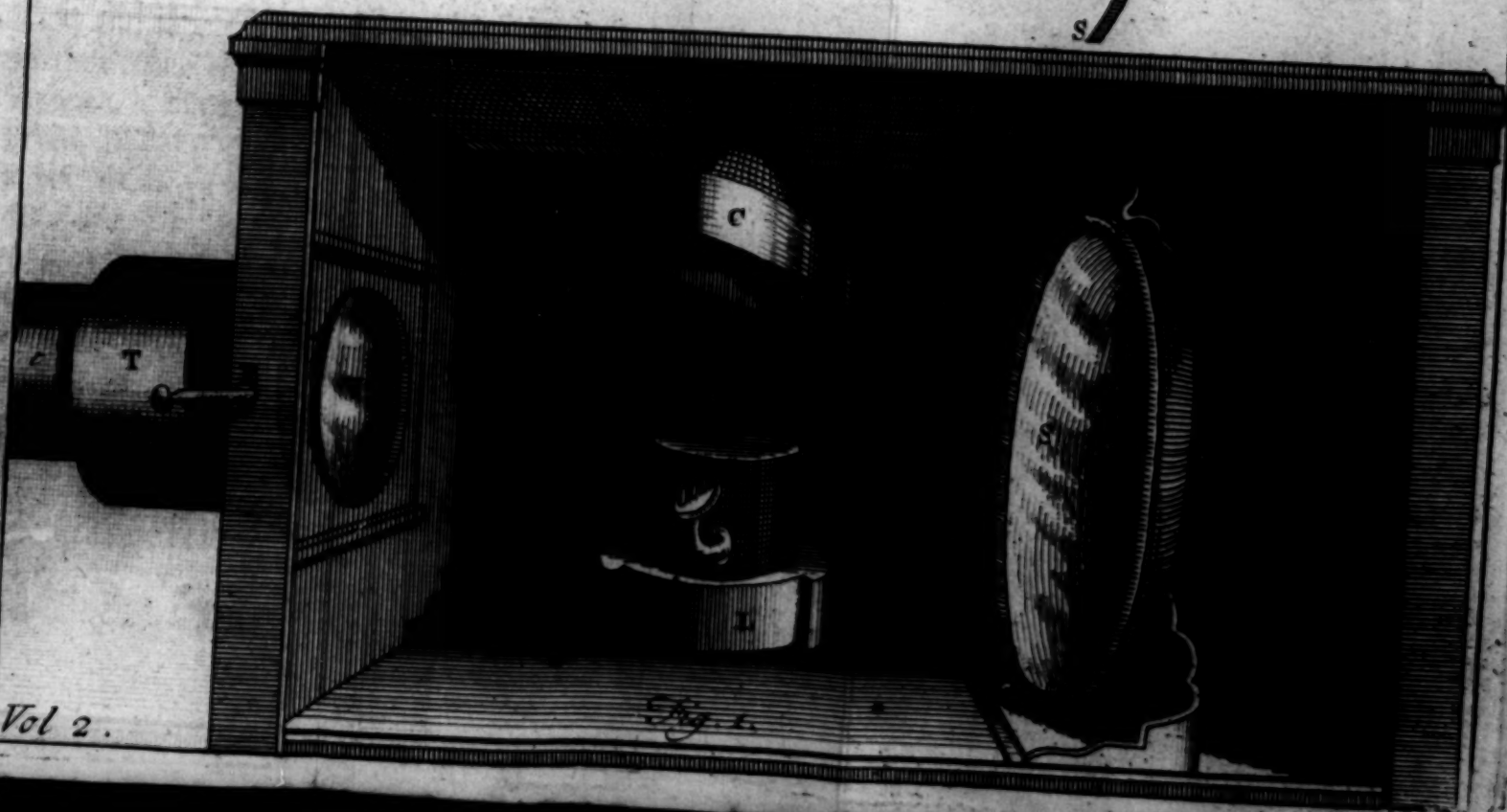
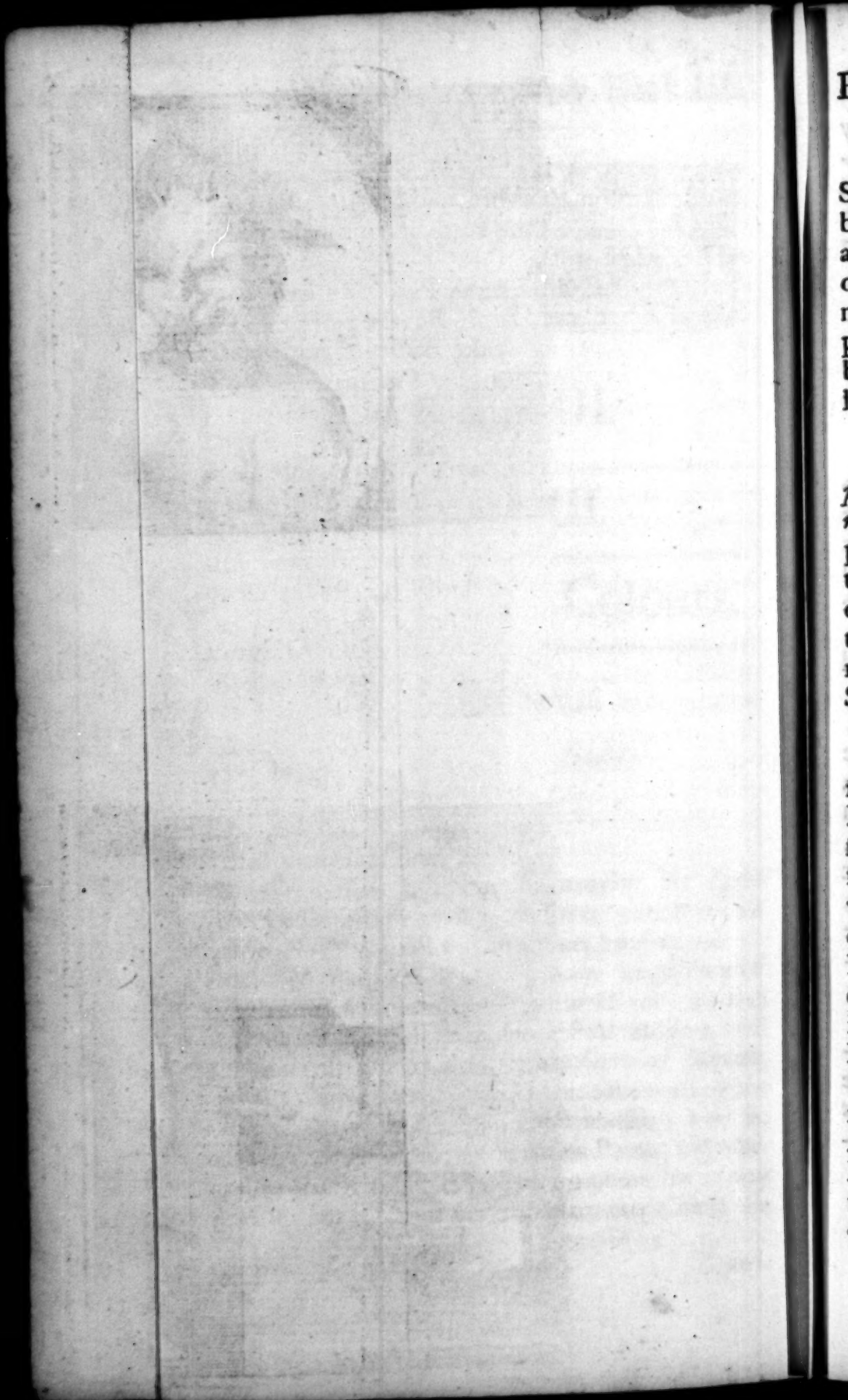


Fig. 1.



Experiment 1.] In a dark Room, in which the Sun's Light comes in thro' an Hole, let that Hole be cover'd with a thin Plate of an opaque Body, and the Light will go thro' it; a Piece of Wood of the thickness of the tenth Part of an Inch, does not intercept all the Light. But the perfect Transparency of Parts in opaque Bodies is not prov'd by this Experiment, for that Transparency is only in very small Parts.

Opacity does not (as is commonly imagin'd) *happen in Bodies, because the way thro' which the Light might pass is stop'd by Particles of Matter*; for Light passes thro' all the smallest Parts of Bodies; neither is such an Interception of Light of any use to cause Opacity: It is required for Opacity that the Light should be reflected and deflected from a right Line, for which there is only required the Separation of two Mediums*.

* 631
796

Let us conceive a Body, consisting of very small Parts, perfectly transparent (such as are the Particles of which Bodies consist*) and separated* 841 from one another by Pores, and that those Interstices are either void, or fill'd with a Medium whose Density is different from that of the Particles; if Light enters such a Body, it will every Moment fall upon a Surface dividing Mediums differing in Density, therefore it will undergo innumerable Reflexions and Refractions in that Body*, so as not to be able to get through it; * 631 therefore we see that *Opacity depends upon the Pores*, 796 for if you fill the Pores with a Medium of the same 843 Density as the Particles of the Bodies themselves, the Light will undergo no Reflexion or Refraction in the Body, but pass directly thro'*, and the Body * 841 will be transparent. 631

Tho' 697

Tho' we cannot make Experiments, whereby to fill the Pores of the Body with a Medium exactly of the same Density as the Particles, yet the following Experiments will clearly enough prove Sir *Isaac Newton's* Doctrine concerning Opacity.

Experiment 2.] Paper becomes more transparent when moisten'd with Water, for it fills the Pores, and differs less in Density from the Particles than Air does. Oil has the same Effect.

Experiment 3.] Take a Piece of Glass two Inches thick, and take several Plates of the same Sort of Glass laid upon one another, yet so as not to be quite two Inches thick, and you will find that these will be less transparent than the solid Piece, because of the Air between the Plates, which does not get into the solid Piece where all the Parts cohere.

Experiment 4.] Take twelve Plates of the same Glass, as near as may be of the same thickness; let six and six of them be laid together; if you take the least transparent of those two Particles, and having dipt it in Water, take it out again, it will become more transparent than the other; because the Water, which in that Case fills the Interstices between the Planes, differs less in Density from the Glass than Air does.

What has been said of Opacity is farther confirm'd, and put out of all Doubt by innumerable
844 Experiments, by which *Bodies perfectly transparent become opaque, by the Separation of their Parts*, without the Intervention of any opaque Body.

Experi-

Experiment 5.] Let any perfectly transparent Liquid, that may be chang'd into Froth, be shak'd till it be full of Bubbles, it will immediately become opake, by reason of the Interstices that are fill'd with Air.

Experiment 6.] Turpentine and Water are transparent Bodies, but when mix'd they become opake.

Experiment 7.] Water and Oil, by being mix'd together, become opake, tho' singly they are transparent.

Experiment 8.] Tho' Glas be transparent, yet if it be reduc'd to Powder it becomes opake, as it also does when it is crack'd.

We clearly see in all these Cases that Opacity is produc'd, because there is a Medium of different Density between the transparent Parts; which may also be observ'd in the Clouds, which are opake on account of the Air interpos'd between the Particles of the Water.

If we add to this what is said in the 22d Chapter following, concerning the Colours of thin Plates, we shall have new Experiments, by which alone it is fully prov'd that Bodies intercept the Light, because they consist of very thin Particles, encompass'd with a Medium differing from them in Density.

Some opake Bodies reflect a little Light, and the rest of the Light, by innumerable Divisions which it undergoes in the Reflexions and Refractions above-mention'd, is extinguish'd in the Body; such are black Bodies; if there were any Bodies perfectly black, they would reflect no Light; 845
for all Bodies, when no Light falls upon them,
and

and therefore they reflect no Rays, appear black.

Other opaque Bodies appear to have various Colours; some transparent Bodies are also ting'd with Colours; and whence those arise we are now to examine.

C H A P. XVIII.

Concerning the different Refrangibility of the Sun's Rays.

Bodies appear differently colour'd, tho' they be enlighten'd by the same Rays of the Sun which are reflected by them. Besides these there are several Phænomena of Light relating to Colours, not to be overlook'd.

846 In these we are to observe three things. 1st, The Rays are to be examin'd. 2^{dly}, Their Reflection is to be observ'd. 3^{dly}, We are to enquire into the Constitution of the Surfaces of Bodies differently colour'd.

847 As to the Rays, the first Property to be taken notice of here is, that *in the same Circumstance all Sorts of Rays do not undergo the same Refraction.*

DEFINITION I.

848 *The Rays which undergo this different Refraction, are said to have a different Refrangibility, and those are said to be most-refrangible, which are most inflected by Refraction.*

DEFINITION II.

849 *Those Rays are said to be homogeneous, which do not differ from one another in Refrangibility.*

DEFINITION III.

And those are called heterogeneous, which under the same Circumstances are not equally inflected by Refraction.

Plate XV. Fig. 1.] Between A B and C D let there be a Sun Beam made up of an innumerable Quantity of parallel Rays; these are not equally refracted; for if they fall obliquely on the Surface B D of the denser Medium, some of them are refracted between B E and D G, and are moved according to that Direction in the denser Medium; others are more inflected, and direct their Motion between B F and D H according to the Direction of those Lines; and indeed no Direction can be conceiv'd between the Mediums, along which the Rays do not move in every Point between B and D; so that the least Beam whatever is, by Refraction, divided into an innumerable Quantity of Rays; because every Sun Beam, however small, is heterogeneous, and consists of an innumerable Quantity of Rays refrangible in all Degrees of Refrangibility.

The above-mention'd parallel Rays falling upon a plane Surface, by Refraction are mov'd between B E and D H; which Lines diverge from each other, and being continued, are more and more separated; so that the Rays above-mentioned are dispersed by Refraction. In N^o. 653. we have consider'd the Rays as homogeneous, as also every where in the foregoing Part; the difference of Refrangibility is small enough in the Rays of the Sun, not to have been worth observing in the foregoing Propositions. Besides, we were first to examine what happens in homogeneous Rays; and what must be chang'd in the Propositions, upon account of the different Refrangibility, will easily appear.

That

852 That this Refrangibility of the Rays may be made visible, the above-mention'd Divergence must be increased; which will be performed, if the Rays above-mentioned fall upon the Surface E H, which terminates a denser Medium, and separates it from the rarer Medium; and which makes any Angle with the Surface B D, and is so inclin'd to it, that the Rays, which are more refrangible, fall more obliquely upon it than those that are less refrangible; so that the former going into a rarer Medium, are more turn'd out of the way, and diverge more from the others upon a double Account, that is, both upon the account of a greater Refrangibility, and of a greater Inclination. The Rays between B E and D G, which are the least refrangible, being refracted a second time, continue their Motion between E I and G L, the others between F M and H N; in which Case, if these Rays fall upon a Plane at the distance of 15 or 20 Foot, those that are most refrangible will be separated from those that are least refrangible, and the whole intermediate Space is enlighten'd with Rays endow'd with a mean Refrangibility.

Experiment 1. Plate XV. Fig. 2.] Let there be an Hole in the Plate of a Metal in the Window-shut of about a quarter of an Inch, thro' which a Sun Beam enters the darken'd Room; let this Beam fall upon a triangular Prism of Glass A A, in such manner that it may be wholly reflected by the lower Surface (see the *Expr.* of N^o. 789.) By the two Refractions which the Light undergoes, Rays that are differently refrangible do not diverge, and being reflected, come upon a second Prism B B, which is likewise of Glass, and triangular, in the same manner as if they had come directly from the Sun. These two Prisms are moveable

moveable about their Axes, as we before explain'd *; the first, A A, is laid upon a piece * 197 of Wood fasten'd to it, yet so as not to hinder it from turning about its Axis; this piece of Wood is fastened to a three-legg'd Staff with a Ball and Socket, such as is used in practical Geometry. The second Prism, B B, is laid on a Frame or Stand S, in whose opposite Sides there are several Slits that answer one another; by these the Prism is sustain'd at different Heights, but always horizontally, the End of the Axis being plac'd in correspondent Slits.

The Light that comes to the Prism B B, moves perpendicular to its Axis, and passes thro', as is demonstrated in *Fig. 1.* in which B D and E H represent the Sides of the Prism; the Light is also equally inclin'd to each side, which will happen by moving the Prism about its Axis; for the Light of the Sun (as you move the Prism) will ascend, and then descend again; and the Situation required is that in which the Light is highest of all: now both the Prisms are to be so disposed, that in this Case the Light may go horizontally out of the Prism B B. This horizontal Beam, at the distance of 15 or 20 Foot, must fall upon the Board T, which is covered with white Paper, and has such a Foot, that upon it it may be raised and fixed at different Heights; the Rays come diverging to the Paper, and upon it form the oblong Image R V, terminated at the Sides by parallel Lines, but semicircular at R and V.

If the Rays of the Sun that go thro' the round Hole, fall upon a Plane at a certain distance, they will form a bright round Spot, so much the greater as the Plane is more distant from the Hole, which arises from the Rays that come from the Sides of the Sun, which making an Angle

Angle with those that come from its Center to the Hole, and intersecting them in the Hole, gives the Image of the Sun upon the Plane.

If the Rays did not go thro' the Prism B B, and fall upon the Plane at the distance of the Board T, the Image of the Sun would have its Diameter equal to the Breadth of the Image V R; which
853 Breadth is not alter'd by the Refraction, because the Rays enter the Prism perpendicular to its Axis, and in respect of the Breadth of the Image, are not inclined to it. But as in another respect the Image of the Sun is oblong, it follows plainly from thence, that all the Rays have not undergone the same Refraction; for homogeneous Rays, tho' refracted, will give a round Image of the Sun. The least refrangible Rays go to R, and the most refrangible to V; and the whole Image R V is terminated with Semicircles at R and V, because the whole Image consists of circular Images. Now between R and V there are innumerable Quantities of circular Images, made by Rays of all possible Refrangibilities; otherwise the Image R V would not be terminated with rectilinear Sides.

854 In most Experiments we have said that the Light is let into the darken'd Room thro' a Slit or an Hole, which we leave to the Contrivance of the Workman; our Method was the following.

Plate XV. Fig. 3.] We made an Hole 4 Inches square in the Window-shut, which on three Sides had Ledges of Wood, A B, B C, C D, so rabbetted as to make Grooves to keep in the square flat Piece of Wood Q L, which is six Inches long, and six Inches wide; it may be drawn out, and

and several Sorts of them serve for several Experiments. That which we use in the following *Experiment* has a Hollow and a Hole in the middle behind to contain a convex Lens, which is the Object-Glass of a Telescope of 16 or 20 Foot; the round Hole in the middle mark'd *f* is above half an Inch in diameter, thro' which the Rays of the Sun passing through the Glass, enter the Room. In the fore-part also the Board *Q* is hollow'd, but not in the middle; this Hollow contains the brass round Plate *L*, which towards its Circumference has an Hole at *f*, which Hole is equal to the Hole in the Board *Q* itself, and agrees with it. Together with *L* there is another concentric Plate, which is less, and moveable about the Center; this Plate has several unequal Holes, which successively come to *f* as the Plate is turn'd round, so that you may at Pleasure let the Light enter a dark Place thro' a bigger or a lesser Hole; which in many Experiments, that are made without the Lens above-mention'd, is of good use. The *Pin m*, join'd with the Plate, serves to turn it by.

Experiment 2.] Let in a Sun-Beam thro' the Lens above-mention'd, thro' an Hole a quarter of an Inch wide, into the darken'd Room; by the Rays thus let in, at the distance where parallel Rays are collected, the Sun is very exactly represented, and its Image terminated with exactly describ'd Bounds; for the Rays that come from the several Points of the Sun, which by reason of its immense distance may be look'd upon as parallel, are collected in one Point at that distance.

Plate XV. Fig. 2.] Now if with these Rays you make the Experiment above-mention'd, the several Images made by the homogeneal Rays,

supposing the Board at a just distance, are exactly terminated; and therefore the oblong Image R V, which is made of all these Images, is likewise regularly terminated.

This Experiment will succeed in the same manner, if the Rays pass thro' a Prism made of any Body which is denser than Air.

Experiment 3. Plate XV. Fig. 2.] Let a triangular Prism be made of Wood and two Glass Planes, and filled with Water, such as is represented at B B (*Plate XVII. Fig. 3.*) If you make use of this Machine instead of the Prism B B in this *Fig.* the Experiment will succeed in the same manner; and in passing thro' Water, heterogeneous Rays in the same manner are separated by Refractions.

Experiment 4.] If any Person standing 15 or 20 Foot off, looks at the Hole thro' which the Light is let in, it will appear round; but if before his Eyes he hold a triangular Glass Prism, or the watery one mention'd in the last Experiment, so that the Rays coming from the Hole (after such Refractions as the Light suffers in the foregoing Experiment) reach the Eyes, the Hole will appear oblong. The Situation of the Prism will be found, if holding it horizontally with one Edge (or the refracting Angle) upwards, it be a little mov'd about its Axis, so as to make the Image of the Hole to ascend and descend; and the Prism be held fast in that Position in which the Hole appears most depress'd.

This Experiment, as well as the foregoing ones, proves the different Refrangibility of the Rays; for by the homogeneous Rays of each Refraction, the Hole appears in the imaginary Foci of the Rays coming from the several Points of the

the Hole *, which Image is round ; the Rays * 737 which undergo a different Refraction, enter the Eyes in different Directions ; and you have several Images, all which form the oblong Image, which is really seen.

But that *this different Refrangibility does not de-* 855 *pend upon the reflecting Medium, but the different Constitution of the Rays themselves*, is prov'd ; because those Rays, which in one Case undergo the greatest Refraction, are in any Refraction turn'd out of the way more than any.

Experiment 5. Plate XV. Fig. 4.] All things being dispos'd, as in the first Experiment, at any distance from the Prism B B, let the oblong Image of the Sun fall upon the vertical Prism C C, which is also moveable about its Axis ; as the Rays go thro' it, let it be turn'd about its Axis, and left fix'd where the Rays are least of all turn'd out of the way by Refraction thro' the Prism ; in that Case the Rays are refracted in the same manner thro' this Prism as thro' the first, but they are not dispos'd the same way, for that would form a square Image ; but here the Image keeping the same Breadth, is inclin'd at R V, the Rays at V being mostly turn'd out of the way, as in the Refraction thro' the first Prism B B.

*The Demonstration, before given *, of the con-* 856 *stant and settled Ratio between the Sines of the Angles * 639 of Incidence and Refraction, may be referr'd to any homogeneous Rays ;* but considering the different Refrangibility of the Rays, this Proportion varies, as it follows from the Experiments of this Chapter.

But *the Refrangibility in all the different Sorts of* 857 *Rays is every way unchangeable*, as will appear by the Experiments to be mention'd hereafter.

C H A P. XIX.

Concerning the Colours of the Rays, and their Unchangeableness.

858 **T**HE different Refrangibility of the Rays goes along with the different Colours; and every Sort of Rays, as they are more or less inflected by Refraction, have a particular Colour of their own, and which is wholly unchangeable.

In respect of the Colours, the same thing is to be observ'd as has been taken notice of in regard to other Sensations⁵⁰²; the Colours are Ideas^{580, 6} which have nothing common with the Rays, by which they are excited; therefore we must define what we mean by colour'd Rays and colour'd Objects.

DEFINITION I.

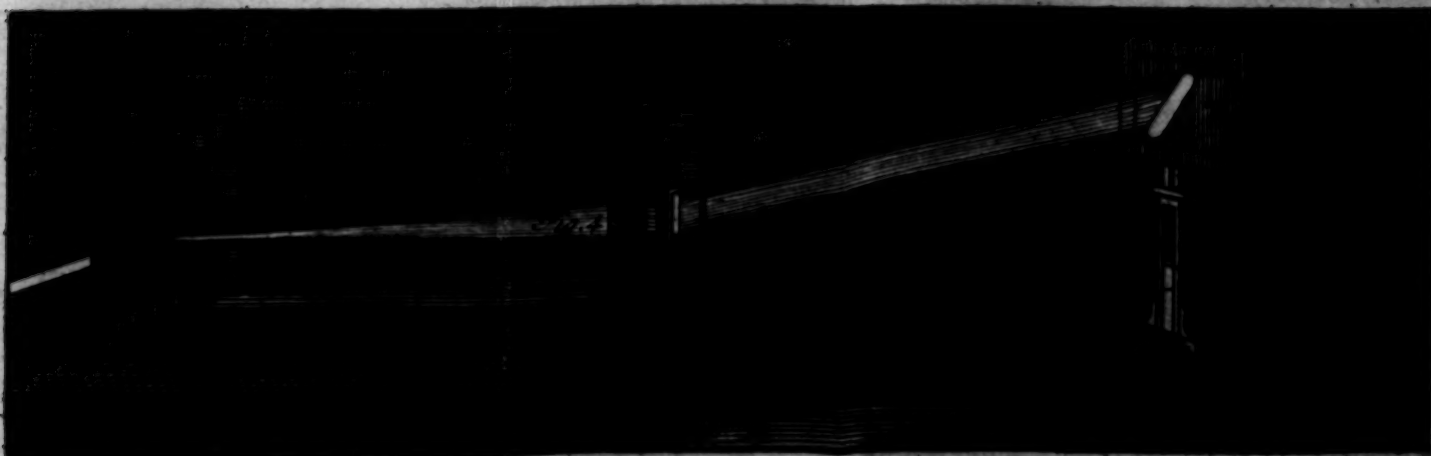
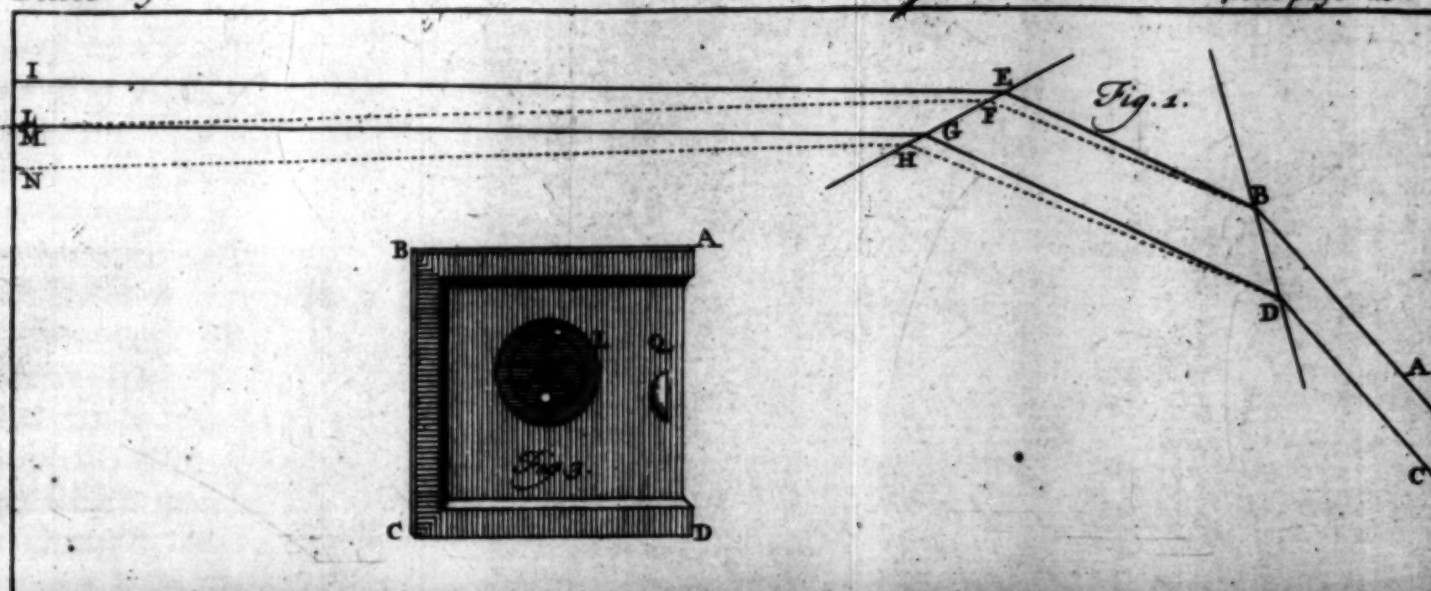
859 An Object is said to be of such a Colour, whose Idea is excited in the Mind by the Rays reflected from the Object.

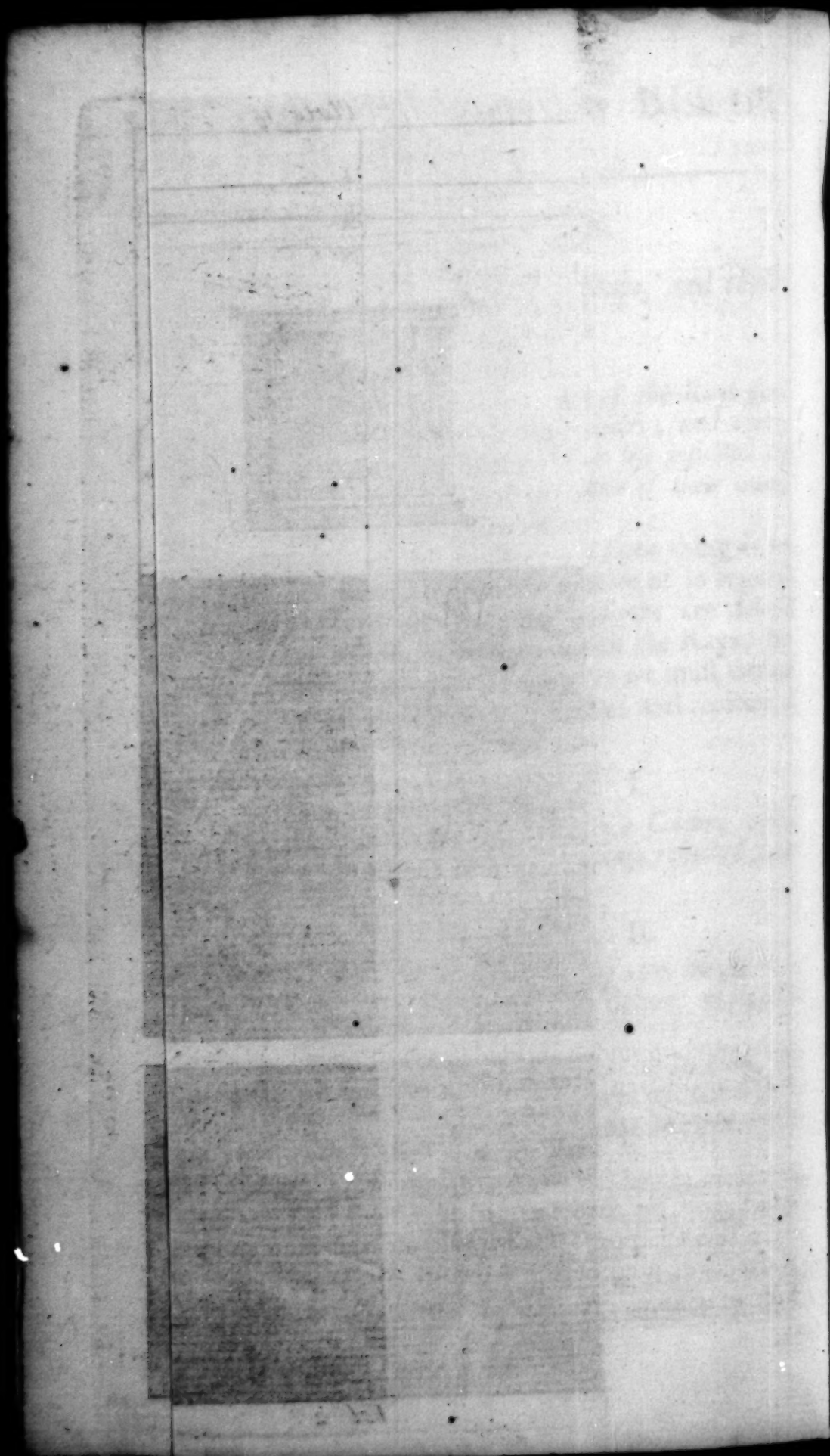
DEFINITION II.

860 Homogeneous Rays, which striking upon the Retina, excite in our Mind an Idea of any Colour, are called Rays of that Colour.

We have said that the Rays excite an Idea, by which we understand that the Rays put the Fibres into a trembling Motion, and that Motion occasions an Idea in the Mind.

[Plate XVI. Fig. 1.] It is very evident from the Experiments of the foregoing Chapter, that Rays of different Refrangibility have a different Colour; for those Experiments shew, that the Image of the Sun, which is oblong, is tinged with different





rent Colours; the Rays which are least turn'd out 861
of the way by Refraction are red, the rest of the Colours are in the following Order; namely, Orange, Yellow, Green, Blue, Purple and Violet; of which last Colours are the Rays that are most refracted. The oblong Image of the Sun above-mention'd, as has been said *, is made up of an innumerable * 853 Quantity of round Images; if their Diameters be diminish'd (which is done by intercepting the Sun's Rays, so that only those that come from the Center of the Sun shall pass thro' the Prism) the Centers of the particular Images which make up the oblong Image will not be chang'd, and therefore the Length of the Image *a b* between parallel Lines is not chang'd; and this alone would remain the same, if the Breadth of the Image should be infinitely diminish'd; and therefore this Length alone is to be consider'd in determining the Limits of the Colours in the Image itself; these Colours are represented in this Figure by the Letters *a, b, c, d, e, f, g, h*; and the Number which is set down against each Colour, denotes the Space taken up by it in the Image, the whole Length of the Image being divided into 360 Parts.

If the Breadth of the oblong Image of the Sun be diminish'd, the heterogeneous Colours are more separated 862
in the Image, because there are fewer particular Images confounded in the several Points, where Rays of different Refrangibilities differ little from each other.

The Colour of any Ray, as also its Refrangibility, 863
cannot be chang'd by any Refractions or Reflexions, or mingling of the Rays together.

In this Chapter I shall speak of Reflexion and Refraction, and of the Mixture of Rays in the following one.

864 That the Refrangibility is not chang'd by Refraction, is prov'd by the 5th Experiment of the foregoing Chapter, which may also be referr'd to Colour, but is more clearly evinc'd by the following Experiment; concerning which it is to be observ'd (as may be also said of what follows) that the Experiments are to be made with Prisms of clear Glass, free from Veins, for they occasion the Light to move irregularly, and the Rays are not duly separated by the Refractions.

Experiment 1. Plate XVI. Fig. 2.] Every thing being as in the first Experiment of the foregoing Chapter, you must make the Experiment with a Sun Beam going thro' an Hole of half an Inch diameter; the Frame or Stand S, in which the Prism is, must be such, that the little flat Board t, to be us'd in this Experiment, may move between its Sides; this Board has an Hole F of one 8th of an Inch Bore, thro' which the Light refracted by the Prism is transmitted, whereby the Rays in the oblong Image are better separated from one another, the Rays being mostly intercepted; this oblong Image R V, at the distance of 10 or 12 Feet from the Prism, falls upon the Board t of a Stand like the former, in which Board also there is a small Hole F like that in the first, thro' this the Rays pass upon a second Prism laid upon this Stand, and are refracted in the same manner as in the first; by moving the first Prism a little about its Axis, the Image R V ascends or descends, whereby the different Rays are successively transmitted thro' the Hole; in every Case, the Rays refracted thro' the second Prism, and striking upon the flat Board T, which is cover'd with white Paper, are not dispers'd at H; but the Image is round, if the Rays fall perpendicularly upon the Paper, and also of the same Colour as the Rays falling

falling upon the second Prism; yet the Image H is so much the more lifted up, as the Rays by Refraction thro' the first Prism are more turn'd out of the way; that is, those that are most refrangible in the one Case, do also suffer the greatest Refraction in the other.

It will appear also by the following Experiments, that the Refrangibility and Colour are not chang'd by Reflexion. 865

Experiment 2.] The Rays which, for Example, make the red Part of the oblong colour'd Image, whatever Body they are reflected from, are always red; that is, all Bodies appear red in that Light; in the Violet Light they are Violet; Green in the Green; and so on in other Colours.

This will appear by trying it with Vermilion, Orpiment, blue Bice, or Cloth of any Colour, &c.

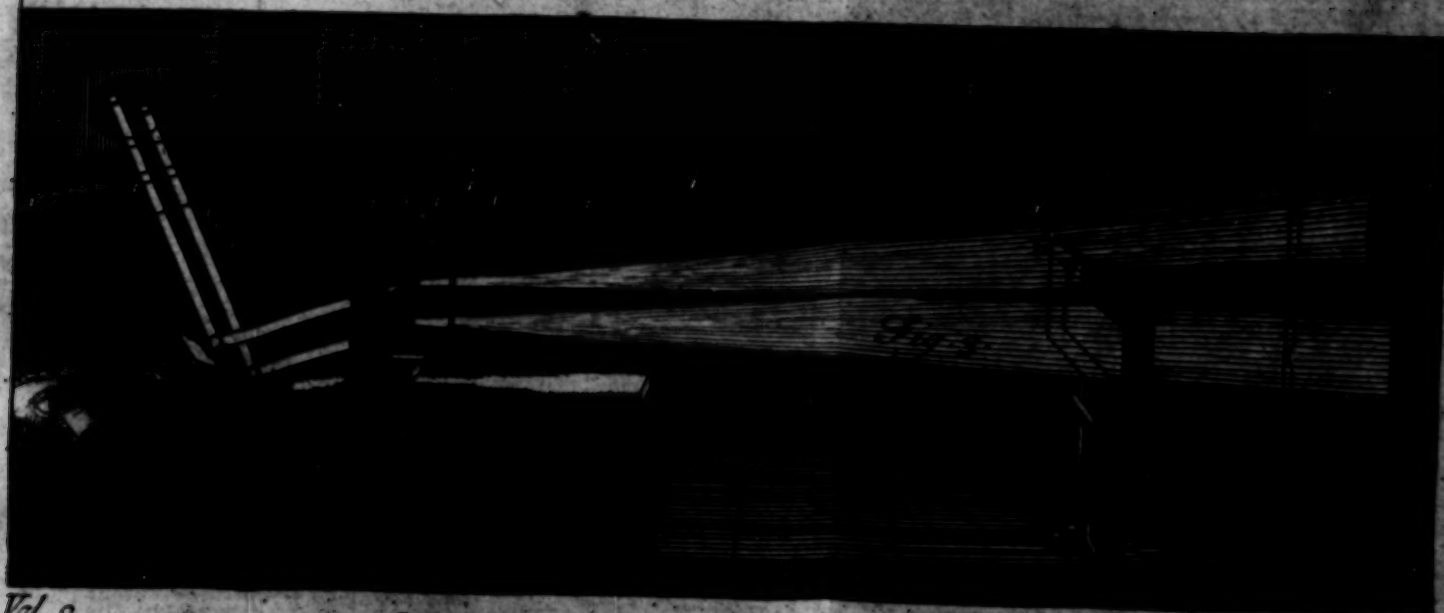
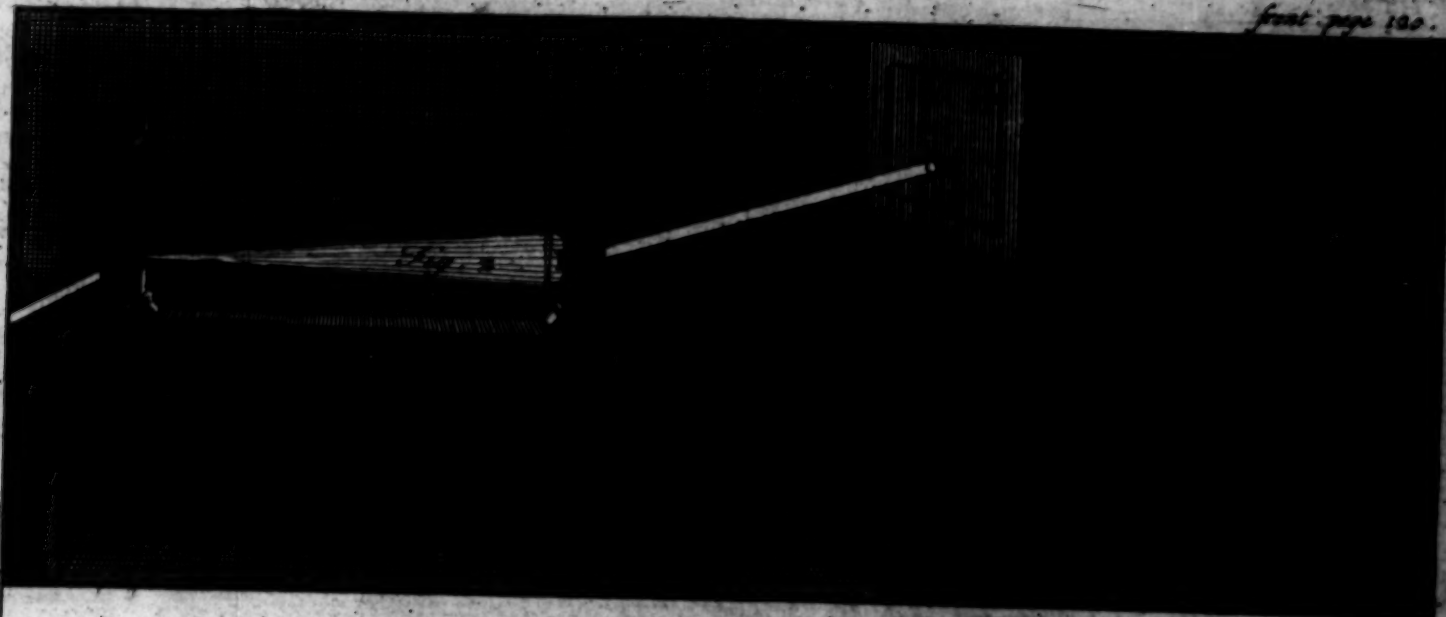
Experiment 3. Plate XVI. Fig. 3.] Let the Light enter the dark Room thro' two Holes of a quarter of an Inch diameter each, in the round Plate moveable in the Window; let these Beams be about two Inches asunder, and reflected by the plane Mirror S.

The Plate and Mirror must be so dispos'd, that the two Beams may fall upon two Prisms A A, which are laid horizontally upon the same Frame, so that the oblong Image, produc'd by the Refraction of those Prisms, may touch one another at their Sides; let one Prism be turn'd a little about its Axis, that the Red of one Image may be just on the side of the Violet of the other; let these Colours, and none of the rest, be intercepted by a wooden Ruler that has a white Paper pasted upon it, so that the Red be at R,

and the Violet at V, the rest of each Image falling upon the Wall, which must be cover'd with a black Cloth. If any Person stand at O, and thro' the Prism B B looks at those Colours R and V (in the manner describ'd concerning the Hole in *Experiment 4.* of the foregoing Chapter) he will see the Colours separated from one another, the Red at *r*, and the Violet at *v*; which therefore being reflected in going thro' the Prism B B, undergoes a greater Refraction.

866 In the first Experiment of this Chapter we gave a Method, whereby to separate the heterogeneous Rays better than in other Experiments; in the following Experiments, Lights of divers Colours become much more homogeneous, which is requir'd in the sixth Experiment of this Chapter.

Experiment 4. Plate XVII. Fig. 1.] Let the Sun's Light enter the dark Room thro' an Hole of one 8th of an Inch, the Sun Beam must be reflected by the Prism A A to the convex Lens V, which stands upon a Foot, and is about 3 or 4 Inches wide; the Length of the incident and reflected Rays taken together is about 7 Feet; the Convexity of the Lens is such, that the Foci of the Rays coming from the several Parts of the Hole, at the distance of 10 or 12 Feet from the Lens, will form the Representation of the Hole, if a Paper be held there*. Just beyond the Lens place the Prism B B, whereby the Rays (as in the first Experiment of the foregoing Chapter) are dispers'd; now placing the flat Board T at the distance at which the Rays coming from the several Points are collected, there will fall upon it a well terminated oblong colour'd Image R V, whose Length will have a greater Proportion to its Breadth than in other Experiments, and which therefore





therefore is made up of Rays more homogeneous*, and so much the less mix'd as the Hole in the Window is the less; by moving forward or backward the Board T, one may find the Distance at which the Image is most distinct and terminated without the Penumbra. 862

Now that the Rays separated in this Experiment are homogeneous enough, is prov'd from this, that they can be no more dispers'd by a new Refraction; and that this is the Property of homogeneous Rays, follows from their equal Refrangibility, which has been already confirm'd by an Experiment*, and is more fully demonstrated by the following one. 864

Experiment 6.] Take two Circles of Paper of an Inch diameter each, and let Light fall upon them in such a manner, that the one may have the homogeneous Rays of any Colour fall upon it, and the other may be enlighten'd by the Rays of the Sun; if both these Papers be look'd at thro' the Prism at the distance of some Feet, as in the third Experiment of this Chapter, the Circle enlighten'd by the heterogeneous Light appears oblong, and ting'd with different Colours, as in the fourth Experiment of the foregoing Chapter; but neither the Colour nor Figure of the other Circle is chang'd.

Experiment 6.] Upon a white Paper draw black Lines parallel to one another, and about the 16th Part of an Inch wide; let these be enlightened, by throwing upon the Paper the oblong Image, which is describ'd in the fourth Experiment, so that the Lines may lie long-wise in the Image; besides this, you must have a convex Lens, about 5 or 6 Inches wide, sustain'd upon a Foot, such as

as are represented at V (*Plate XIX. Fig. 2.*) which collects the red Rays that come from a radiant Point six Foot distant from the Glass, at an equal distance on the other side. If this Lens be plac'd at the distance of six Feet from the Image above-mention'd, the Parts of the black Lines that are enlighten'd by the Red falling upon the Paper, by means of the Rays which are collected by the Lens, will be exactly represented in the red Image at the distance of six Feet; but you must bring the Paper forward about 3 Inches and a half, to make those Parts in the Lines which are enlightened by the Purple, to appear distinct in that Part of the Image which is of the same Colour. The intermediate Colours give Images at intermediate Distances; the Violet is so weak, that the Threads cannot be represented in it.

This Experiment does also confirm, that the Colour of reflected Rays is not chang'd by a new Refraction thro' the Lens; as also that the most refrangible Rays are more inflected than the others in passing thro' the Lens.

867 This last Experiment does also prove, *that the different Refrangibility of the Rays is the Cause that hinders the Perfection of Telescopes*; for the Foci of the Points that are equally distant, fall at different Distances from the Lens, according to their different Colours; whence all the Representations of the Points are unequally distant from the Eye-Glass, and therefore they cannot be all perfectly seen thro' it.

868 Concerning the Reflexion of the Rays, it is to be observ'd, *that those Rays are more easily reflected totally which have the greatest Refrangibility*; for the greater is the Refraction of the Rays, the less Obliquity

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Obliquity is required to make them be totally reflected * 793

Experiment 7.] Take a Prism, plac'd as before *, in the Experiment where it was observ'd, * 790 that by moving the Prism about its Axis, the Rays that first went thro', when their Inclination is increas'd, become wholly reflected; but if the Prism be gently mov'd in this Case, we shall perceive that the Violet Rays are the first which will be wholly reflected, then the Purple Rays, and all the rest in the same Order as they are in the oblong Image of the Sun so often mention'd; which appears if the reflected Rays be separated by the Refraction of the Prism.

C H A P. XX.

Of the Mixture of Colours, where we shall speak of Whiteness.

THAT the Refrangibility and Colour of the 869 Rays are not chang'd by the Mixture of Rays of different Refrangibility, has already been said *, and now we must prove it by Experiments. • 863

Experiment 1. Plate XVI. Fig. 3.] This Experiment must be made in the same manner as the third of the foregoing Chapter; here you must make the Red R, and the Violet V be confounded and mix'd together, by throwing them upon the same Part of the Ruler LL, which thereby will appear of a Purple Colour in that Place; but if a Person looks at them thro' a Prism, the Colours will appear separated; and therefore neither the Colour nor Refrangibility are chang'd by this Confusion of Colours.

Expe-

Experiment 2. Plate XVII. Fig. 2.] If the oblong colour'd Image of the Sun (of which mention was made in the first Experiment of the 18th Chap.) falls at R V upon the convex Lens L, mention'd in *Experiment 4.* of the foregoing Chap. which must be plac'd at 6 or 7 Foot from the Prism BB, the divergent Rays will converge by the Refraction of the Lens, and intersect one another at the distance of 9 or 10 Feet at A; if the Board T be plac'd at a greater distance, the Rays, which diverge again after the Intersection, will fall upon it dispers'd, and you will again have an oblong colour'd Image; but the Colours, by reason of the Intersection at A, will appear in a contrary Order, and will not be chang'd by having been mix'd together at A.

Experiment 3. Plate XVII. Fig. 2.] Every thing still remaining as in the former Experiment, if with a black Paper you intercept some of the Rays of the Image R V, which changes the Mixture (which this way may be vary'd in any manner) the Colours of the other Rays, that are again separated, are no way chang'd.

- 870 If the Rays of the Sun, as they come to us, are wholly reflected by any Body, that Body appears white; but these Rays are an Heap, or Parcel of
- 847 Rays of various Colours*; whence we deduce,
- 858 that a Mixture of different Colours makes a Whiteness;
- 871 for if the Colours which are observ'd in the oblong Image of the Sun, so often mention'd, be mix'd and confounded together, in the same Proportion as they are in that Image, a Whiteness will be produc'd; which also proves, that in that respect the Rays are unchangeable: the Rays that come from the Sun appear white; but if they are separated, their Colours are discover'd; and if they

they be mix'd again, the Whiteness will be restored.

Experiment 4. Plate XVII. Fig. 2.] Things being dispos'd as in the two foregoing Experiments, let the Board T be plac'd at A, in the very Place where all the Rays of the Image R V are confounded together, there will be a Whiteness at A; if with a black Paper you intercept the Red of the Image R V, the White will vanish, and the Colour at A be bluish; but if you intercept the violet and blue Rays, the White becomes reddish.

*Experiment 5. Plate XVII. Fig. 3.] Take three triangular Prisms, made of Wood and Plates of Glass which contain Water, as B B, D D, D D; the Plates of Glass in each of them make an Angle of about 70 Degrees; the Length of the Plate is of 6 or 7 Inches, and their Breadth of 3 Inches; these Plates are fix'd in another manner in the Prisms D D, D D, than in the Prism B B, so as to make their Bases bigger; that is, these Prisms are shorter and deeper than the other. Let the Sun's Rays be refracted thro' the Prism B B, as in *Experiment 3. of Chap. 18.* and let the oblong Image of the Sun at the distance of 3 or 4 Foot fall upon the Surface of the Prism D D, plac'd parallel to the Surface of the Prism B B, out of which the Rays go. In the second Prism the Rays undergo a contrary Refraction than in the first, because of the Parallelism above-mention'd; and because the Edge of the Angle, form'd by the Glass Plates in the Prism B B, is turn'd upwards, and that of D D downwards; therefore the first Refraction is destroy'd by the second, and the Rays go out of the Prism D D parallel to one another, and fall upon R V; for if the Prisms be brought together, so that the parallel Surfaces may*

- may touch one another, the Light will pass thro' a Medium terminated with parallel Planes (which is form'd by the two Prisms join'd together) thro' which Light, of any degree of Refrangibility, will pass without change of Direction*. Now the Prisms are separated, that the heterogeneous Rays may be separated before they again become parallel; if these colour'd Rays fall upon the third Prism D D, and in passing thro' it undergo a Refraction like that which they have undergone in passing thro' the first or second Prism, the Rays that go out at *r v* converge, on account of unequal Refractions in Rays of different Colours, and concur at A, in which Place also Whiteness will be produc'd, as in the foregoing Experiment.

Experiment 6.] If an oblong colour'd Image of the Sun be made after the manner describ'd in *Experiment 1. Chap. 18.* and a Person standing at the distance of the Prism that refracts the Light, looks at the Image thro' the said Prism, as was done in respect of the Hole in the fourth Experiment of the same Chapter, he will see a round and white Image, the second Refraction destroying the first; for thereby the Rays being again mixed, enter the Eye, and in this Case make the Object appear white.

- 872 *The Mixture of all the Colours which are observ'd in the oblong Image of the Sun is not necessary for producing White; the Whiteness of the Sun's Rays is a little inclin'd to yellow; and therefore if part of the yellow Rays be taken out of the Mixture, the White will be the more perfect. From the Mixture of four or five Colours in a just Proportion White will be produc'd.*

Primary,

Primary, that is, homogeneous, Colours being mix'd, produce innumerable Colours, different from the primary or homogeneous ones. From a Mixture of other Colours one may produce a Colour like that which is homogeneous; but when there can be no difference observ'd by the naked Eye between an homogeneous and a mix'd Colour, one may perceive a sensible difference thro' a Prism. 873

Experiment 7.] Thro' a Prism look at any small Objects, such as Letters upon Paper, Flies, and other such things; if they be expos'd to the common open Light, they will appear confus'd; but if they be enlighten'd by the homogeneous Light of the fourth Experiment of this Chapter, they will appear distinct when seen thro' the Prism. 874

C H A P. XXI.

Of the Rainbow.

HAVING made an end of considering what relates to the Rays, whereby Bodies are enlighten'd; before we leave this Subject, we must explain a Phænomenon, which is too remarkable and common to be pass'd by in Silence.

The *Iris* or *Rainbow* is what every Body has often seen; we must explain what is the Cause of it, having first laid down some things for that Purpose.

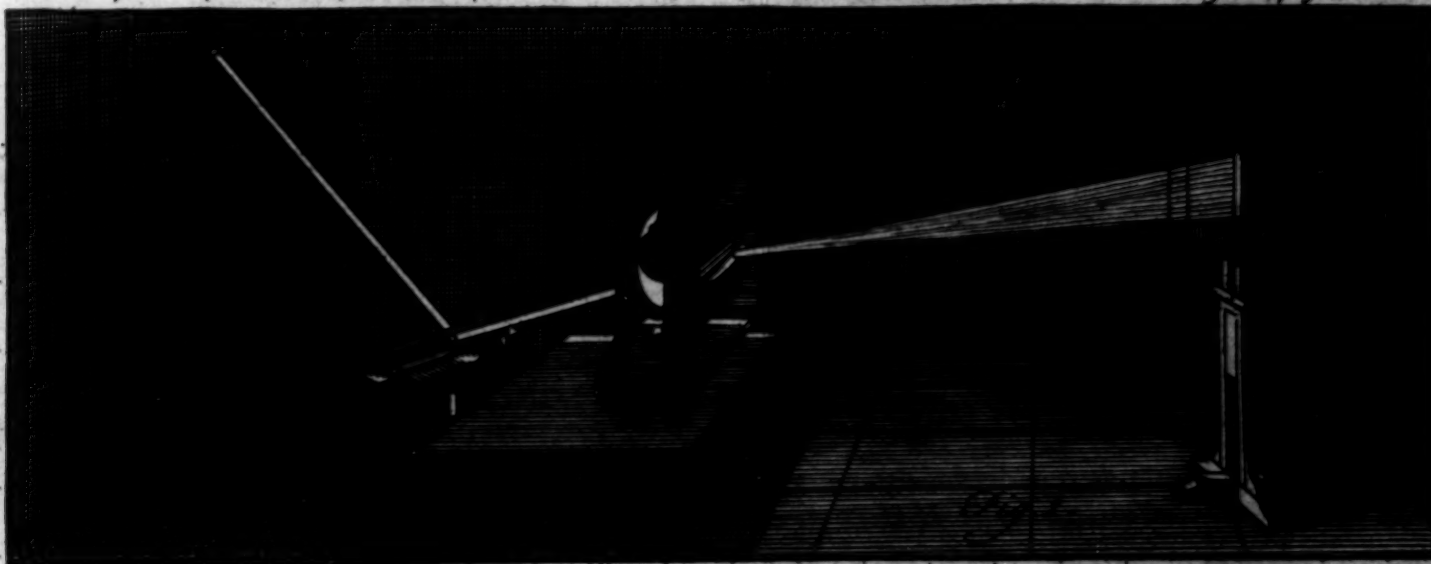
Plate XVIII. Fig. 1.] Let there be a dense Medium encompass'd with a rarer, terminated by the Circle BDFH; let homogeneous Rays that are parallel to one another fall upon it, and let AB be one of these Rays; let the Semidiameter CB be drawn and continu'd to N, it will be perpendicular to the 875

- the Surface separating the Mediums; ABN therefore is the Angle of Incidence; this Angle is equal to the opposite vertical Angle CBL , whose Sine is CL , perpendicular to BL , and going thro' the Center; the Ray is refracted to-
- 624 wards the Perpendicular*, and CBM is the Angle of Refraction, whose Sine is CM drawn from C perpendicular to BD : there is the same Pro-
 - 639 portion between such Lines as CL and CM *, for every Ray, as there is for AB . The Ray BD does in part penetrate into the rarer Medium in the Direction DE , and is in part reflected along DF , and makes the Angle of Reflexion CDF
 - 805 equal to the Angle of Incidence* BDC ; whence BD and DF are equal. The Ray DF does also in part go out of the denser Medium along FG , and is in part reflected along FH ; which in the same manner does in part go thro' HI , and is in part reflected: But this Reflexion, and other farther Reflexions and Refractions, we shall not consider; they are too weak, on account of the several Divisions that the Light has undergone.

Plate XVIII. Fig. 1.] The Ray FG , which after one Reflexion goes out of a denser Medium, makes the Angle GPA with the incident Ray AB , which varies in different incident Rays; therefore tho' these Rays should be parallel, *they will be scatter'd when they go out, after one Reflexion, as may be seen by the second Figure.*

- 806 The Ray EE , which being continu'd goes thro' the Center C , is not turn'd out of its way either by Reflexion or Refraction*.

629 As you recede from this Ray, the returning Ray is continually less and less inclin'd to the incident one. So the Ray DD , which goes out of a denser Medium along dd , and returns along the Line, makes a greater Angle with dd ,
than



than the intermediate Rays between DD and EE do make with their returning Rays, which go out of the denser Medium.

Let BB be a Ray, in respect of which this 876 Inclination is the least of all, that is, which makes the Angle G P A (Fig. 1.) the biggest of all. Beyond BB the returning Rays are more inclin'd to the incident ones; thus AA returns along *aa*.

From this Disperſion of the returning Rays re- 877 ceding from the denser Medium, they become continually weaker and weaker, and their Colour cannot be perceived throughout the whole Space which they fill, tho' the Colour of the incident Rays be vivid. The Colour in the returning Rays is only ſenſible, where the neighbouring Rays are parallel, and thoſe next to them diverge out very little; ſo that at a great Diſtance they are denſe enough to be perceiv'd. Theſe only are ſaid to be efficacious, and will be ſuch, when the incident Rays which are near one another, being refracted, concur in the very Point of Reflexion.

Plate XVIII. Fig. 3.] Let AB, *ab*, be neighbouring Rays parallel to one another, falling upon a circular Surface that terminates a denser Medium; if theſe being refracted along BD, *bd*, do concur in D the Point of Reflexion, the reflected ones DF, *Df*, will make the ſame Angles with one another as the incident ones DB, *Db*; therefore the refracted Rays FG, *fg*, will be parallel * and efficacious *. In this Caſe the follow- 877 ing Method will ſerve to determine the Angle made by the incident Ray with the returning one; that is, the Angle APG, which here is the greateſt of all.

Let us call the Ratio of I to R, that which 878 is found between the Sines of the Angle of Incidence and Refraction, when the Light goes

out of a rarer Medium, by which a denser is encompass'd, into a denser contain'd in the Circle itself; therefore having drawn Cm perpendicular to bD , and the Arc mn with the Center C and Semidiameter Cm ,

$$I, R :: CL, CM :: Cl, Cm :: CL - Cl = Ll, CM - Cm = Mn.$$

Draw Bo perpendicular to BL , and also Bp perpendicular to BD ; and draw bp , so that it may make a right Angle with Bp ; lastly, join together by Lines the Points B, C , and M, m , the Triangles Bbo , BCl are similar; for they are rectangular, and the Angles oBb and CBL (the difference of each of which from a right Angle is the Angle oBC) are equal.

It may be prov'd in the same manner, that the Triangle $BM C$ and Bbp are similar; the Triangle Mmn , which is rectangular at n , is also similar to this; for the Sides $Mn Bp$, which are perpendicular to the Line BD , are parallel; as also Mm and Bb , because the Lines BD, bD , are bisected at M and m into equal Parts; therefore also Bb is the Double of Mm , and Bp the Double of mn .

Hence we deduce

$$BC, BL :: Bb, Bo.$$

$$BC, BM :: Bb, Bp.$$

Therefore

$$BL, BM :: Bo = Ll, Bp = 2 Mn :: I, 2 R :: CL, 2 CM, \text{ by comparing these Proportions with the aforesaid Proportion.}$$

Now since the Squares of proportional Quantities are themselves proportional, you will have

$$BL^2, CL^2 :: BM^2, 4 CM^2.$$

Whence we deduce

$$BL^2 + CL^2 :: BC^2, BL^2 :: BM^2 + 4 CM^2 = BC^2 + 3 CM^2, BM^2 = BC^2 - CM^2 = BL \times LC = CM^2.$$

By

By subtracting the first and second Term from the third and fourth (which does not change the Proportion) you will have

$BC^a, BL^a, :: 3 CM^a, LC^a - CM^a, :: 3 R^a, I^a - R^a$; for there is the same Ratio between CM and LC as between R and I .

If therefore you know the Ratio between R and I , you have the Ratio between the Semidiameter BC , and the Line BL , which is the Sine of the Angle $BC L$, which Angle therefore is given, and by consequence you have the Arc BN , and also FG , for they are equal.

Having the Sine BL , you have also the Sine of the Angle BCM ; because (as we have shewn before)

$$BL, BM :: I, 2 R.$$

Therefore the Arc BD is determin'd, to which DF is equal.

Hence we may easily find the Arcs NH and BF ; if the former be taken out of the latter, and the Remainder be divided into two equal Parts, we shall have, as is well known, the Measure of the Angle APG .

When the Ratio between I and R varies, the Angle APG is chang'd, which therefore becomes different, according to the different Refrangibility of the Rays.

If the abovesaid Surface be enlighten'd by heterogeneous Rays, as they flow from the Sun, the efficacious Rays of different Colours do not make equal Angles with the incident ones, and so by the help of this Refraction the Colours are separated.

Plate XVIII. Fig. 4. Experiment 1.] Let the Sun's Light enter the dark Chamber thro' a Slit in the moveable round Plate O , and being horizontally reflected from the Looking-glass S , let it pass thro' a Slit in the little Board or Stand T ;

* 630 as has been before explain'd. * Take a Phial exactly cylindric, made of clear Glass, and full of Water; let the Beam or Ray at fg fall upon the Surface of the Phial, it will be refracted in the Water towards bi , and there reflected, and will go out of the Phial at lm ; the Phial may be easily so plac'd as to make these Rays efficacious; and because of the Breadth of the incident Ray or Beam, efficacious Rays of all Colours will go out of the Phial in the same time, for they are very little distant from one another in their Incidence; if these efficacious Rays fall upon a white Paper, at the distance of four or five Feet from the Phial, they will produce vertical *Fasciæ* or Pillars, of various homogeneous Colours, arising from the efficacious Rays of each Colour; if also the Eye be plac'd any where at the distance of some Feet from the Phial, as at N , in these efficacious Rays, it will see in the Phial that Colour whose Rays enter the Eye, and by a successive Motion of the Eye it will perceive all the afore-mention'd Co-

* 861 lours. * *Plate XVIII. Fig. 5.*]

881 But as to those Rays, which after a double Refraction in a denser Medium emerge, they will be efficacious if they are parallel after the first Reflexion; for then FH , fb , are in the same manner inclin'd to Hb as BD , bd , to Bb ; and therefore supposing the incident Rays AB , ab , to be parallel, the emerging Rays HI , bi , will also be parallel *.

* 927 In this Case dD is half of the difference between the Arcs DE and df , or DB and db , but their difference is $Bb - Dd$; if therefore this last be subtracted from the other, there will remain the Double of the Arc Dd , whose Triple therefore is Bb . If by Lines the Points D , d , and B , b , be join'd, the Triangles BEb and DEd will be similar, as is known; which therefore obtains, tho'

tho' the Arcs Bb , Dd , be so very small, that they may be taken for right Lines.

There is therefore that Ratio between ED and Eb , which obtains between these Arcs, that is, ED is the third Part of Eb , or EB ; because we suppose the Arc Bb to be extremely small. MD therefore is divided into two equal Parts at E ; and ME is a third Part of EB .

Now if (as in *Fig. 3.*) the Triangles Bob , Bpb , and Mmn , be form'd, Mm will be the third Part of Bb , and Bp the Triple of Mn ; now if, *mutatis mutandis*, we apply to this Figure what was demonstrated with respect to *Fig. 3.* * 878 because in this Bp is equal to $3 Mn$, whose Square is $9 Mn^2$, we shall have

$$BC^2, BL^2 :: 8 R^2, I^2 - R^2.$$

From which Proportion, as was said of *Fig. 3.* the Arc BN may be discover'd to which HG is equal; and because in this Case

$$BL, BM :: I, 3 R,$$

You have also the Arc BD , to which (because the Angles of Reflexion are equal to the Angles of Incidence) * DF and FH are equal. * 805

From which things being found, we may easily 882 deduce the Arcs $GFDN$, and BH , whose half Difference is the Measure of the Angle HPB , which is form'd by the emerging Ray with the incident one; which Angle in this Case is the least of all that are like it, and is different according to the different Refrangibility of the Rays; whence also in this Case the efficacious Rays of va- 883 rious Colours, supposing the incident ones parallel, are separated after a double Reflexion.

Plate XVIII. *Fig. 4. Experiment 2.*] This Experiment is perform'd after the same manner as the former, only the Situation of the Phial must be a little chang'd, that after two Reflexions in

the Phial, the Rays may come to the Eye or Paper towards N.

884 What has been hitherto explain'd, may be applied to the Rainbow; for which Phænomenon it is required, that Drops of Water should be suspended in the Air; that the Spectator should be placed with his Back towards the Sun, between it and the Drops; and that there should be a dark Cloud behind the Drops, that the Colours may be more sensible, which are scarcely perceiv'd if vivid Light enters the Eye at the same time.

If, these things being suppos'd, we conceive each Drop to be cut by Planes passing thro' the Sun and the Eye of the Spectator, what has been demonstrated of a Medium terminated with a circular Surface, * may be applied to each of these

* 878, 879, Sections.

880, Now here we speak of Rays that penetrate out
881, of Air into Water. In red Rays, that is, those
882, which are least of all refrangible, the Ratio be-
883, tween the Sine of the Angle of Incidence and the Sine of the Angle of Refraction, that is, between I and R, is that of 108 to 81; or which is the same, of 4 to 3; with which Numbers if the Computation be made, the Angle A P G (Fig. 3.)

* 879 will be 42 Deg. 2 Min. *, and the Angle A P I

* 882 (Fig. 4.) will be 50 Deg. 57 Min. * If we speak of the Violet Rays, I and R are to one another as 109 to 81; which Numbers give the Angles

* 879 A P G (Fig. 3.) of 40 Deg. 17 Min. *, and A P I

* 882 (Fig. 4.) of 54 Deg. 7 Min. *

Plate XIX. Fig. 1.] Let the Drops be suppos'd to be diffus'd in the Air, and enlighten'd by the Sun's Rays, which are parallel to one another, and to the Line OF, that passes thro' the Eye of the Spectator. Let the Lines *e* O, EO, *b* O, BO, be conceiv'd to be drawn; and let the Quantity

of

of the following Angles be thus, (*viz.*) eOF of 40 Deg. 17 Min. EOF 42 Deg. 2 Min. bOF 50 Deg. 57 Min. BOF 54 Deg. 7 Min. These same Lines, with the incident Rays de , DE , ab , AB , form Angles that are respectively equal to the afore-mention'd; therefore if the Drops be conceiv'd at e , E , b , B , the Violet efficacious Rays, after one Reflexion in the Drop e , enter the Eye, and the red efficacious Rays come to the Eye from the Drop E ; in like manner, after one Reflexion the other intermediate Colours are observ'd between e and E in the above-mention'd Order*. After two Reflexions in the Drop, the efficacious red Rays come to the Eye from the Drop b ; and the efficacious Violet ones from the Drop B ; the intermediate Colours appear between these Drops after the same manner as between E , e ; but they are dispos'd in a contrary Order, and by reason of their double Reflexion are also weaker. * 861

Let us conceive a Line at Oe to be revolv'd about a fixed Line OF , preserving the Angle eOF , and to form a Cone, or part of a conic Surface; in every Situation the Line eO will, with the Rays of the Sun that are parallel to one another, and to the Line OF , form an Angle of 40 Deg. 17 Min. If therefore the Drops be diffus'd near part of the Surface of this Cone, at the same or different Distances, the Eye will see a Violet Arc or Bow. The same may be said of the other Colours; and therefore Drops being suspended in the Air, it sees an Arc or Bow of the Breadth eE , ting'd with the homogeneous Colours before mention'd*, that are dispos'd in the same Order as in the Experiments with Prisms; because the heterogeneous Rays are separated as much in the Drops as in the Prism*. * 852

880

886 By the same way of reasoning it is plain *there will be a broader Arc or Bow surrounding the first, in which the same Colours will appear in a contrary Order, and weaker.*

Experiment 3.] Hang up a black Cloth in the Light of the Sun, and let a Spectator look at it standing between the Sun and the Cloth; then if Water be made to fall in small Drops between the Spectator and the Cloth, the Spectator will see a Rainbow, at least the inner one.

C H A P. XXII.

Of the Colours of thin Plates.

WE pass on to the Colours of natural Bodies; and before we go any farther, we think it proper to examine thin Plates. Whoever attentively has observ'd very thin Glass, or Bubbles made of Water thicken'd with Soap, must have perceiv'd several Colours in them. Rays of Light, by help of a thin transparent Plate, are
887 separated from one another, and according to the different thickness of the Plate, the Rays of some Colours are transmitted, and those of others are reflected; and the same very thin Plate is of another Colour, when seen by the transmitted Rays, than when seen by the reflected ones.

Experiment 1. Plate XVIII. Fig. 6.] Take two Object-Glasses belonging to long Telescopes, A B and C D, and let one of them be laid upon the other; then let them be press'd together hard, and in the middle, where the Glasses touch one another, you will see a transparent Spot, which is encompass'd with colour'd Circles. If the
Light

Light reflected by the Air that is between the Glasses comes to the Eye at O, there will appear a black Spot; and the Colours, which, as you recede from the Center, are so disposed, that by reason of the same Colours coming over again, they may be referr'd to several Orders, will be as follows: BLACK, Blue, White, Yellow, Red: VIOLET, Blue, Green, Yellow, Red: PURPLE, Blue, Green, Yellow, Red: GREEN, Red; which Colours are all encompass'd with other Colours; but as you recede from the Center, grow continually weaker and weaker.

If the Light passeth thro' the Glass to the Eye at O, the transparent Spot which transmits all the Rays is white; and by that Series, as you recede from the Center, there will appear Colours, which are also referr'd to several Orders contrary to those above-mention'd: WHITE, yellowish Red, Black, Violet, Blue: WHITE, Yellow, Red, Violet, Blue: GREEN, Yellow, Red, bluish Green: RED, bluish Green, which are also encompass'd with weaker Colours.

Experiment 2.] Blow up Soap'd-water into a Bubble, to form a thin Plate of Water; let this be cover'd with a very clear Glass, left by the Agitation of the Air the Colours to be observ'd in that Bubble should be confounded by the Motion of the Water; such a Bubble, because the Water continually runs down every way, is very thin at top, and the thickness in going down is continually increas'd; and for the same reason the thickness of the whole is continually diminish'd; before the Bubble breaks, the top of it becomes so thin as to transmit the whole Light, and appear black; if in that Case the Bubble be observ'd by reflected Light, when it is enlighten'd

en'd by the Reflexion of a whitish Sky, and the extraneous Light is intercepted by placing any black Body behind the Bubble, the black Spot above-mention'd will appear, and be encompass'd with the same colour'd Circles, disposed in the same Order as about the black Spot in the former Experiment; as the Water descends, the Rings are continually dilated till the Bubble bursts.

When the extreme Circumference of the Bubble appears red by the reflected Rays, if it be look'd at so as to be seen by the transmitted Rays, it will appear blue; and generally the Colours produc'd by transmitted and reflected Rays, in the same manner as in the foregoing Experiment, are opposite to one another.

- 888 By comparing these Experiments, it follows, that if we increase the thickness of a very thin Plate, its Colour will be chang'd, and there will be the same Changes successively, and in the same Order, whether the Plate be form'd out of a rarer or a denser Medium; for in the Plate of Air, between the Glasses, and the watery one in the Bubble, whose thickness increases as it goes farther from the middle, the Colours will be in the same Order; yet in a
- 889 denser Plate a less thickness is requir'd than in a rarer to have it ting'd with the same Colour.

Experiment 3.] Every thing being disposed as in Experiment 1. if you wet the Edges of the Glasses a little on one side, the Water will by degrees insinuate itself between the Glasses, and there will be observ'd the same colour'd Circles as in the Air, neither will their Order be chang'd, but the Circles will be less; when the Water is got as far as the Center, all the Portions of the Circles in the Water will be separated from the Portions of the Circles in the Air, and be all reduc'd into a less Space.

The

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*The Colour of a Plate depends upon its Thickness * 890
and Density, and not upon the encompassing Medium. 887
889*

Experiment 4.] If you take a Piece of Ising-glass or Talk, so thin as to have it colour'd, the Colours will not be chang'd by wetting it; that is, if instead of Air the Plate is encompass'd with Water.

*The Colour of the same Plate is so much the more 891
vivid, as its Density differs more from the Density of the circumambient Medium. This is prov'd by the foregoing Experiment, in which the Colours of the Plate, when wet, are more languid than those of the same Plate encompass'd with Air. In the third Experiment also the Colours are less vivid than in the second; in both there is a Plate of Water; but in the second Experiment it is inclosed with Air, and in the other with Glass; but Water and Glass differ less in Density than Air and Water.*

*If Mediums equally differ in Density, the Colours 892
will be more vivid if the denser be encompass'd with the rarer; for a very thin Glass Plate, which is colour'd on account of its thinness, being encompass'd with Air, the Colours will be more vivid than in Experiment 1. in which a Plate of Air is encompass'd with Glass.*

*A Plate of the same Density, encompass'd with the 893
same Medium, will reflect so much the more Light as it is thinner; yet if the thickness be too much diminished, it does not reflect the Light. All this is plain 894
from the foregoing Experiments; in the three first, the colour'd Circles, which are the least, and which are also the thinnest, reflect Light best of any; but in the Center, where the Spot is the thinnest of all, there is no sensible Reflexion; as this clearly appears in the second. In the first there is also a very thin Plate of Air, which does
not*

not reflect Light; for the transparent Spot in the Center is much bigger than those Parts of the Surfaces of the Glasses, which immediately touch by the yielding inwards of the compress'd Parts.

- 895 *If there are Plates of the same Medium, whose Thickneses are in an arithmetical Progression of the natural Numbers 1, 2, 3, 4, 5, 6, 7, &c. if the thinnest of them all reflects any homogeneous Rays, the second will transmit the same, the third again will reflect them, and the Rays will be alternately reflected and transmitted; that is, the Plates whose Thickneses in the above-mention'd Progression answer to the odd Numbers 1, 3, 5, 7, &c. reflect the Rays which the others transmit, whose Thickneses answer to the even Numbers 2, 4, 6, 8, &c.*

- This Property of the Plates obtains in respect of any Sort of homogeneous Rays, with this difference, that different Thickneses are required
 • 887 for different Colours, as has been said before*; the thinnest of all for reflecting Violet, and they must be the thickest for reflecting Red; if the Thickneses are intermediate, the Rays of an intermediate Refrangibility are reflected; that is,
 896 *as the Refrangibility of the Ray increases, the Thickness of the Plate that reflects it is diminish'd.*

Experiment 5.] Let the Experiment be made in a dark Room to produce the oblong Image of the Sun upon a Paper, such as is mention'd in Experiment 1. Chap. 18. Take two Object-glasses of long Telescopes (such as were mention'd in the first Experiment of this Chapter) let them be press'd together, and so disposed, that every single Colour of the Image above-mention'd may be successively seen in them as in a Looking-glass; that is, that the Glasses may be successively enlighten'd by several homogeneous Rays; which may

may be done by gently moving about its Axis the Prism that separates the Rays to make the oblong Image. The Rings mention'd in the first Experiment appear, but more in Number, and only of one Colour, by reason of the Unchangeableness in homogeneous Rays * ; in the Interstices of those Rings the Rays are transmitted, as plainly appears by holding a Paper behind, upon which the transmitted Rays will come ; the Rings are least of any when they are violet, then are successively dilated, considering the following Colours, quite to the Red ; the Rings being of any Colour, if you measure exactly the Diameters of the Circles that may be conceiv'd to be in the middle of the Breadth of the Rings, the Squares of their Diameters will be to one another as the odd Numbers 1, 3, 5, &c. and measuring in the same manner the Diameters of the Circles in the middle of every one of the Interstices between the Rings, the Squares of their Diameters will be as the even Numbers 2, 4, 6, &c. Now in using spherical Glasses, the Thicknesses of the Plate of Air in the Circle above-mention'd are as the even and odd Numbers. • 863

DEFINITION.

An homogeneous Colour, in a Plate of any Medium, is said to be of the first Order, if the Plate be the thinnest of all those that reflect that Colour; in a Plate whose thickness is triple, it is said to be of the second Order, &c. 897

*A Colour of the first Order is the most vivid of any; and successively in the following Orders, in the second, third, &c. it is less and less vivid. ** • 893

When a Plate of Air is enlighten'd with heterogeneous Rays, as that between the Glasses of the Telescopes, or any Plate like it of any other Substance, as in *Experiment 1. and 2.* several of the

the Rings seen in the last Experiment are confounded together, and that Colour is seen which is made of their Mixture; for *the same Thickness*
 899 *of a Plate is often required for reflecting different Colours of various Orders*; so a Plate which reflects the Violet of the third Order, does also reflect the Red of the second Order, as may be deduc'd from the last Experiment, if you attentively consider it; therefore in the first and second Experiment, the third violet Ring is confounded with the outward Edge of the second red Ring, and Purple Colour is produc'd; yet all the Red of the second Order is not absorb'd, because the red Ring is wider than the violet one.

900 *The more the thickness of a Plate is increas'd, the more Colours it reflects, and different ones, of different Orders*; the violet Plate of the tenth Order falls in with the blue one of the ninth Order, and the yellow one of the eighth Order; and lastly, with the red one of the seventh Order; and the Colour of the Plate is made up of a Mixture of those Colours.

901 *If in the first and second Experiments a Spectator look obliquely upon the Plates, such as that made of Air, and that which is made of Water, the Rings will be dilated as they are seen more obliquely*; that is, in that Motion of the Eye *the Colour of the Plate in a determinate Place is chang'd*; yet the Dilatation is greater in the first Experiment; which proves, that *the Colour is more changed by the Obliquity of the Rays, if the Plane be encompass'd with a denser Medium, than if it be inclos'd by a rarer Medium*.

Plate XVIII. Fig. 7.] The Demonstration of which Proposition is easily deduc'd from the Laws of Refraction. Let L and l be thin Plates, the last encompass'd with a denser, and the first with a rarer Medium; let them both be of the same thickness;

thickness; if the Rays AB , ab , equally inclin'd to the Plates, fall upon them, there will be at L a Refraction towards the Perpendicular*; on the contrary, at I the Rays are refracted from the Perpendicular*; and tho' BD and bd are equal, bc is longer than BC ; and therefore there is a greater Change in the Motion of the Light in the Plate I than in the Plate L . *The Density of the Plate L being increas'd, the Medium with which it is compass'd remaining the same, there will be a less Difference between BC and BD , and therefore a less Change of Colour; and if the refracting Power of the Plate be so increas'd, that the refracted Rays (whatever be the Obliquity of the incident ones) shall not sensibly differ, there will be no sensible Difference in the Colour of the Plate, whatever Situation the Eye is plac'd in.*

Hence we may easily deduce, that *the Colour of some Plates will vary by changing the Position of the Eye, and that the Colour of others is permanent.*

C H A P. XXIII.

Concerning the Colours of natural Bodies.

WHAT relates to the Colours of all Sorts of Bodies, may be easily deduc'd from what has hitherto been explain'd.

We have shewn that the Rays of Light have Colours peculiar to themselves, and unchangeable, so as not to be chang'd by Reflexion.*

Therefore *the Rays reflected from Bodies, according as a greater or less Refrangibility is proper to the Colour of the Body itself, have a greater or less Refrangibility.*

Experiment 1.] In the middle of a black Paper fix two square Pieces of Ribbon, the one red, and the other violet, which must be join'd so as to touch one another at their Sides, in the same manner as the Red and Violet Colours in the third Experiment of Chap. 19. The black Paper must be so plac'd, that the Ribbons may be well enlighten'd by the Light that comes into the Room thro' the Window; if a Spectator looks at these Ribbons thro' a Prism, as in the Experiment above-mention'd, the Colours will appear separated, in the same manner as in that Experiment.

Experiment 2. Plate XIX. Fig. 2.] Place the two Ribbons mention'd in the former Experiment at R and V; let the last be violet, the first red; let them be enlighten'd by the Flame of a Candle; at six Feet distance place the convex Lens V (of which mention has been made in *Exper. 6. Cap. 19.*) at the distance of about six Feet, you will have the Representation of the Ribbon R upon a white Paper at r ; at a less distance you will have an exact Representation of the other at v . You may determine when the Representations are exact, by binding black Threads upon the Surface of the Ribbons; for these Threads appear distinctly where the Representation is exact.

907 That Bodies have various Colours, because different Rays are reflected from Bodies differently colour'd; and that a Body appears of that Colour which arises from the Mixture of the reflected Rays, may not only be deduc'd from the foregoing Experiments, but may be also demonstrated directly by others.

Experi-

Plate 18.

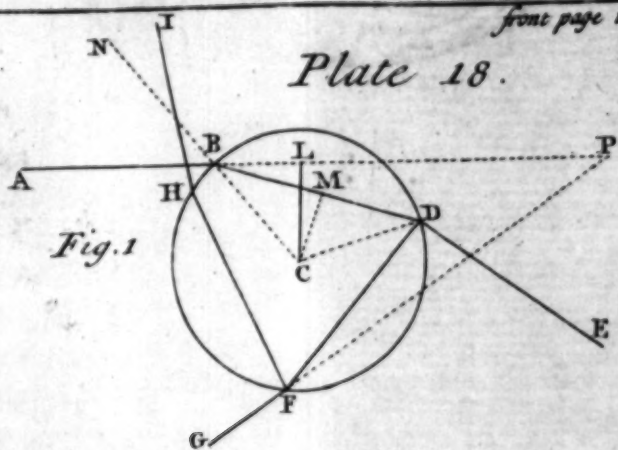


Fig. 1

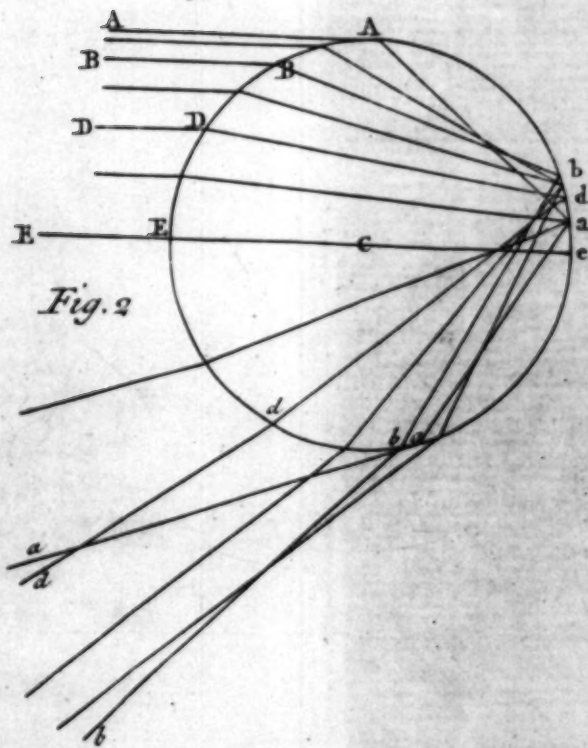


Fig. 2

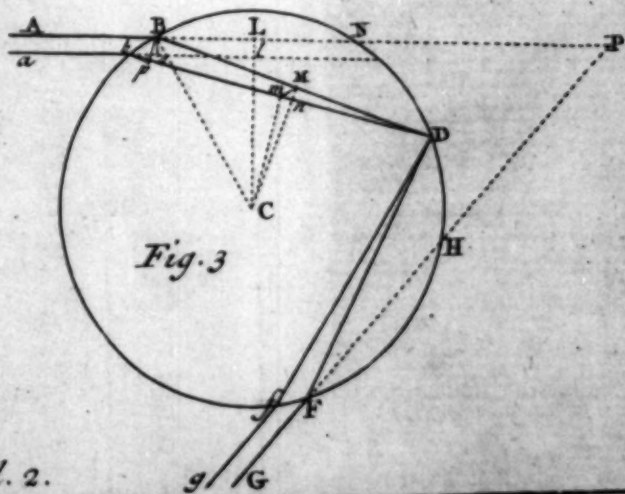


Fig. 3

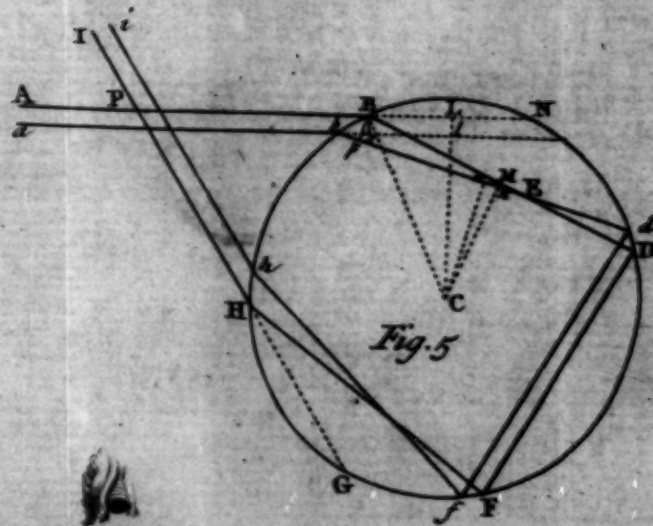


Fig. 5



Fig. 6

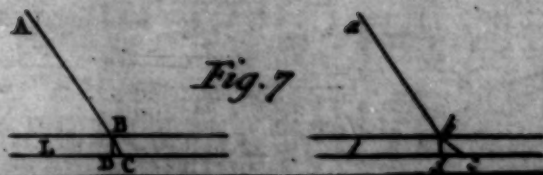


Fig. 7

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Experiment 3.] Take two Bodies of any Kind, the one red, the other blue, let them be enlighten'd successively in a dark Place by the Colours of the colour'd Image, which is made by the Refraction of the Prism, every one of the Colours will be reflected by both; but the red Rays are copiously reflected from the red Body, whilst the blue Body reflects but few of them, which plainly appears by comparing both Bodies when they are enlighten'd by the red; the contrary may be observ'd in blue Colours, which are copiously reflected from the blue Body, whilst only a few are reflected from the red one.

The Rays which are not reflected from a Body, penetrate into it, and there suffer innumerable Reflexions and Refractions, as we have explain'd between N^o 842. and 843. till at length they unite themselves to the Particles of the Body itself*; therefore *a Body grows hot so much the sooner* * 546 *as it reflects Light less copiously*; for which reason 908 *a white Body*, which reflects almost all the Rays 909 with which it is enlighten'd*, *heats the slowest*; * 870 *whilst a black Body*, into which almost all the Rays penetrate, because only few are reflected*, *ac-* * 845 *quires Heat sooner than any other.*

To determine that Constitution of the Surfaces of Bodies upon which their Colour depends, we must take notice of the smallest Particles of which these Surfaces are made up; these Particles are transparent*, and are separated by a Medium of * 841 different Density from the Particles themselves; * 843 they are also thin, otherwise the Surface would as it were be cover'd by a transparent Body*, and * 843 the Colour would depend upon the Particles under these; therefore in the Surface of every colour'd Body there are innumerable small thin Plates; but by lessening the Plate, keeping the

same thickness, its Properties, as to the Reflexion of Light, are not chang'd; for the least Plate, in respect of the Rays of Light, is very large; wherefore what has been demonstrated in the preceding Chapter, may be applied to these Plates in the Surfaces of Bodies; from whence we deduce the following Conclusions.

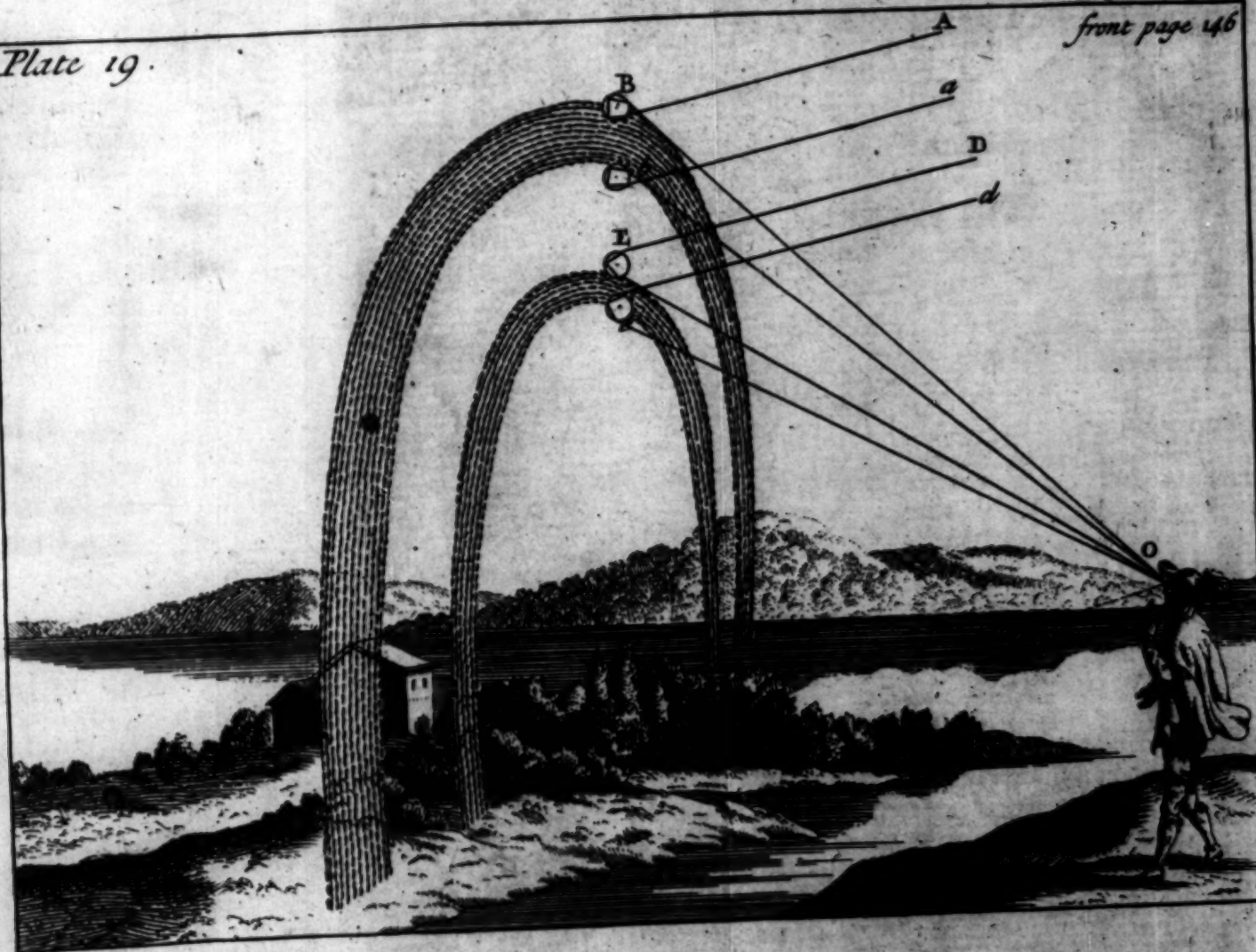
- 910 *The Colour of a Body depends upon the Thickness*
 • 890 *and Density of the Parts of the Body, which are in*
 911 *the Surface, between the Pores of the Body*.*
 • 898, *The Colour is so much the more vivid and homoge-*
 899, *neous, as the Parts are thinner*.*
 900.
 912 *Cæteris paribus, the aforesaid Parts are of the*
 • 896 *greatest thickness when the Body is red, and of the*
 861 *least when violet*.*
 913 *The Parts of Bodies are much denser than the Me-*
 • 902, *dium contained in their Interstices*.*
 903,
 904. *This Density is less in the Tails of Peacocks, and*
 914 *in some Silks, and in general in all Bodies whose*
 • 901 *Colour varies according to the different Situation of*
 903 *the Eye*.*
 915 *The Colour of a Body is more obscure and darker*
 • 891 *when a denser Medium enters its Pores*, for then*
 the Parts, upon which its Colour depends, are
 surrounded by a denser Medium than before.

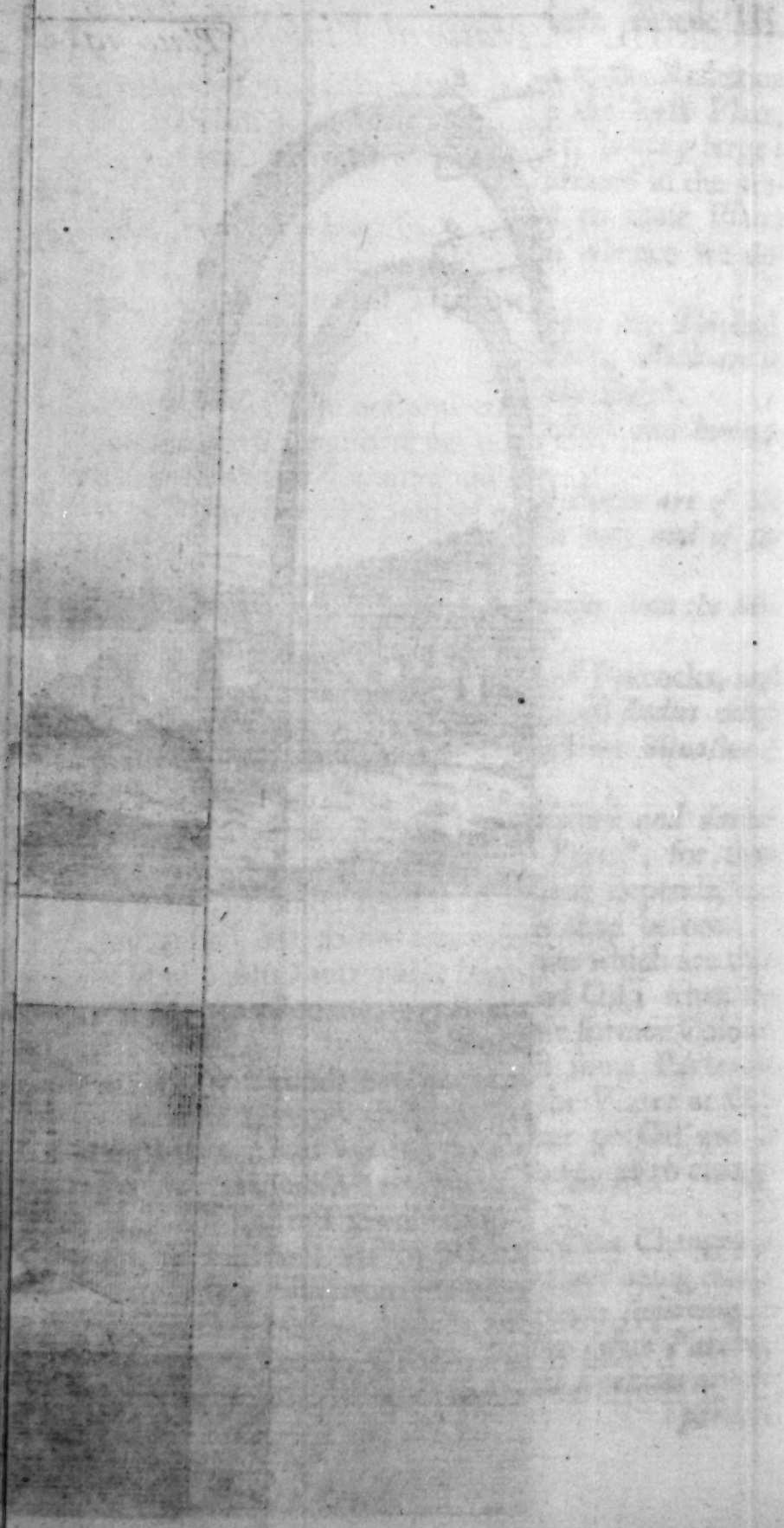
We experience this in all Bodies which are thoroughly penetrated by Water and Oil; when the Bodies are dry they recover their former Colour, unless in some Cases, in which some Parts are carried away by the Action of the Water or Oil, or when some Parts of the Water or Oil are so united with the Parts of the Body, as to change the Thickness of the Plates.

- From such a Cause are deduc'd the Changes in the Colours of some Liquors, by their being mix'd
 916 *with others; often the saline Particles swimming in*
one Liquid, unite themselves to the saline Particles
swimming in another; or the united Particles are se-
 parated

Plate 19.

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parated by the Actions of others that are superadded; on all which Accounts the thickness of the Particles is chang'd, and together with it the Colour of the Liquids*.

* 888

Sometimes the Colour of a Liquid is different, when seen by reflected Rays, from what it is when seen by transmitted ones; we have shewn before whence this arises*.

917

* 887

Experiment 4.] An Infusion of *Lignum Nephriticum*, that is not too much tinged, appears blue by reflected Rays; but yellow if the Phial which contains the Infusion be plac'd between the Light and the Eye.

Experiment 5.] If you pour Spirit of Vinegar into the Infusion of *Lignum Nephriticum*, it will appear yellow in any Position whatever.

In this Case the thickness of the Particles is chang'd, and the Rays that were transmitted thro' every one of them, are now intercepted; but tho' the Liquor is plac'd between the Eye and the Light, it is seen by reflected Rays; for we may easily conceive that such Rays come to the Eye by the various Reflexions which the Light undergoes in the Liquid; but this Colour only is sensible, because the Rays cannot penetrate directly thro' the Liquid.

From this we may deduce the reason why a colour'd Liquid in a Glass, of the Figure of an inverted Cone, if it be placed between the Eye and the Light, appears of a different Colour in different Parts of the Vessel; in the lower Part all the Rays which are transmitted thro' the Particles are not intercepted, then they are more and more intercepted, according as there is a greater Quantity of the Liquid between the Eye and the Light, till at length they come to be all intercepted, and

918

only those Rays which are reflected by the Particles penetrate the Liquid ; in which Case the Colour coincides with the Colour of the Liquid seen by the reflected Rays.

- 919 *Clouds often appear very beautifully coloured ; they consist of aqueous Particles, between which Air is interspersed ; therefore according to the various thickness of those aqueous Particles, the Cloud*
 920 *will be of a different Colour*.*

The End of the Third Book.





Mathematical ELEMENTS

O F

Natural Philosophy,

Confirm'd by

EXPERIMENTS.

BOOK IV.

PART I. *Of the System of the World.*

CHAP. I.

A general Idea of the Planetary System.



* **H**E who attentively considers that * 15
Space can be terminated by no
Bounds, will scarce be able to
deny, that the Supreme Almighty
Intelligence has every where
manifested the same Wisdom which he has shewn
to the Inhabitants of the Earth in a small Com-
pass. What I here call a small Compass, im-

menly exceeds our Comprehension ; yet it is but small, compared with infinite Space.

- 921 *Our Earth, with sixteen other Bodies* (for we have no Knowledge of any more) *moves in a determinate Space* ; neither do these Bodies recede from, or come nearer to one another, beyond their set Bounds ; and their Motions are perform'd according to unchangeable Laws.

DEFINITION I.

- 922 *This Collection of seventeen Bodies is called the Planetary System.*

The whole Art of *Astronomy* is almost employed concerning these alone, and these will be chiefly my Subject in this Work ; the other Bodies that constitute the Universe are too far distant from us for their Motions (if they are mov'd) to fall under our Observations ; of these only the lucid Bodies can be perceiv'd by us, and of those only the more remarkable ones, and which are less distant from us than the rest ; even most of such as are seen by the Telescope are invisible to the naked Eye.

DEFINITION II.

- 923 All these Bodies are called *fixed Stars*.

They are called fixed, because, as far as can be perceiv'd, they keep the same Position with respect to one another ; we must take notice of something peculiar concerning these hereafter.

- 924 But as to the Planetary System, *in this* we have said *there are seventeen Bodies, which are all spherical ; one only shines by its own Light, the rest are opaque, and are visible only by the Light which they borrow from that.*

- 925 *The Sun is that lucid Body, and far the greatest of all in the Planetary System ; it is quiescent in the middle*

dle of it, at least is agitated by a very small Motion.

DEFINITION III.

The other Sixteen are called *Planets*.

These are divided into two Classes; six are called 926
the primary Planets; ten are called the secondary
Planets. When we speak of the Planets without
any distinction, we always understand the pri-
mary ones.

The primary Planets move round the Sun, and are 927
carried at different Distances from it, in Curves
that return into themselves.

A secondary Planet revolves round a primary one, 928
and accompanies it in its Motion round the Sun.

The Planets in their Motions describe elliptic Lines, 929
not much different from Circles; and all those
Lines are fixed; at least there is but a small
Change to be observed in a long Time in their
Situation.

Plate XXIV. Fig. 3.] An elliptic Line is form- 930
ed, if a Thread, whose Extremities are fix'd in the
two Points F and f, is mov'd, remaining stretch-
ed; as is to be seen in this Figure, in which the
Thread is represented at F d f, F E f, F B f;
the Points F, f, are called the Foci; the Line
A a, which passes thro' them, and is terminated
on each side by the Circumference of the Ellipse,
is called the greater Axis, and it is the greatest
Line that can be drawn in the Ellipsis; the mid-
dle Point C of the Axis is the Center of the El-
lipse; and the lesser Axis D d falls perpendicularly
thro' this Point upon the greater Axis.

The Orbits of all the primary Planets are in such 931
a Position, that one of their Foci falls in with the
Center of the Sun; let the Ellipse A D a d repre-
sent the Orbit of a Planet, F will be the Center
of the Sun.

DEFINITION IV.

932 *The Distance between the Center of the Sun and the Center of the Orbit is called the Eccentricity of the Planet ; as F C.*

933 *In every Revolution the Planet approaches once to the Sun, and once recedes from it ; and at its greatest distance is at a , the Extremity of the greater Axis of the Orbit ; and at its least distance in the opposite Extremity A.*

DEFINITION V.

934 *That distance of the Planet from the Sun is called the mean distance, which is equally different from the greatest and the least ; at this distance the Planet is in the Extremities D, d , of the smaller Axis.*

DEFINITION VI.

935 *The Point of the Orbit, in which the Planet is at its greatest distance from the Sun, is called the Aphelium ; as a .*

DEFINITION VII.

936 *The Point of the Orbit, in which the Planet is at its least distance from the Sun, is called the Perihelium,*

DEFINITION VIII.

937 *These Points are commonly called the *Auges*, or *Apsides*.*

DEFINITION IX.

938 *The Line which joins the Apsides, that is, the greater Axis of the Orbit, is called the Line of the Apsides.*

939 *Every Orbit is in a Plane which passes thro' the Center of the Sun.*

DEFI.

DEFINITION X.

The Plane of the Orbit of the Earth is called the 940
Plane of the Ecliptic.

This Plane is to be continu'd every way ; and
Astronomers consider the Position of the Planes of
the other Orbits with respect to this.

DEFINITION XI.

The Points in which the other Orbits cut the Plane 941
of the Ecliptic, are called the Nodes.

DEFINITION XII.

The Line which joins the Nodes of any Orbit, that 942
is, the common Section of the Plane of the Orbit
with the Plane of the Ecliptic, is called the Line
of Nodes.

A Planet is not carried with an equal Celerity in 943
all the Points of its Orbit ; the less it is distant from 944
the Sun, the swifter is its Motion ; and the Times in
which the several Arcs of its Orbit are run thro', are
to one another as the Area's formed by Lines drawn
from the Planet to the Center of the Sun ; the Arcs
A B and a E are run thro' in Times, which are
to one another as the Area's of the mix'd Trian-
gles A F B, a F E.

All the Planets are carried the same way, and 945
their Motion in their Orbits is contrary to that
Motion which we observe daily in all the celestial
Bodies, by which in one Day they seem to be car-
ried round the Earth ; of which hereafter.

DEFINITION XIII.

A Motion, such as is that of the Planets in their 946
Orbits, is said to be in consequentia, and direct.

DEFINITION XIV.

947 *The contrary Motion is called a Motion in antecedentia ; and sometimes retrograde.*

948 *The more distant the Planets are from the Sun, the slower they move in their Orbits ; so that the periodical Times of the most distant ones are greater, both because they have a greater Orbit to run thro', and a slower Motion.*

DEFINITION XV.

949 *The Line which passes thro' the Center of the Planet, and about which it moves, is called the Axis of the Planet.*

950 *The Planets, at least most of them, and the Sun itself, move round their Axis ; there are two, of which, in this respect, Astronomers have been able to make no Observations, but which in all Probability have this Motion.*

951 *This Motion agrees or conspires with the Motions of the Planets in their Orbits ; that is, it is in consequentia.*

952 *The Axes themselves are mov'd by a parallel Motion, so that all the Points of the Axis of a Planet describe equal and similar Lines.*

DEFINITION XVI.

953 *The Extremities of the Axis are called the Poles of the Planet.*

954 *Plate XX. Fig. 1.] Astronomers compare together accurately enough the Distances of the Planets from the Sun, to give us an Idea of the whole System. The Dimensions of the Orbits are represented in this Scheme, in which the Points N N shew the Nodes of each Orbit.*

955 *Nevertheless we cannot compare the Dimensions of this System with any that we know upon the Surface of the Earth ; for no Astronomers will assert, that*

that the Observations made concerning such a Comparison are free from Error.

But that the several Parts of the System may 956 be compared together, we suppose the mean Distance of the Earth from the Sun divided into 1000 equal Parts, which we make use of in measuring the other Distances.

The Sun ☉, as was said before, is agitated by 957 a small Motion in the middle of the System; it moves round its Axis in the Space of 25 Days; and its Axis is inclin'd to the Plane of the Ecliptic, making an Angle of 87 Deg. 30 Min.

Mercury ☿ is the least distant from the Sun of 958 any of the Planets; its mean Distance from the Sun is 387, its Eccentricity 80; the Inclination of its Orbit, that is, the Angle form'd by the Plane of its Orbit with the Plane of the Ecliptic, is 6 Degr. 52 Min. it performs its Revolution round the Sun in 87 Days, 23 Hours.

The next is *Venus* ♀, whose Distance from the 959 Sun is 723, its Eccentricity 5, the Inclination of its Orbit 3 Deg. 23 Min. It performs its periodical Motion in 224 Days, 17 Hours; and its Motion round its Axis is perform'd in 23 Hours.

The third Planet in order from the Sun is our 960 Earth ⊕; its mean Distance from the Sun is 1000, its Eccentricity 169; it is moved in the Plane of the Ecliptic; its periodical Time is 365 Days, 5 Hours, 51 Min. and the Motion about its Axis is perform'd in 23 Hours, 56 Min. 4 Sec. Its Axis makes an Angle with the Plane of the Ecliptic of 66 Deg. 31 Min.

The mean Distance of *Mars* ♂ from the Sun 961 is 1524, its Eccentricity 141, the Inclination of its Orbit 1 Deg. 52 Min. its periodical Time 686 Days, 23 Hours; its Revolution about its Axis is perform'd in 24 Hours, 40 Min.

Jupiter

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962 *Jupiter* ♃, the biggest of all the Planets, is distant from the Sun, at a mean distance, 5201 ; its Eccentricity 250 ; the Inclination of its Orbit 1 Deg. 20 Min. the periodical Time 4332 Days, 12 Hours ; and its Revolution about its Axis in 9 Hours and 56 Min.

963 The mean Distance of *Saturn* ♄, the most distant Planet from the Sun, is 9538 ; its Eccentricity 547 ; the Inclination of its Orbit 2 Deg. 30 Min. the periodical Time 10759 Days, 7 Hours ; it is encompass'd with a Ring, which does not touch the Planet, but never leaves it ; this Ring is not visible without a Telescope.

The mean Distance being given, if you add the Eccentricity, you will have the greatest Distance ; but if you subtract the Eccentricity from the mean distance, you will have the least di-

• 836 stance. •

964 The three Planets, *Mars*, *Jupiter*, and *Saturn*, which are more distant from the Sun than the Earth, are call'd the *superior Planets* ; *Venus* and *Mercury* are call'd the *inferior ones*.

965 Of the *primary Planets*, three are accompanied by *secondary ones*.

Five Planets call'd *Satellites*, move about *Saturn* ; four about *Jupiter* ; one about the Earth (*viz.*) the Moon.

The *secondary Planets*, except the Moon, are not visible to the naked Eye.

966 The *Satellites*, by Lines drawn to the Center of the *primary Planets*, describe Area's about them proportional to the Times ; as has been said of the *primary Planets* with respect to the Center of the

• 944 Sun. •

967 The Moon moves about the Earth in an Ellipse,
968 one of whose Foci is in the Center of the Earth, from
969 which the mean distance of the Moon is 60 Semi-diameters of the Earth and a half ; its Eccentricity

is liable to change; the mean one is of 3 Semidiameters and a Third; the Plane of its Orbit forms an Angle with the Plane of the Ecliptic of five Degrees; but this Inclination is not constant, or always the same. In the Motion of the Moon round the Earth, neither the Line of the Nodes, nor the Line of the Apsides, is carried in a parallel Motion, but the Line of the Nodes Westward, or in antecedentia; the Lines of the Apsides Eastward, or in consequentia; the first performs its Revolution in about 9 Years, the second in 19; the periodical Time of the Moon's Motion about the Earth is 27 Days and about 7 Hours; and it is turn'd about its own Axis exactly in the same Time.

Plate XX. Fig. 2.] The first, or inmost of the 971
Satellites of Jupiter, is distant from Jupiter's Center $2\frac{1}{2}$ Diameters of Jupiter; it is mov'd round Jupiter in one Day, 18 Hours, 28 Min.

The Distance of the second is 4 and an half Diameters of Jupiter; its periodical Time is 13 Hours, 18 Min.

The Distance of the third is $7\frac{1}{2}$ Diameters; its periodical Time 7 Days, 4 Hours.

The fourth is distant $12\frac{1}{2}$ Diameters; it performs its Motion in 16 Days, 18 Hours, 5 Min.

Plate XX. Fig. 3.] The first, or inmost Satellite of Saturn, is distant from Saturn's Center $\frac{2}{3}$ of a Diameter of the Ring; its periodical Time is 1 Day, 21 Hours, 18 Min. 972

The Distance of the second is $1\frac{1}{2}$ Diameter of the Ring; its periodical Time is two Days, 17 Hours, 41 Min.

The Distance of the third is $1\frac{1}{2}$ Diameter of the Ring, its periodical Motion is 4 Days, 13 Hours, 47 Min.

The

The Distance of the fourth is 4 Diameters of the Ring, its periodical Time 15 Days, 22 Hours, 41 Min.

The distance of the fifth is 12 Diameters of the Ring; its periodical Time is 79 Days, 7 Hours, 53 Min.

973 Concerning the Motion of these, as also of the Satellites of *Jupiter* about their Axes, we can hitherto determine nothing certain from astronomical Observations.

If we take notice of the Distances and periodical Times of the Planets, we shall find that the following Rule holds good in our System; wherever several Bodies are moved round the same Point, that is, *about the Sun, Saturn, and*

974 *Jupiter (viz.) the Squares of the periodical Times are to one another as the Cubes of the mean Distances from the Center.*

975 *Plate XX. Fig. 4.] To give a Notion of the Dimensions of the Bodies themselves in our System, we have contriv'd the fourth Figure, in which all the primary Planets and Saturn's Ring are describ'd according to their Dimensions; the Sun, whose Magnitude exceeds all the rest, is represented by the greatest Circle (Fig. 1.) that is, the Circle which terminates the Figure.*

These Dimensions represent the Proportions of the Bodies one to another exactly enough, if you except the Earth, which, for the Reason already mention'd*, cannot be so compar'd with the other Bodies, as to leave no room to doubt what Proportion it bears to them.

976 Yet the Earth's Diameter may be measur'd, and contains 3.400669 Perches, each of which is equal to 12 Rhynland Feet; but altho' the Diameters of the other Planets may be compar'd together, and with the Sun's Diameter, yet it can't be determin'd how many Feet they contain, till after

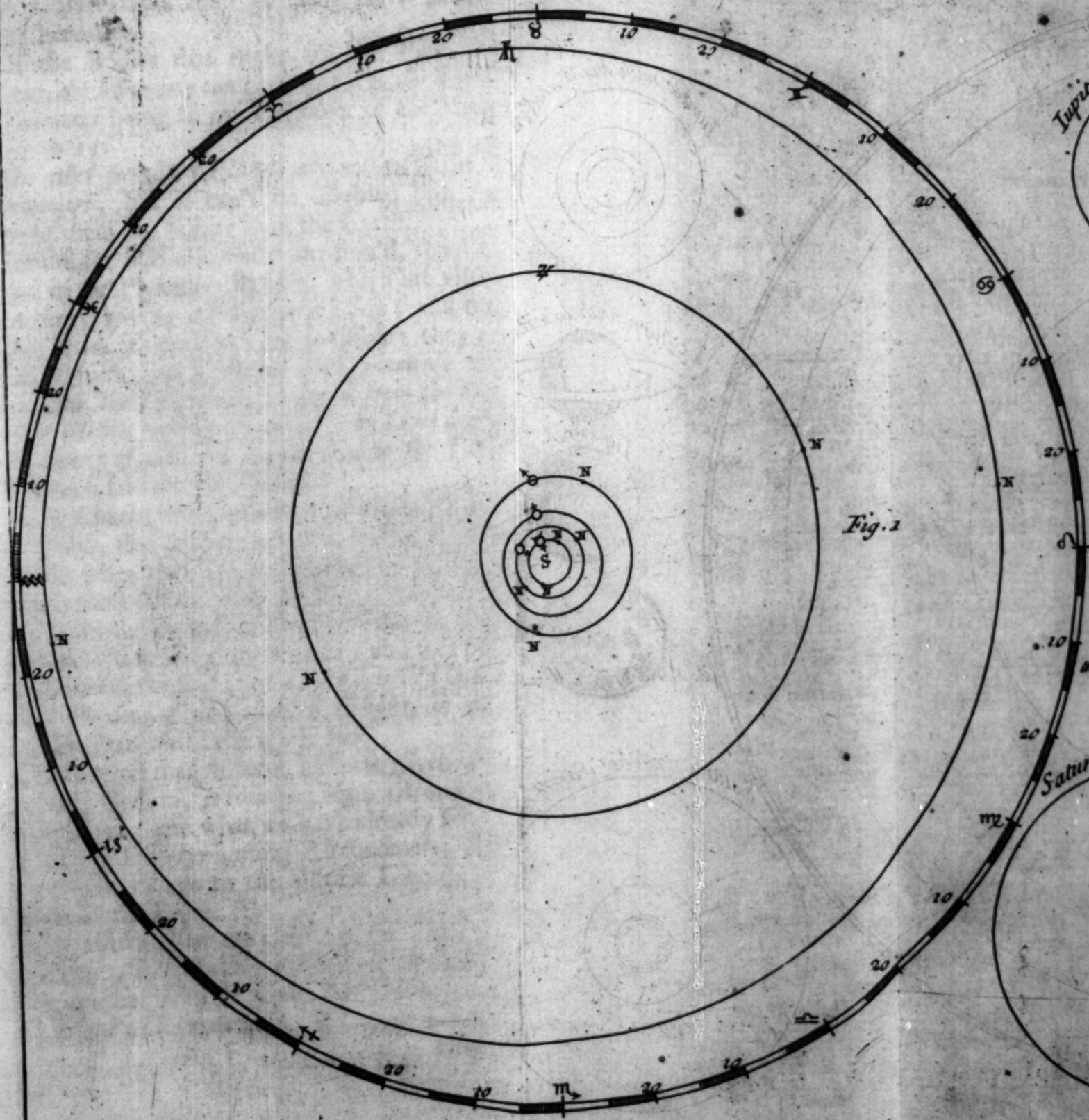


Fig. 1

Jupiter with his Satellites



Fig. 2

Saturn with his Ring



Jup-ter



Fig. 4

Mars

Venus

the Earth

Mercury

Saturn with his Satellites

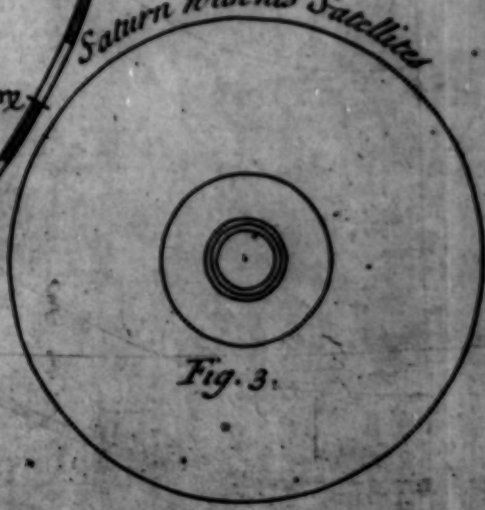
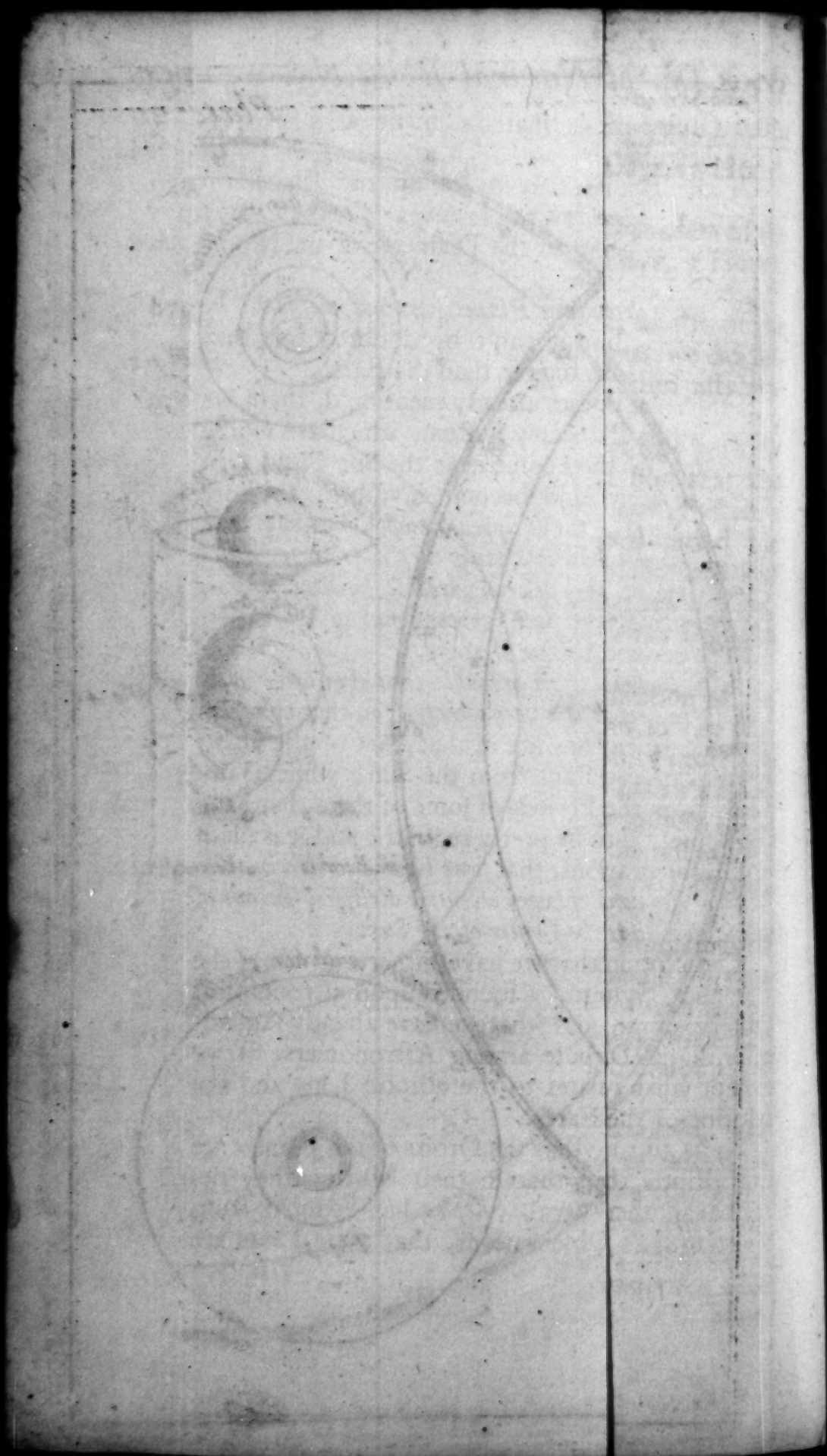


Fig. 3



B
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after Observations shall be made at a proper Time hereafter.

Of the Bodies that make up the Planetary System, *the Moon only can be compar'd to the Earth*, its Diameter being to the Diameter of the Moon as 40 to 11.

The other secondary Planets are not measur'd by Astronomers, but it can't be doubted but that some of them are bigger than the Earth.

Besides the Bodies already mention'd, there are others in the Planetary System, which are visible for a time, as they come near the Sun; and then recede from it, and become invisible; they are called *Comets*; they appear most commonly with Tails, and the Tail is always turn'd from the Sun; in their Motion they describe Area's, by Lines drawn to the Center of the Sun, proportional to the Times, as has been said of the Planets. *

As to Comets, it is probable that they move in elliptic Orbits, that are very eccentric, so that they are invisible when they are in that Part of the Orbit which is most distant from the Sun; which is deduc'd from the Periods of some of them that have been observ'd to be pretty regular; and it is plain from Observations, that *some have describ'd in their Motion Portions of very eccentric Eclipses, in one of whose Foci was the Center of the Sun.*

The Notion that we have hitherto given of the Planetary System, is founded upon astronomical Observations; and what we have already said admits of no Dispute among Astronomers, if we except what relates to the elliptic Line and the Motion of the Earth.

Some affirm, that the Orbits of the Planets are not elliptic, but that in their Motion they describe another Oval. *Kepler* has deduc'd from *Tycho Brabé's* Observations, that these Lines are elliptic;

elliptic ; and we shall shew in the following Part that no other Curves can be describ'd by the Planets.

Those that say the Earth is at rest, have no astronomical or physical Argument for a Foundation of their Opinion ; that is, don't reason from *Phænomena* ; neglecting the Simplicity of the System, and the Analogy of the Motions, they assert, that their Opinion is not contrary to Observations ; in which they err, as we shall shew in the following Part.

CH A P. II.

Concerning the apparent Motion.

WHoever, after having read the former Chapter, looks upon the Heavens, will scarcely believe that he beholds the System which is explain'd there ; and a more exact Consideration of the heavenly Motions will increase his
 983 Doubt : no wonder, *since we can observe very little in the Heavens but false Appearances.*

The common Observer of the Heavens is a Spectator, who thinking himself to be at rest, is carried about by various Motions, and beholds Bodies, concerning whose Distance and Magnitude he makes false Judgments. The true System of the World was unknown for many Ages, even to the most exact Observers of the Heavens.

But

But we must explain how all things which are observ'd in the heavenly Bodies have Place in the System that has been explain'd, in respect of a Spectator upon the Earth; that is, we shall deduce the Appearances from the true Motions; which cannot be done, unless we first lay down some general things concerning the apparent Motion in general.

It is certain that we have no Art by which we can discover the true Motion, only the relative Motion can be perceiv'd by the Senses; and it is that only concerning which we treated in the former Chapter. Who can reasonably affirm or deny, that all the Bodies which are known to us, are not carried in a common Motion thro' the immense Spaces?

The relative Motion is to be distinguish'd from the apparent one; for the apparent Motion is the Change which appears to be in the Situation of the Bodies, and depends upon the Change of the Picture in the bottom of the Eye; for Objects have the same apparent Relation to one another as their Representations have in the Eye, for they are seen as they are painted in the Eye*; and the Change in that Picture, from the Motion of the Bodies, most commonly differs from the Change of the Relation between the Bodies themselves; as follows from the Formation of that Picture.

The Heavens are nothing but an immense Space, which cannot be seen, and would appear black*, if innumerable Rays of Light, flowing from the heavenly Bodies, did not continually penetrate our Atmosphere; most of them come to us from the Bodies in right Lines, yet a great many suffer various Reflexions in the Atmosphere, and enlighten the whole Atmosphere; which is the reason that in the Day Bodies are enlighten'd even

without the Reflexion of the Clouds, to which the Rays of the Sun cannot come directly.

- These Rays are heterogeneous and white, for there are Bodies enlighten'd by these Rays which appear white; and these Bodies seen thro' Prisms, at the Extremities, are ting'd with Colours, which does not happen in an homogeneous Colour*;
 • 874 also a Circle of white Paper, of half an Inch diameter, being put upon black Cloth, if it is enlighten'd by these Rays, it will appear oblong
 • 861 thro' the Prism*; and the same Colours which are observ'd in the Rays of the Sun, are seen here in the same manner; all which things would not happen, if the Air, as many think, was a blue Liquid, that is, thro' which only the Sun's blue Rays, at least mostly such, pass.

- 987 *When we look at the black Sky, the white Rays before mention'd enter our Eyes, whence the blue Colour of the Sky arises; because we are accusom'd to see a Colour where there is a colour'd Object, we also refer the Colour of the Heavens to an Object; but since this is seen equally towards all*
 988 *Parts, we conceive a concave spherical Surface, in whose Center we are placed; we imagine this Surface to be opaque, and therefore distant from us beyond all visible Bodies.*

When a Body is between a Plane and the Eye, of whose distance we cannot judge, the Body appears to us to be applied to the Plane, whatsoever the distance is between that and the Plane; for there is no reason why the Parts of the Plane, which are painted at the Sides of the Image of the Body in the Eye, should not appear at the same distance with the Body.

- 989 Thence also all the celestial Bodies (of which the least distant from us (*viz.*) the Moon) is yet so remov'd, that we can give no Judgment of its
 • 732 distance*, are refer'd to that imaginary Sphere
 above.

above-mention'd, and they all appear equally distant, and seem to move in the Surface of a concave Sphere; so the Moon appears to be among the fixed Stars, altho' its distance bears scarcely any sensible Proportion to the distance of Saturn; and the distance even of Saturn itself is nothing, compar'd with the immense distance of the fixed Stars; it is no wonder therefore that the common People know nothing of the Magnitude of the celestial Bodies and the Immenstity of the Heavens.

We see from what has been said, how the Motion of any Body being given, and the Motion of the Earth being known, the apparent Motion may be determin'd.

We have said that a Sphere is imagin'd beyond the fixed Stars, in whose Center is the Spectator*. • 988
The Orbit of the Earth is so small in respect of the Diameter of this Sphere, that the Center of the Sphere is not sensibly chang'd by the Alteration of the Place of the Spectator, whilst he is carried along with the Earth; wherefore in all the Points of the Earth's Surface, and at any Time, the Inhabitants of the Earth imagine the same Sphere; to which they refer the heavenly Bodies; and which hereafter we shall call the Sphere of the fixed Stars. 998

These things being laid down, if we conceive a Line to be drawn thro' the Earth and a Body, which being continu'd beyond the Body, cuts the aforesaid Sphere, we have a Point, to which the above-mention'd Body is referr'd, and which is the apparent Place of that Body. 991

Whilst the Body, or the Earth, or both are mov'd, this Line is mov'd also, and the apparent Motion is the Line, which is describ'd amongst the fixed Stars by the Extremity of the Line above mention'd, which goes thro' the Earth and the Body; whose apparent Motion is observ'd. 992

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993 Therefore the same Appearances will follow from the Earth's being mov'd out of its Place, as if the Body had been mov'd; and the same also may be deduc'd from the Motion of both.

994 But if the Body and the Earth be so mov'd, that the Line which passes thro' these Bodies be carried in a parallel Motion, the Body will seem to be at rest among the fixed Stars; because in this Case the Space gone thro' by the End of the Line among the fixed Stars, cannot exceed the Space gone thro' by the Earth; but the Line that is equal to the whole Space which the Earth can go thro' at so great a distance as the fixed Stars, is not sensible to us.

995 From the Motion of the Earth round its Axis there is also produc'd an apparent Motion, which will be easily deduc'd in its proper Place from the Foundation laid down in this Chapter.

That the apparent Motion differs from the relative, and is varied by the Motion of the Spectator, is what Sailors every Day observe.

CH A P. III.

Of the Phenomena or Appearances of the Sun from the Motion of the Earth in its Orbit.

Plate XXI. **L** E T the Sun be at S, and the Earth in its Orbit at T, and let

$r s$ be the Sphere of the fixed Stars; the apparent

• 991 Place of the Sun will be at s^* . When the Earth is

996 carried in its Orbit from T to t , the Sun seems to

move in consequentia, and to run thro' the Arc

• 992 $s r^*$, which measures the Angle $r S s$ equal to the

Angle T S t ; so that the Celerity of the apparent Motion of the Sun depends upon the Celerity of the angular Motion of the Earth, with respect

to the Center of the Sun; which Motion increases upon a double Account; on account of the distance from the Sun being lessen'd, and the Celerity of the Earth being increas'd; both which Causes always concur; * *wherefore the Inequality* * 944
of the apparent Motion of the Sun is sensible. In a 997
whole Revolution of the Earth, the Sun also seems to 998
run thro' a whole Circle.

DEFINITION I.

This apparent Way of the Sun is called the Ecliptic 999
Line; it is the Section of the Sphere of the
fixed Stars with the Plane of the Ecliptic, suppo-
sed to be continu'd to this Sphere.

This Way is divided into 12 equal Parts, each of which contains 30 Degrees; these Parts are called the Signs, and are distinguish'd by these Names: *Aries* ♈, *Taurus* ♉, *Gemini* ♊, *Cancer* ♋, *Leo* ♌, *Virgo* ♍, *Libra* ♎, *Scorpius* ♏, *Sagittarius* ♐, *Capricorn* ♑, *Aquarius* ♒, *Pisces* ♓. Whence these Parts have their Names, we shall explain when we treat of the fixed Stars.

The Sun is longer a going thro' the six first Signs 1000
than the six last, and the difference is nine Days.

Altho' a Circle has neither Beginning nor End, 1001
yet when several Points must be determin'd in it, some Point must be taken as *the Beginning*; this, *in the Ecliptic Line, is the first Point of Aries*; we shall shew how it may be determin'd hereafter; it is not fixed to one Place among the fixed Stars; therefore *the Orbits of the Planets, which alter so* 1002
little, that they may be look'd upon as unchange-
*able, * don't preserve the same Situation, in respect* * 929
of this Point.

DEFINITION II.

1003 *The Distance of the Sun from the first Point of Aries, measured in consequentia, is called the Sun's Longitude.*

1004 *The Longitudes of the other heavenly Bodies are measured after the same manner in the Ecliptic; they*

1005 *are referr'd to this Line, by conceiving a great Circle to pass thro' the Body, and cut the Ecliptic perpendicularly; for the Point in which the Ecliptic is cut by this Circle, determines the Longitude of the Body.*

DEFINITION III.

1006 *The Distance of a heavenly Body from the Ecliptic is called its Latitude; it is measur'd by an Arc of a great Circle perpendicular to the Ecliptic, intercepted between the Body and the Ecliptic.*

DEFINITION IV.

1007 *If we imagine a Line to go thro' the Center of the Sphere of the fixed Stars, and perpendicular to the Ecliptic, the Points in which this cuts the above-mentioned Sphere, are called the Poles of the Ecliptic.*

DEFINITION V.

1008 *The Zodiac is a Zone which is imagin'd in the Heavens, which the Ecliptic Line cuts into two equal Parts, and which, on either side, is terminated by a Circle parallel to the Ecliptic Line, and eight Degrees distant from it. On account of the small Inclinations of the Orbits of the Planets, and the Moon to the Plane of the Ecliptic, no Bodies of the*

1009 *Planetary System appear without the Zodiac.*

DEFINITION VI.

Those of them that have the same Longitude are 1010
said to be in Conjunction.

DEFINITION VII.

Those whose Longitudes differ 180 Degrees, are 1011
said to be in Opposition.

CHAP. IV.

Of the Phænomena of the inferior Planets,
arising from the Earth's and their own
Motions in their Orbits.

Plate XXI. **L**ET S be the Sun, A V B *v* the
Fig. 2.] Orbit of an inferior Planet; let
T be the Earth in its Orbit, and *a v b* Part of the
Sphere of the fixed Stars; the apparent Place of
the Sun is *v* *. * 991

If from the Earth there be drawn to the Orbit
of the Planet the Tangents T A *a*, T B *b*, it is
evident that the Planet, in its apparent Motion,
is never remov'd farther from the Sun than the
Distance *v a*, *v b*; and that the Planet accom-
panies it in its apparent Motion round the Earth.

DEFINITION I.

The apparent Distance of the Planet from the Sun 1012
is called its Elongation; *v a* or *v b* is the greatest
Elongation; this varies upon two Accounts (*viz.*) 1013
because the Earth and the Planet revolve in el-
liptic Lines *. * 929

The Planet performs its Motion sooner than 1014
the Earth, * therefore in its Motion it passes between * 948
the Earth and the Sun, and then moves beyond the

Sun, in respect of the Earth; so that it is in Conjunction with the Sun in two Manners, but never in Opposition.

That we may have an Idea of the apparent Motion of the Planet, we must conceive the Lines $T B b$, $T S v$, $T A a$, to move along with the Earth, so that the Points A , V , B , and v , whilst the Earth performs its Revolution, may run thro' the Orbit of the Planet; but the Planet, which moves swifter, passes successively through these Points over and over.

1015 *When it is carried in its Orbit from V to D , it seems to move among the fixed Stars from v to d ; in this Case the apparent Motion is in anteceden-*

1016 *tia, and the Planet is retrograde; in D it is said to be stationary, because it appears for some Time in the same Place among the fixed Stars; this obtains when the Orbit of the Planet, in the Place in which the Planet is, is so inclin'd to the Orbit of the Earth, in the Place in which the Earth is, that if the Line d be drawn parallel to the Line $T D$, and at a small distance from it $D d$ be to $T t$ as the Celerity of the Planet in its Orbit to the Celerity of the Earth; these Lines are run thro' in the same Time; * and the Line which is drawn thro' the Earth and the Planet is carried in a parallel Motion; for which Reason the apparent Place of the Planet is not chang'd*.*

* 992 *Between d and B the Orbit of the Planet is more inclin'd to the Orbit of the Earth, wherefore the Extremity of the Line passing thro' the Earth and the Planet (altho' the Planet moves*
1017 *swifter than the Earth) is carried in consequentia, towards which Part also the apparent Motion of the*
* 992 *Planet is directed* ; yet since the apparent Motion of the Sun exceeds the apparent Motion of the Planet, the Elongation is increas'd, which becomes greatest when the Planet is at B . Whilst the*

the

the Planet goes thro' the Arc Bv , its apparent Motion is also *in consequentia*, and exceeds the apparent Motion of the Sun, to which it is coming, and then goes beyond it, receding from it till it comes to A. Between A and E the Motion *in consequentia* is continu'd; but the Sun, whose apparent Motion in this Case is swifter, as has been explain'd concerning the Arc dB , comes towards the Planet, and the Elongation is diminish'd. At E, in the same manner as at D, the Planet is stationary; between E and V it is again retrograde. 1018

The Orbit of the Planet is inclin'd to the Plane of the Ecliptic, * therefore it *does not seem to move* * 958 *in the Ecliptic Line, but* sometimes less, sometimes 959 *more distant from it, and appears to be carried in an irregular Curve, which sometimes cuts the Ecliptic.*

[Plate XXI. Fig. 3.] Let NVN be the Orbit of the Planet, whose Nodes are NN ; let S be the Sun; Tt the Orbit of the Earth in the Plane of the Ecliptic; the Earth T ; the Planet V . If VA be imagin'd to pass thro' the Planet, and to be perpendicular to the Plane of the Ecliptic, the Angle VTA , or rather the Arc which measures it, is the Latitude of the Planet; * this * 1006 is called *the geocentric Latitude*, to distinguish it from the Latitude of the Planet seen from the Sun, which is called *the heliocentric Latitude*, and is in that Case the Angle VSA . Here we speak of the *geocentric Latitude*, because we examine the Phænomena as they appear from the Earth.

When a Planet appears in the Node, it appears in 1019 *the Ecliptic Line*; and the Curve, which is described by the Planet, by its apparent Motion in the Zodiac, cuts the Ecliptic Line; as the Planet re- 1120 *cedes from the Node, its Latitude is increased, which is also different, according to the Situation of the Earth;*

Earth; so the Planet remaining at V, the Latitude is greater if the Earth be at T, than if it was at *t*. Now, if the Earth remaining in its Place, we imagine the Planet to be carried from V to *v*, the Angle *v* T B will be less than the Angle V T A upon a double Account, from the Planet coming nearer the Node, and the Spectator being mov'd farther off.

Now if we consider that both the Earth and the Planet are continually mov'd, we shall easily conceive that the Latitude is chang'd every Moment from each Cause, which sometimes act contrariwise, and sometimes conspire in increasing and diminishing the Latitude; whence it necessarily follows, that the apparent Motion is perform'd in an irregular Curve, which, as was said before, cuts the Ecliptic as often as the Planet passes the Node, that is, twice in each of its Revolutions; this Curve also does not recede from the Ecliptic, on either side, beyond certain Limits in the Zodiac.

We discover also some remarkable Phænomena of the inferior Planets by means of the Telescope, which are owing to their Opacity.

[*Plate XXI. Fig. 4.*] Let S be the Sun, T the Earth, A, B, C, *v*, D, E, F, V, an inferior Planet, *ex gr. Venus* in its Orbit; this Planet shines with Light borrow'd from the Sun, and that Hemisphere only which is turn'd to the Sun is enlighten'd, the other Hemisphere is invisible; therefore that Part only of the enlighten'd Hemisphere which is turn'd to the Earth can be seen from it; in V the Planet cannot be seen; in *v* it would appear round, if the Sun's Rays did not hinder it from being seen.

7021 Going from *v* the Planet continually decreases; at D it has the Figure *d*; at *e* and *f* it is drawn as it appears at E and F, and continues to decrease

crease till it vanishes at V, and then again increases successively, changing its Figure till the whole enlighten'd Hemisphere be turned towards the Earth.

When the Node is at V, or near it, the Planet 1022 appears in the very Disk of the Sun, and as it were applied to it, and is observ'd as a black Spot which moves on the Sun's Surface; in this Case, properly speaking, we don't see the Planet, but we perceive where it intercepts the Sun's Rays.

The less distant the Planet is from the Earth, the 1023 greater it appears*, and the more lucid; but as it * 733 comes nearer the Earth, the lucid Part that is visible is less; so that on one account the Light increases, and on another it is diminish'd; and there is a Distance at which the reflected Light is greatest.

CHAP. V.

Concerning the Phænomena of the superior Planets, arising from their Motions and the Motion of the Earth in their respective Orbits.

THE apparent Motions of the superior Planets do in many things agree with what has been explain'd in respect of the inferior Planets, and in many things disagree; they do not 1024 always accompany the Sun, but are often observed in Opposition; but in their Opposition (as has been said of the inferior Planets) they do not always seem to be carried in consequentia, but often appear stationary, and often retrograde. 1025

1026 *Plate XXII. Fig. 1.]* Let M be a superior Planet, for Example, *Mars* in its Orbit, A T H B the Orbit of the Earth; the periodical Time of the Earth is shorter than the periodical Time of *Mars**, therefore the Earth in its Motion goes between it and the Sun; in which Case the Planet appears at F, among the fixed Stars, opposite to the Sun; thro' M draw the Lines B M, A M, that touch the Earth's Orbit, which being continu'd go to G and D in the Sphere of the fixed Stars; let us imagine that whilst the Planet is carried about in its Orbit those Lines are also moved, so that the Points A and B, in which the Lines that go thro' the Planet touch the Orbit of the Earth, perform a Revolution in the periodical Time of the Planet; now since the Earth revolves faster, it passes thro' the Points A and B in its Motion; in this Motion the apparent Place of the Planet seen from the Earth, is not remov'd from the Place of the Planet seen from the Sun beyond F D and F G. Let T be such a Point in the Orbit of the Earth, that the Line *t m* being drawn, may be parallel to T M; let T *z* be to M *m* as the Celerity of the Earth to the Celerity of the Planet, in which Case these small Lines are gone thro' in the same Time; * in the mean time the Planet seems to be at rest, * and is said to be stationary; in the same manner it is stationary when the Earth is at H. In the Motion of the Earth between T and H, the Planet appears to move in *antecedentia* from E thro' F, and is said to be retrograde; whilst the Earth goes thro' the rest of its Orbit the Planet is direct.

1027 The Phenomena which relate to the Latitude, are like those which have been explain'd in respect to the inferior Planets*.

1028 *Jupiter* and *Saturn* encompass the Orbit of the Earth at a great distance, wherefore almost their whole

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Plate 21.

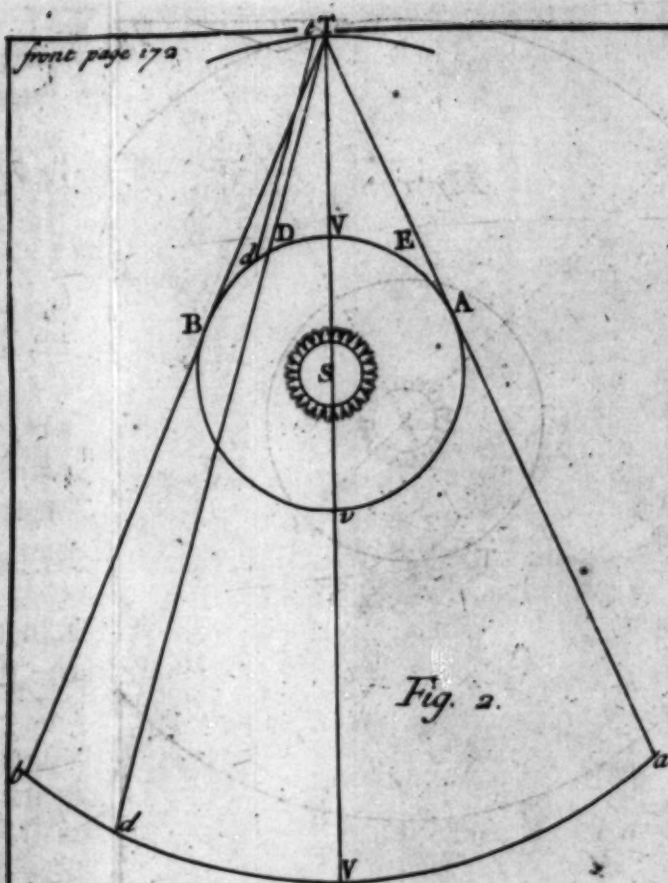


Fig. 2.

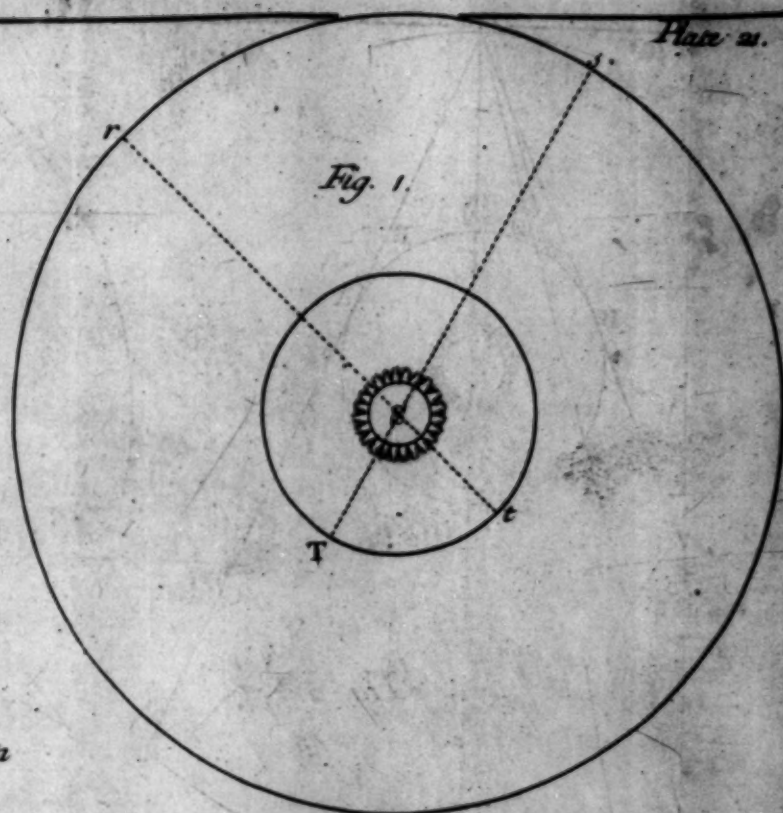


Fig. 1.

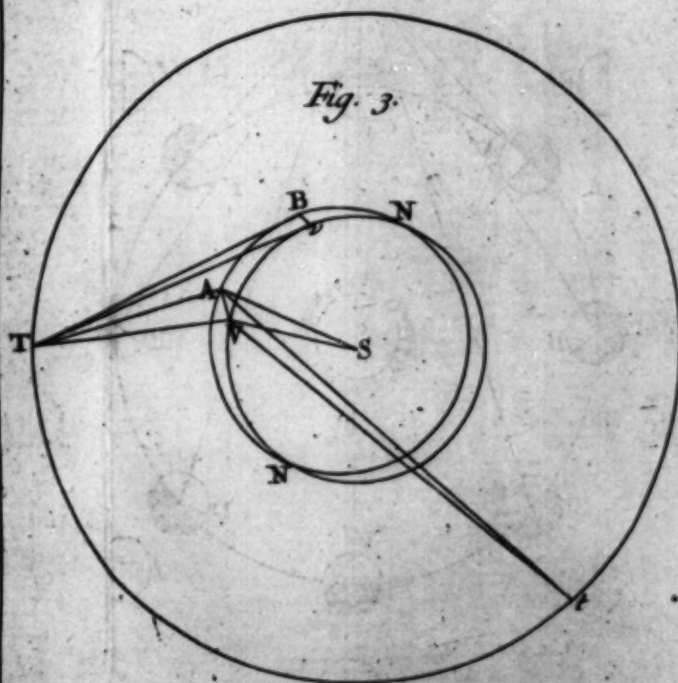


Fig. 3.

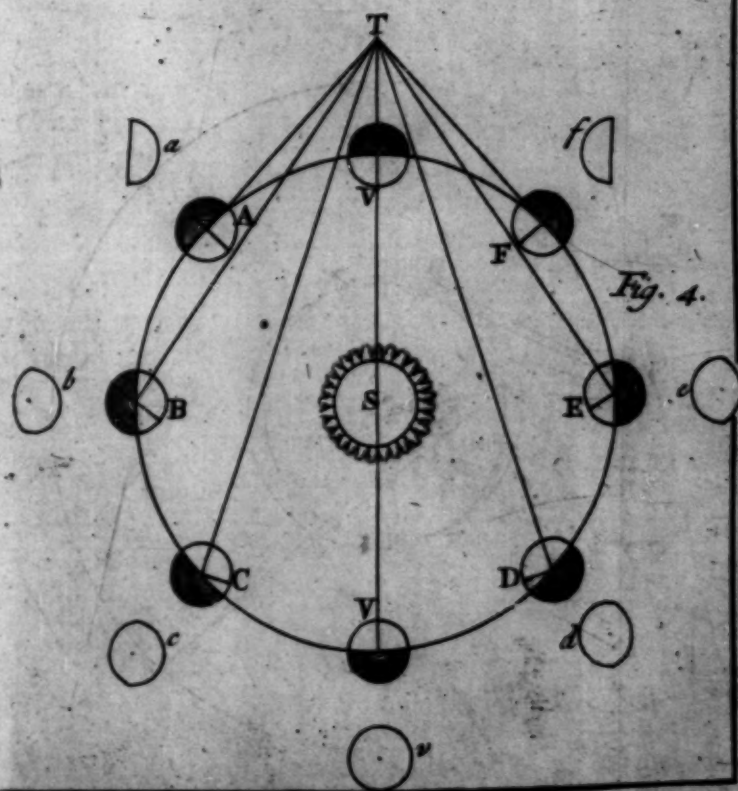
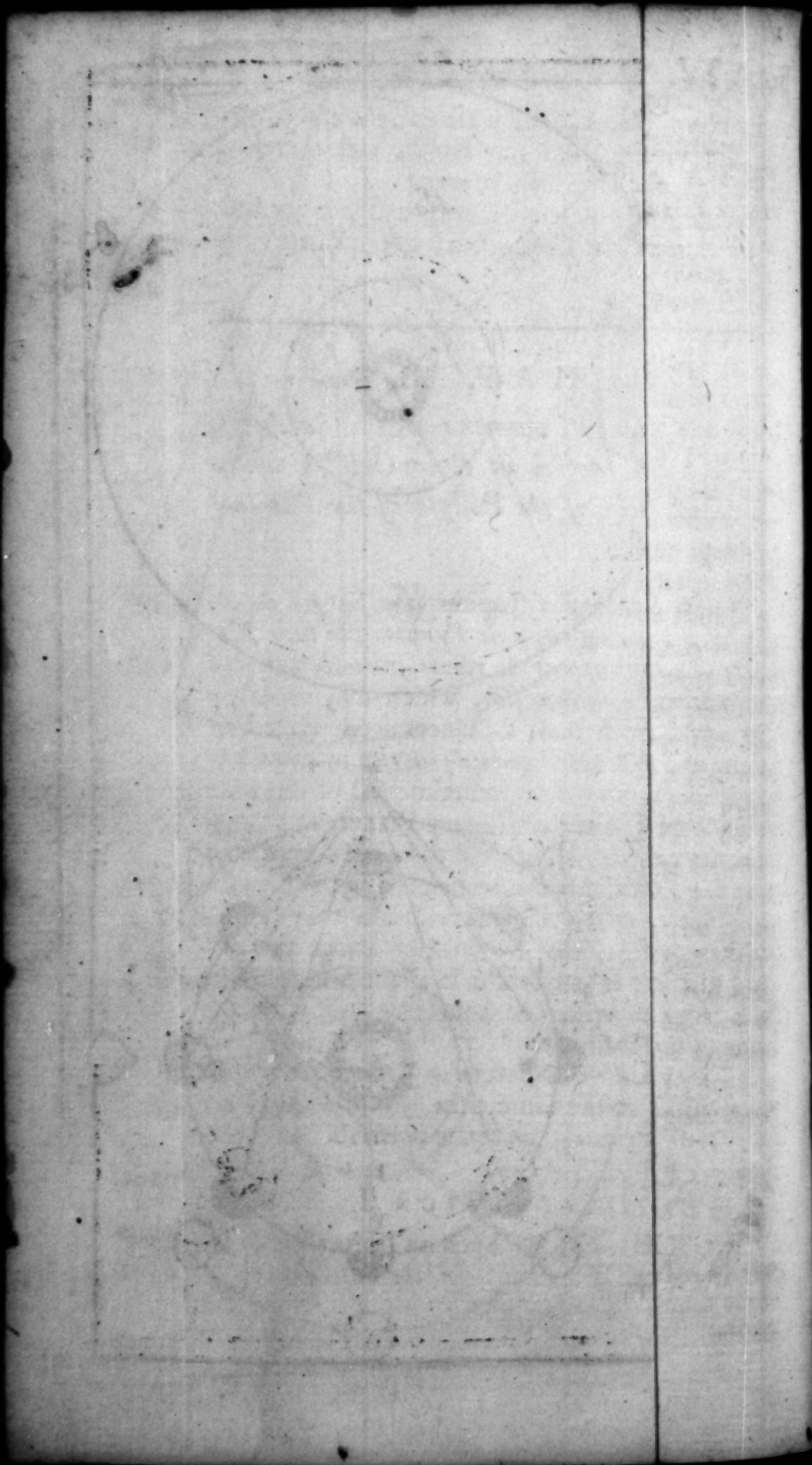


Fig. 4.

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whole Hemispheres, which are enlighten'd by the Sun, are visible from the Earth, and therefore these Planets *always appear round.*

Because Mars is less distant, it *appears a little gibbous between the Conjunction and Opposition with the Sun.*

C H A P. VI.

Concerning the Phænomena of the Satellites, from their Motion in their Orbits; where we shall speak of the Eclipses of the Sun and Moon.

THE Satellites of Jupiter and Saturn do *always accompany their Primaries in their Motion, and never appear to recede from them beyond certain Limits on either side, which may be easily determin'd from their Distances from their Primaries, and they are alternately carried in antecessoria and in consequentia; sometimes all of them are on the same side of the primary Planet, and sometimes the primary is observ'd to be between them; they are all always in the same right Line, or very little distant from it; all which things may be deduc'd from this, that the Motion about the primary Planets is perform'd in Planets that make small Angles with one another, and with the Plane of the Ecliptic.*

All the Satellites of Saturn or Jupiter are not always visible at the same time, sometimes they are hid by their Primary, and often immers'd in its Shadow.

DEFINITION I.

Plate XXII. Fig. 2.] Such an Immersion in the Shadow is call'd an Eclipse of the Satellite.

Let

Let S be the Sun, T the Orbit of the Earth, I Jupiter, M m the Orbit of a Secondary of Jupiter; whilst the Secondary moves from M to m it undergoes an Eclipse, and not being enlighten'd by the Sun, becomes invisible; if the Earth be at T, the Emerfion into the Shadow is easily observed; on the contrary, the Emerfion is more sensible if the Earth be placed at t.

- 1034 Among the Bodies that accompany Saturn we
 * 963 have said that there is a Ring, * concerning which it is to be observ'd, *that an Observer upon the Earth never sees it wider than it is represented in the 4th Fig. of Plate XX. and that sometimes it is invisible*; namely, when the Plane of the Ring being continu'd goes thro' the Earth, for the thicknefs of the Ring is not sensible; the Ring is also invisible when its Plane continu'd passes between the Earth and the Sun, for then the enlighten'd Surface of the Ring is turn'd from the Earth; in each Case Saturn appears round; yet in the last Case, by reason of the Rays that are intercepted by the Ring, there appears a black Belt upon the Surface of the Planet, like that which is occasion'd by the Shadow of the Ring.

The Phænomena of the Earth's Satellite, namely, of the Moon, are very remarkable in respect to us, and therefore particularly to be explain'd.

- 1035 It is very often in Conjunction with the Sun; and as often in Opposition to it, but not at every Revolution of the Moon in its Orbit; for whilst the Moon, after one entire Revolution of 27 Days and 7 Hours, returns again to the Place among
 1036 the fixed Stars, in which it was in Conjunction
 * 960 with the Sun, the Sun is gone from that Place,
 996 and is about 27 Deg. distant from it; * therefore
 998 the neighbouring Conjunctions are 29 Days and a half
 distant from one another.

DEFINITION II.

The lunar periodical Month is the Time of one 1037
Revolution of the Moon in its Orbit.

DEFINITION III.

The Moon's synodical Month, or a Lunation, 1038
*is the Time that the Moon spends between the two
 next Conjunctions with the Sun.*

The Moon is invisible in its Conjunction with the 1039
 Sun, because the enlighten'd Hemisphere is turn'd
 from the Earth. Let T (Plate XXII. Fig. 3.) be
 the Earth, N the Moon between the Sun and the
 Earth, the enlighten'd Hemisphere will be *mli*,
 which cannot be seen from the Earth.

Whilst the Moon is carried in its Orbit from the 1040
 Conjunction to the Opposition, the enlighten'd Part,
 which is directed towards the Sun, does continually
 become more and more visible to the Inhabitants of the
 Earth; and in the Points A B C the Moon does
 successively acquire the Figures *a b c*.

At P, in its Opposition with the Sun, it appears 1041
 round; then going thro' D E F it decreases, as it is
 represented at *d e f*.

DEFINITION IV.

The Conjunction of the Moon with the Sun is called 1042
 the New-Moon.

After the Conjunction, the Moon is as it were
 renew'd.

DEFINITION V.

The Opposition of the Moon with the Sun is called 1043
 the Full-Moon, because the whole Moon appears en-
 lighten'd.

DEFI-

DEFINITION VI.

1044 *The Conjunction and Opposition of a Satellite with the Sun, are called by the common Name Syzygies.*

1045 *At A and F the dark Part of the Moon is a little enlighten'd by the Rays that are reflected from the Earth, and therefore it is seen by a Spectator to whom the Sun is not visible; that is, in the first Case, after the setting of the Sun; and in the second, before its Rise.*

DEFINITION VII.

1046 *When the Light of the Sun is intercepted by the Moon, so that, in respect of any Observer upon the Earth, the Sun is partly or wholly cover'd, the Sun is said to undergo an Eclipse.*

Properly speaking this is an Eclipse of the Earth, on whose Surface the Shadow or *Penumbra* of the Moon falls.

DEFINITION VIII.

1047 *An Eclipse of the Moon is the Obscuration of the Moon by the Shadow of the Earth.*

1048 *The Eclipse of the Sun is never observ'd, except at the time of the New-Moon.*

1049 *The Moon is never eclips'd but at the time of the Full-Moon.*

1050 *Yet the Luminaries are not eclips'd at every one of the Syzygies, because the Moon does not*
 • 969 *move in the Plane of the Ecliptic, * in which the Sun and Earth always are; wherefore, upon account of the Moon's Latitude, its Shadow at the New-Moon often does not touch the Earth; and itself at the Full-Moon passes beside the Shadow of the Earth.*

1051 *But when the Moon has no Latitude, or but very little; that is, when it is in the Node, or near it, at its Syzygies, an Eclipse is observed; in that*

Case the Moon appears to be in the Ecliptic; or very near it; and this it is that has given the Name to that Line.

Plate XXIII. Fig. 1.] That what relates to the Eclipse of the Moon may appear the more plainly, let O O be the Way of the Moon, R R the Plane of the Ecliptic, the Center of the Earth's Shadow is always in it; * N is the Node of the Moon's Orbit. 904 939

If the Center of the Earth's Shadow be at A, the Moon that goes by at F will not be darken'd.

If the Moon be less distant from the Node at 1052 the Full-moon, as at G, the Shadow of the Earth is at B, and *the Moon is darken'd in part; this is called a partial Eclipse.*

If, supposing the Shadow at D, the Moon be 1053 full, *the Moon will be wholly darken'd at I; it runs into the Shadow at L, and goes out of it at H; and the Eclipse is said to be total.*

The Eclipse is said to be central when the Center of 1054 the Moon goes thro' the Center of the Shadow, which only happens in the very Node N.

We have hitherto spoken of the Shadow of the Earth, because when we mention the Earth we understand its Atmosphere, which is join'd to it, of which we have spoken elsewhere; * *the Shadow of the Atmosphere is properly consider'd in lunar 418 1055 Eclipses, for the Shadow of the Earth itself does not reach the Moon.*

Plate XXIV. Fig. 1.] Let T be the Earth, the Atmosphere about it F D G G D F; the Sun's Rays B D, B D, touching the Atmosphere; these go strait on, and terminate the Shadow of the Atmosphere, out of which if the Moon be, it is immediately enlighten'd by the Sun's Rays; but it is not enlighten'd in the same manner all the while it is between B D and B D.

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1056 *The Rays which enter the Atmosphere obliquely are*
 • 617 refracted, * and as they come towards the Earth,
 they continually penetrate into a Medium, which
 • 424 is denser and denser, * and therefore are every
 429 Moment inflected, and *move into Curves*; so the
 • 617 Rays E F E F penetrate the Atmosphere in the
 Curves F G F G that touch the Earth; all the
 Light between E F E F is intercepted by the
 Earth, and the Rays G A G A terminate the
 Earth's Shadow.

The Light between E F and B D being refracted by the Atmosphere, is scatter'd between G A and B D continu'd; and beyond A, the Point of the Earth's Shadow, the Lights that
 1057 come from all Parts are confounded, but are continually weaker and weaker the farther from the Earth; so that *the Shadow of the Atmosphere is not a perfect Shadow, but a weak Light*, whereby the Moon is visible in an Eclipse.

1058 *The Shadow of the Atmosphere is conical, because the Sun's Diameter is greater than the Diameter of the Atmosphere, which is scarce bigger than that of the Earth; and this Cone does not reach quite to Mars, as appears from immediate Observations; but the Shadow of the Diameter, in the Place where it is cut by the Moon's Orbit, is scarce one Fourth less than the Diameter of the Earth.*

With the same Reasoning that we have prov'd that the Moon comes into the Shadow of the Atmosphere, when the Moon in its full is in the Node, or near it, it is also prov'd that the Moon's Shadow falls upon the Earth *at the New-Moon, when the Moon is in the Node, or near the Node*;
 1059 therefore in that Case *the Sun undergoes an Eclipse*, concerning which several things are to be observed.

Plate

Plate XXII. Fig. 4.] Let S be the Sun, T the Moon, let the Shadow of it fall upon any Plane GH; this Shadow is encompass'd with a *Penumbra*, for beyond L and E that Plane is enlighten'd by one entire Hemisphere of the Sun; as you go from L to H, and from E to G, the Light is continually diminish'd; and near G and H the Rays come to the Plane only from a small Part of the Sun's Surface.

DEFINITION IX.

This diminish'd Light, which encompasses the Shadow GH every way, is called the Penumbra. 1060

In the Eclipse of the Moon the Shadow of the Earth is encompass'd with the like Penumbra, but this is only sensible near the Shadow, and therefore has but a small Breadth; but if an Observer be plac'd upon a Plane upon which the Shadow falls, he may observe the whole Penumbra; as is the Case in the Eclipse of the Sun. 1061
An Observer I or F can only see the Semidiameter of the Sun, the rest of the Diameter is hid by the Moon; and going from L towards H, the Sun is continually more and more hid by the Moon, till it becomes wholly invisible in the Shadow itself. 1062

Hence it follows, that *there is a solar Eclipse, tho' the Shadow of the Moon does not touch the Earth, provided the Penumbra comes to its Surface; and also, that the Eclipse is not observ'd in all the Places in which the Sun is visible; and that it is different, according as the Shadow or a different Part of the Penumbra goes thro' the Place, in the Places in which it is observ'd.* 1063 1064 1065

But the Eclipse of the Moon is every where the same, where the Moon is visible, during the Eclipse. 1066

But when the Shadow itself of the Moon falls upon the Earth, the Sun's Eclipse is said to be total; if only the Penumbra reaches the Earth, it is said to be 1067
N 2 partial;

partial; and this is what happens when we consider an Eclipse in general.

1068 But as to particular Places, it is said to be total in those Places where the Shadow passes; central in those where the Center of the Shadow passes; that is, where the Center of the Moon covers the Sun's Center; and lastly, *partial*, where the Penumbra only goes by; and this is drawn in Fig. 6.

1069 Plate XXII. Fig. 4.] The wider the Shadow GH is, the more Places is the Eclipse of the Sun total in, and the longer is the Sun wholly obscur'd; but the Breadth of the Shadow is different, according to the different Distances of the Moon from the Earth, and of the Earth from the Sun.

1070 If there be an Eclipse of the Moon, supposing the Earth in the Perihelion, and the Moon in the Apogæum, that is, at the greatest Distance from the Earth, the Shadow of the Moon does not reach the Earth, and the Moon does not cover the whole Sun; such an one is call'd an annular Eclipse, and is represented in Fig. 5.

C H A P. VII.

Of the Phænomena arising from the Motion of the Sun, the Planets, and the Moon, about their Axes.

1071 **T**HE Sun's Motion about its Axis is sensible, by observing the Spots, which are observ'd very often upon the Sun's Surface; these Spots seem to change their Figure and Situation every Day, and sometimes to move swifter, sometimes slower; all which things may be easily deduc'd from the Motion of a spherical Surface; and the Sun, which, if it was not moved by such a Motion, would only once in a Year successively turn its whole

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whole Surface to the Earth, now shews it to the Inhabitants of the Earth in less than the Space of one Month.

Such like Phænomena arise from the Rotation of 1072 *Jupiter, Mars, and Venus, about their Axis, which Motions become sensible, by observing the Spots in the Surface of the Planets.*

Whilst the Earth is mov'd round its Axis, the Observer, who is carried round, imagines himself to be at rest, and all the heavenly Bodies to be in motion.

DEFINITION I.

The Points in which the Axis of the Earth, being 1073 continued both ways, touches the Sphere of the fixed Stars, are called the Poles of the World.

DEFINITION II.

The apparent Motion arising from the Motion of 1074 the Earth about its Axis, is called the diurnal Motion.

DEFINITION III.

A Plane is conceiv'd to pass thro' the Center of the 1075 Earth, perpendicular to its Axis, and continu'd every way; and the Circle, in which it cuts the Sphere of the fixed Stars, is called the celestial Æquator.

In the Motion of the Earth round the Sun the 1076 Æquator is moved; but since the Plane of this Circle is carried by a parallel Motion, the celestial Æquator is not moved.* * 994

DEFINITION IV.

Circles, whose Planes go through the Axis of the 1077 Earth, are called Meridians.

They all pass thro' the Poles of the World, and are 1078 perpendicular to the Æquator.

DEFINITION V.

1079 *The Arc of any Meridian, which is intercepted between the Æquator and a Star, is called the Declination of that Star.*

Plate XXIII. Fig. 2.] Let an Observer be upon the Earth T, who directs his Sight along TA; after a little Time, when the Line TA shall be carried by the Motion of the Earth to Ta, if the Spectator directs his Sight thro' the same Line, the Body A will appear to have been carried thro' the Arc aA; but when the Line has return'd to its former Situation TA, the Body will seem to have perform'd one whole Revolution; but if he directs his Sight along the Axis of the Earth produc'd, because that is at rest, the Body which is seen in the Axis will appear not to
 1080 *have moved, therefore in the Poles of the World the*
 *1073 *diurnal Motion is not observ'd**; but that Bodies which are near the Poles are moved round them, is plain, and that the Body by its diurnal Motion describes so much a greater Circle round the immoveable Pole as it is farther distant from it;
 1081 *therefore the whole Sphere of the fixed Stars seems to revolve about the Axis of the Earth continu'd, in antecedentia, in that Time in which the Earth really turns about its Axis; therefore the diurnal Motion is common to all the celestial Bodies, except so far as it is disturb'd by the Motions above-mentioned.*

The Æquator is equally distant from both Poles, and divides the Heavens into two Hemispheres, whose middle Points are the Poles, which therefore are equally distant from the several
 1082 *Points of the Æquator; therefore the heavenly Bodies which are in the Æquator, by their diurnal Motion seem to describe the Æquator itself, the greatest Circle of all that can be describ'd by the diurnal*

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urnal Motion; the other Bodies describe Circles parallel to the *Æquator*. 1083

The Axis of the Earth is inclin'd to the Plane of the *Ecliptic* in an Angle of 66 Deg. 31 Min. * * 967
 the Poles of the World therefore are distant from the Poles of the *Ecliptic* 23 Deg. 29 Min. and the Plane of the *Æquator* makes an Angle with the Plane of the *Ecliptic* of 23 Deg. 29 Min. Both Planes pass thro' the Center of the Earth; but since this may be look'd upon as the Center of the fixed Stars, * it follows, that the *Æquator* and the *Ecliptic Line* are great Circles, which are inclin'd to each other, and cut one another in two opposite Points, in the beginning of Aries, and the beginning of Libra; which Points, in the Way of the Sun, are determined by these Intersections. * 1085 988 990 1001

When the Sun is in those Points, it seems to describe the *Æquator* by its diurnal Motion; * when it is carried about in the *Ecliptic* by its apparent Motion, it continually recedes more and more from the *Æquator*, and its Declination is increased, and it describes less Circles every Day; * till it comes to its greatest distance from the *Equator*, which is 23 Deg. 29 Min. * then it comes back to the *Equator* again, and goes beyond it also 23 Deg. 29 Min. advancing towards the opposite Pole. 1086 1087 1082 1083 1088 1084

DEFINITION VI.

Those Circles, describ'd by the Sun in its diurnal Motion, which are most distant from the *Equator*, that is, 23 Deg. 19 Min. are called the Tropics. 1089

One touches the *Ecliptic Line* in the first Degree of Cancer, and is call'd the Tropic of Cancer; the other is call'd the Tropic of Capricorn, and passes thro' the first Point of the Sign Capricorn, and there touches the *Ecliptic Line*.

DEFINITION VII.

- 1090 *The Pole of the World, which is next to the Tropic of Cancer, is called the Arctic Pole, and also the North Pole; the opposite is called the Antarctic, and also the South Pole.*

DEFINITION VIII.

- 1091 *The Circles that are describ'd in the diurnal Motion by the Poles of the Ecliptic, that is, by the Points which are distant from the Poles of the World 23 Deg. 29 Min. are called the Polar Circles.*

The Arctic Polar Circle is that which surrounds the Arctic Pole, the other opposite one borrows its Name from the Antarctic Pole.

- 1092 *There remains to be explain'd the Moon's Motion about its Axis, whose Effect is, that the same Face of the Moon is always turn'd towards the Earth.*

Plate XXII. Fig. 3.] Let the Moon be at N, the Face which is turn'd to the Earth is mni ; if the Moon did not turn about its Axis, and all its Points were carried thro' parallel Lines, the Line mi would coincide with the Line ln in the Situation of the Moon at B, and the aforesaid Hemisphere mni would be at lmn ; but because whilst the Moon describes a fourth Part of its Orbit, it performs likewise a Fourth of its Revolution round its Axis, the Face, which would be at lmn , is now at mni , that is, again turn'd to the Earth. After the same manner it is prov'd, that this same Face mni , when the Moon is at P, is seen by an Observer on the Earth, and that it is turn'd towards the Earth at E; as also in all other Points of the Moon's Orbit.

- 1093 *The Axis of the Moon is not perpendicular to the Plane of its Orbit, but a little inclin'd to it; the Axis keeps its Parallelism in its Motion round the Earth, as has been said of the primary Planets; therefore*

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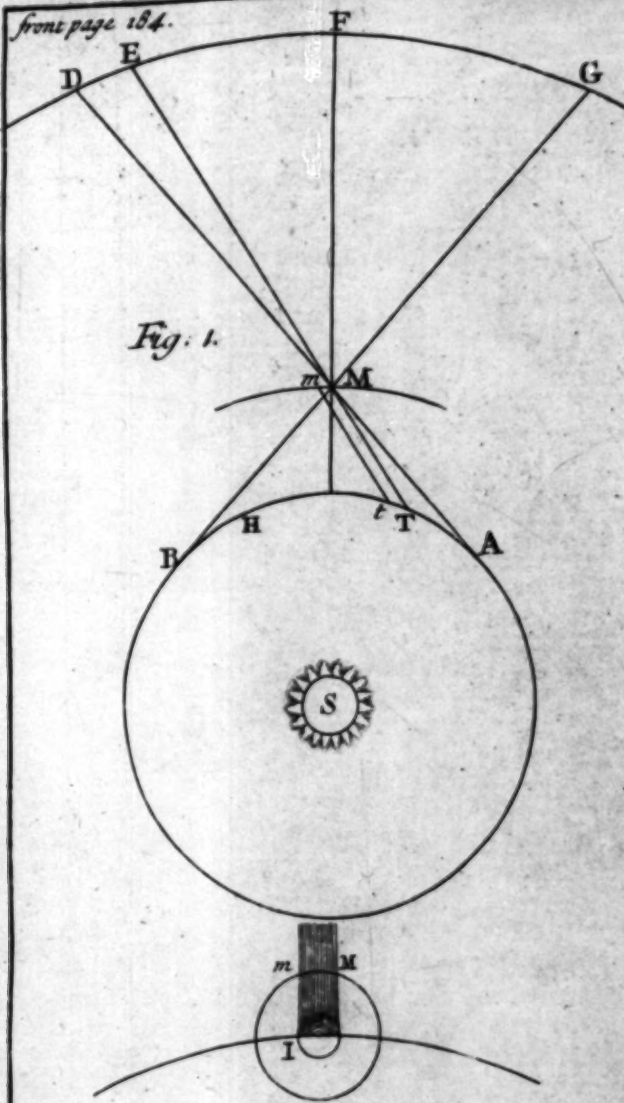


Fig. 1.

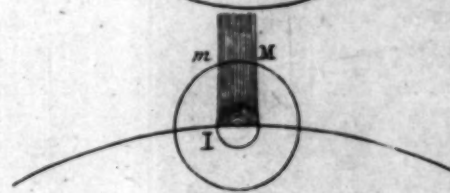
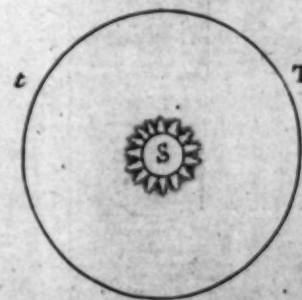


Fig. 2.



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Plate 22.

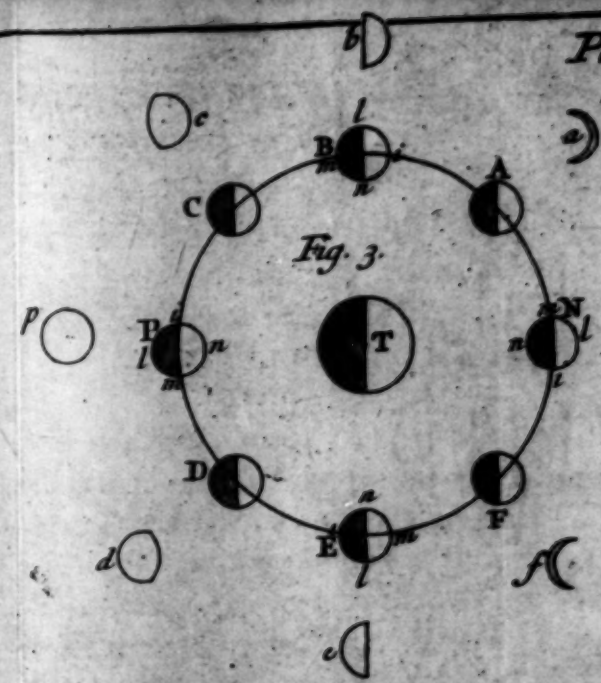


Fig. 3.

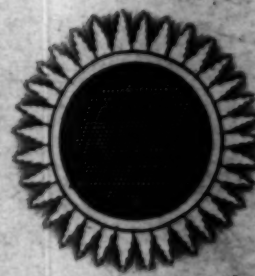


Fig. 5.

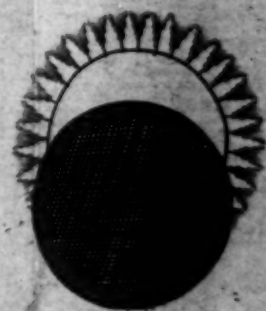


Fig. 6.

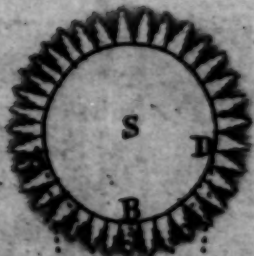
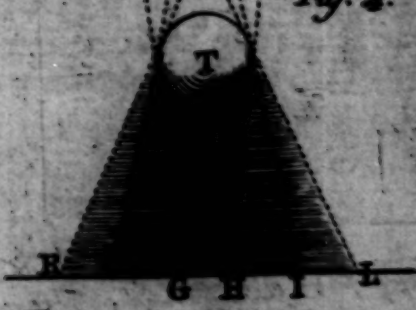
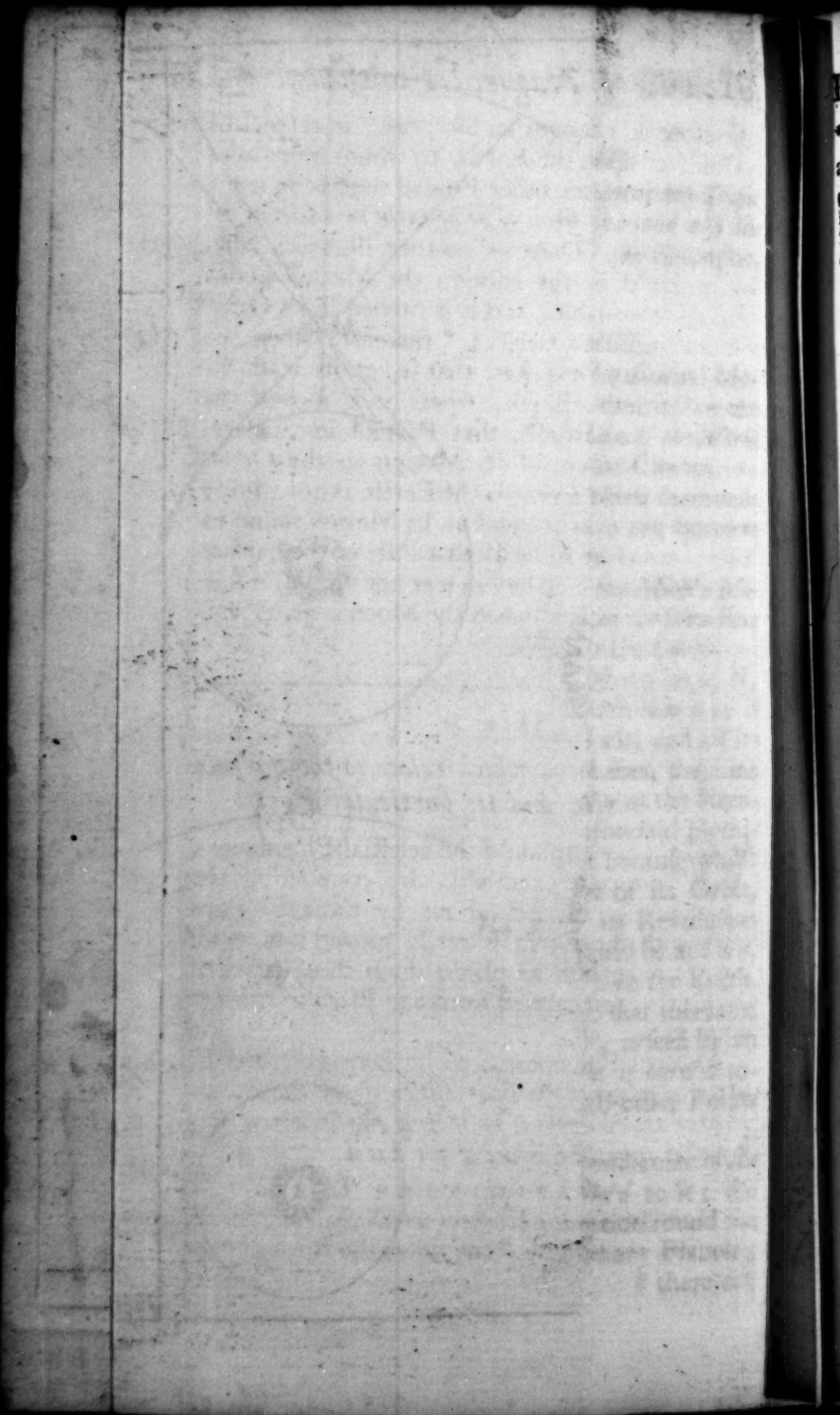


Fig. 4.





* therefore it changes its Situation, in respect of * 957
 an Observer upon the Earth, to whom sometimes
 one, sometimes the other Pole of the Moon is vi-
 sible; whence it *seems to be agitated by a sort of li-*
bratory Motion. There is another libratory Mo- 1094
 tion observ'd in the Moon; the Motion about
 the Axis is equable, and it is carried in its Orbit
 with an unequal Celerity; * therefore when the * 966
 Moon is at its *Perigæum*, that is, at its least di-
 stance from the Earth, where it is moved the
 swiftest in its Orbit*, that Part of its Surface, * 966
 which, on account of its Motion in the Orbit,
 would be turn'd towards the Earth, is not wholly
 turn'd from it on account of its Motion round its
 Axis; therefore some of that Part of the Surface
 of the Moon, which before was not visible, is seen
 at the side; which, when the Moon is at its *Apo-*
gæum, is again visible.

CHAP. V.

*Of the Phænomena which relate to the Surface
 of the Earth, and its particular Parts.*

WE have explain'd the celestial Phænomena
 hitherto examin'd, by considering the
 Spectator as carried about by those Motions
 wherewith the Earth is really moved; now we
 shall consider him as plac'd upon the Surface of
 the Earth, and carried from one Place to another
 upon it.

The first Phænomena to be here observed is, 1095
 that, *by reason of the Interposition of the Earth, one*
Half of the Heavens is invisible to the Observer, who
is plac'd upon the Surface of the Earth.

DEFINITION I.

That Circle in the Heavens which separates the vi- 1096
sible from the invisible Part, when the Rays are not
inter-

intercepted by the Inequalities of the Earth's Surface, is called the Horizon.

When the Height, to which the Spectator can be raised above the Surface of the Earth, is very small, compared with the Semidiameter of the Earth, the Eye of the Spectator may be look'd upon as placed in the Surface itself.

Plate XXIII. Fig. 4.] Let the Earth be at T, and the Observer at S, and P E p e the Sphere of the fixed Stars; if you conceive a Plane at H H to touch the Earth and go thro' S, it will be the Plane of the Horizon, whose Section with the Sphere of the fixed Stars is the Horizon. A Plane, as b b, is conceiv'd to go thro' the Center of the Earth, parallel to H H; the distance b H is insensible, by reason of the immense distance of the fixed Stars; therefore the Section of that Plane, with the Sphere above-mention'd, may be taken
* 994 for the Horizon.*

DEFINITION II.

1097 *The Ascent of the Stars above the Horizon is called their Rise.*

DEFINITION III.

1098 *The Descent below the Horizon is called the setting of the Stars.*

DEFINITION IV.

1099 *If we conceive a Line drawn thro' the Observer and the Center of the Earth, which must necessarily be perpendicular to the Horizon, it will reach the Point Z among the fixed Stars, which is called the Zenith.*

DEFINITION V.

1100 *The Point N, opposite to it, is called the Nadir.*

DEFINITION VI.

1101 *The Section which a Plane of the Meridian, that goes thro' the Observer, makes with the Horizon, is called the Meridian Line, and is directed from North to South,*

DEFI-

DEFINITION VII.

The Eastern Part of the Heavens is that, from 1102
which we see the Bodies rise above the Horizon; and
the East Point is that, in which a Line directed East-
wards, perpendicular to the Meridian Line, and going
thro' the Observer, cuts the Sphere of the fixed Stars.

DEFINITION VIII.

The Point opposite to this is called the West Point, 1103
and the Western Point of the Heavens is opposite to
the Eastern Part.

DEFINITION IX.

The Amplitude is an Arc of the Horizon, which 1104
is contain'd between the East or West Point, and the
Point in which the Star rises or sets; the first is cal-
led the rising, and the other the setting Ampli-
tude; and each is either Northern or Southern
Amplitude.

DEFINITION X.

The Height or Altitude of a Star above the 1105
Horizon, is the Arc of a Circle perpendicular to the
Horizon, in whose Center the Spectator is, terminated
by the Horizon and the Star.

DEFINITION XI.

The difference of the height of a Star, according to 1106
the different Position of the Observer, as he is supposed
in the Center, or on the Surface of the Earth, is cal-
led the Parallax of the Star.

There is only the Parallax of the Moon which can 1107
be determin'd by Observations; the distance of the
rest of the Bodies in the planetary System is too
great to be compared with the Semidiameter of
the Earth; and the Parallax depends upon the
Ratio which the Semidiameter of the Earth has to
the distance of a Planet; therefore even the Pa-
rallax

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1108 *Parallax of Mars, in Opposition with the Sun, is too small for the nicest Observations.*

1109 *Where there is a Parallax, it diminishes as a Body ascends above the Horizon, and vanishes in the Zenith.*

The apparent height of the Stars is also chang'd upon another Account, which equally affects all the heavenly Bodies; the Rays are inflected by the Refraction of the Atmosphere*, and the Stars appear bigger than they are*; yet the bigger they are the less is that Inflexion*, because the Rays fall less obliquely on the Surface of the Atmosphere; in the Zenith there is no Refraction*, even at the distance of 20 or 30 Degrees from the Zenith it is not sensible.

1113 *Since the Stars are rais'd by this Refraction, they are visible before they come to the Horizon.*

Plate XXIV. Fig. 2.] All these things relate to the Surface of the Earth in general; now we must examine the several Parts of it, these are determin'd by referring to the Earth the several Circles which we have before consider'd in the Heavens; so on the Earth we consider the *Æquator*, the *Meridians*, the *Tropics*, and *polar Circles*; and these Circles divide the Surface of the Earth in the same manner as the Sphere of the fixed Stars is divided by the Circles in the Heavens; and therefore the Circles in the Heavens, and those upon the Earth, do so mutually correspond with each other, that a Line being drawn from the Center of the Earth to a Circle in the Heavens, it will go thro' the same Circle in the Earth. If the Poles are *P p*, the *Æquator* will be *E e*, the *Tropics* *T T*, *t t*, and the *polar Circles* *A A*, *a a*.

DEFINITION XII.

1115 *The Meridian, which goes thro' a Place, is called the Meridian of the Place.*

The

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The Plane of it is perpendicular to the Horizon, 1116
because it goes thro' the Center of the Earth and
the Observer.

A Meridian Line drawn in any Place, is part of 1117
the Meridian of the Place.* 1104

DEFINITION XIII.

The Latitude of a Place is its distance from the 1118
Æquator, that is, the Arc intercepted between
that Place and the Æquator.

DEFINITION XIV.

Circles parallel to the Æquator are called Circles 1119
of Latitude.

By determining the Latitude of the Place, we
determine the Circle of Latitude, which goes
thro' the Place; now to determine the Situation
of several Places in respect of each other, we must
determine Places upon the several Circles; which
is done, by supposing a Meridian to pass thro'
any remarkable Place, which by its Section de-
termines a Point upon each Circle of Latitude,
from which the Distances of Places are measur'd.

DEFINITION XV.

The Meridian above-mention'd, taken at pleasure, 1120
is called the first Meridian.

DEFINITION XVI.

The Distance of a Place from the first Meridian, 1121
measur'd on a Circle of Latitude that goes thro' a
Place, is called the Longitude of the Place.

Astronomers refer every thing to the Meridian of 1122
the Place in which they make their Observations.

In explaining the Phænomena which relate to
the several Parts of the Surface of the Earth, we
shall consider the Observer going from the Pole

to the Equator; and first, only take notice of the diurnal Motion.

1123 *Plate XXIII. Fig. 3.] When the Spectator is at S, in the very Pole of the Earth T, the celestial Equator Ee coincides with the Horizon, and the Pole of the World P is in the Zenith; in that Case, because the Circles which are parallel to the Horizon, are also parallel to the Equator, all the heavenly Bodies appear to be carried by a Motion parallel to the Horizon, * in Circles which are represented by the Lines Aa, Bb. The heavenly Bodies in the Hemisphere EPe never set, and 1124 the others are never visible. The Horizon in this Situation is said to be parallel, or this Situation is called a parallel Sphere.*

1125 *Plate XXIII. Fig. 4.] If an Observer upon the Earth T recedes from the Pole, and is at S, the Horizon is said to be oblique, or the Sphere is oblique; then the Axis Pp is inclin'd to the Horizon bb, so much the more as the Observer is farther from the Pole.*

DEFINITION XVII.

1126 *The Angle which the Axis of the Earth makes with*
 *1105 *the Horizon, is called the Height of the Pole. **

1127 *This Height of the Pole is equal to the Latitude.*

OS11 *The Height of the Pole is the Angle PTh, whose Measure is the Arc Pb; the Latitude is measur'd by an Arc, which upon the Earth corresponds to the Arc ZE in the Heavens; * but it 1128 is equal to the Arc Pb; for the Complement of each of them, to a Quarter of a Circle, is the Arc ZP.*

1128 *In this Position of the Observer, because the Equator is inclin'd to the Horizon, all the heavenly Bodies are carried by the diurnal Motion in Circles inclined to the Horizon, represented by the Lines Aa, Bb.*

Some

Some of the heavenly Bodies rise and set at every Revolution of the Earth; namely, those which are between the Parallels to the Equator B b and b i, because all the Parallels between those two are cut by the Horizon. 1129

The Planes of the Equator and the Horizon go thro' the Center of the Earth, therefore these Circles cut one another mutually into two equal Parts, and half of the Equator is above the Horizon; therefore *the heavenly Bodies which are in the Equator, are above the Horizon during half a Revolution of the Earth about its Axis**; and on account of the Equability of the Motion about the Axis, *are invisible during an equal Time.* 1130 *1082

These also rise due East, and set due West (that is, in the very Points of the East and West) for the Section of the Planes of the Equator and the Horizon is perpendicular to a Plane perpendicular to both these Planes, and this last Plane is the Plane of the Meridian of the Place*; wherefore the above-mention'd Section is perpendicular to the Meridian Line*, and consequently goes thro' the East and West Points.* 1131 *1078 1116 *1117 *1102

Bodies between the Equator and a Parallel B b, which touches the Horizon, as in the Circle A a, continue longer above than below the Horizon; and this Difference is so much the greater, the more the Circle A a approaches that Pole which is above the Horizon; on the contrary, as the Body goes towards the opposite Pole, its Time of Continuance above the Horizon is the longer. 1103 1132 1133

This Inequality of the Time that a Body is above and below the Horizon, increases as the Height of the Pole does, because of the Diminution of the Angle made with the Horizon by the Equator and its Parallels. 1134

Bodies whose Distance from the Pole is equal to the Height of it, never set; for such is the Distance of the 1135 the

the Circle Bb , which touches the Horizon, but has no Part of it below the Horizon.

Bodies less distant from the Pole do not so much as come down to the Horizon.

1136 It appears by the same Reasoning, that Bodies whose Distance from the opposite Pole does not exceed the Height of the Pole, never rise above the Horizon, and are always invisible.

1137 Bodies whose Distance EZ from the Equator is equal to the Height of the Pole, go thro' the Zenith Z ; for EZ is equal to the Latitude of the Place to which the Height of the Pole is equal*.

*1126 Plate XXIII. Fig. 5.] When a Spectator S has
1138 receded as far as he can from the Pole, he comes to the Equator, whose Points are equally distant from each Pole*; then the Axis Pp is in the Horizon, with which the Equator makes a right Angle*, for which reason the Horizon is said to be
*1075 right, or this is called a right Sphere.
1114

The Horizon cuts into two equal Parts all the Circles that are parallel to the Equator, which are represented by the Lines Aa , Bb ; therefore
1139 all the heavenly Bodies, at every Revolution of the Earth, rise and set, and are visible and invisible during equal Times.

1140 The Equator itself goes thro' the Zenith, and therefore all the Bodies that are in it pass thro' it also.

If what we have explain'd concerning the diurnal Motion, be applied to the Bodies of whose other apparent Motions we have spoken before, the Phænomena will be easily determin'd from the Motions join'd together; those that relate to the Sun are more remarkable than the rest, and therefore more particularly to be explain'd.

DEFINITION XVIII.

We call a natural Day the Time elapsed between 1141
the Recess of the Sun from the Meridian of a Place,
and its next Return to the same Meridian.

This Day differs from the Time of the Revolution 1142
of the Earth about its Axis, which Times would be
equal if the Sun appear'd immovable among the
fixed Stars; but whilst by the diurnal Motion,
in the Time of one Revolution of the Earth about
its Axis, the Sun is carried round from East to
West, that is, in antecedentia, it is carried by a* *1080
contrary Motion in the Ecliptic, whereby it* *996
comes later to the Meridian.

But as the Sun does not every Day go thro' an 1143
 equal Space in the Ecliptic*, all the *natural Days* *997
 do not equally exceed the Revolution of the
 Earth about its Axis; therefore these Days are
unequal to one another.

Natural Days are unequal also upon another
 Account, namely, by reason of the Inclination
 of the Ecliptic in respect to the Equator; whence
 it follows, that the annual Course of the Sun is un-
 equally inclin'd to the Equator in different Points;
 and tho' the Sun should equally go forward every
 Day in the Ecliptic, the natural Days would not
 equally exceed the Time of the Revolution about
 the Axis; for if the Motion of the Sun be resol-
 ved into two Motions*, of which the one is paral- *192
 lel to the Equator, and the other perpendicular
 to it, the first is only to be consider'd in determin-
 ing the Excess above-mention'd; and that it is
 unequal, is plain from the different Inclination
 above-mention'd.

These two Causes of Inequality often concur,
 and often act contrariwise.

Every natural Day is divided into 24 equal Parts, 1144
which are called Hours; each Hour is divided into

60 Minutes, and each Minute into 60 second Minutes, or Seconds; and so on. That these Parts of

Time vary in different Days, appears plainly from
 1143 what has been * said; they are by Astronomers
 reduc'd to Equality, by considering the Number
 of Hours in the whole Revolution of the Sun in
 the Ecliptic, and dividing the whole Time into
 as many equal Parts as there are Hours, 24 of
 1145 which are taken for one Day. The Time, whose
 Parts are by this Method reduc'd to Equality, is
 called mean Time; and that Reduction is called the
 Equation of Time.

1146 We always make use of the Days and Hours of the
 mean Time in determining the Periods of the heavenly
 Motions.

DEFINITION XIX.

1147 The artificial Day is the Time that the Sun stays
 above the Horizon.

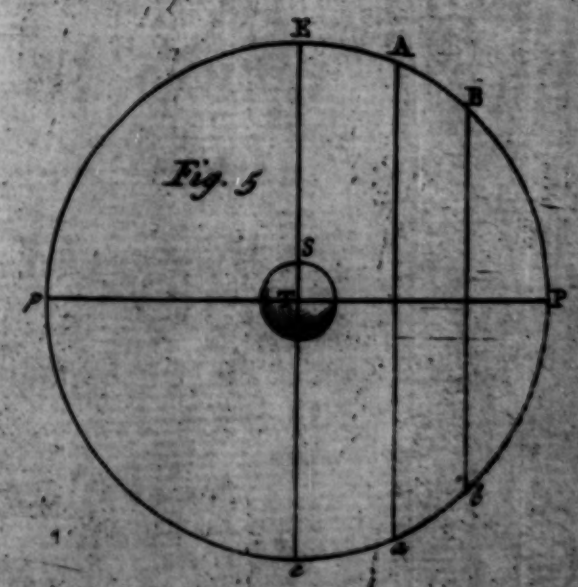
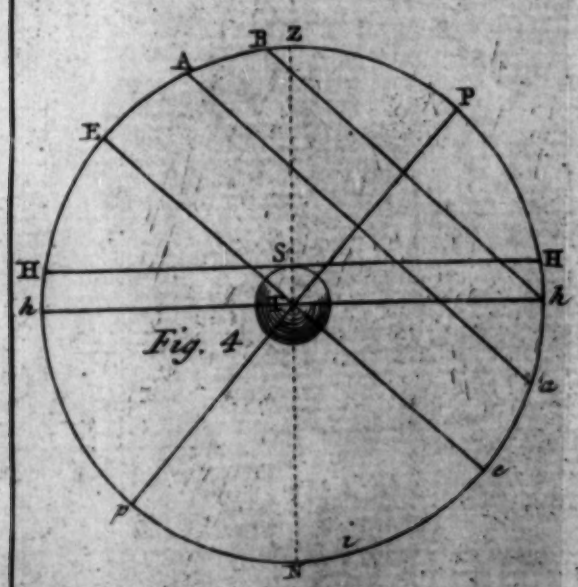
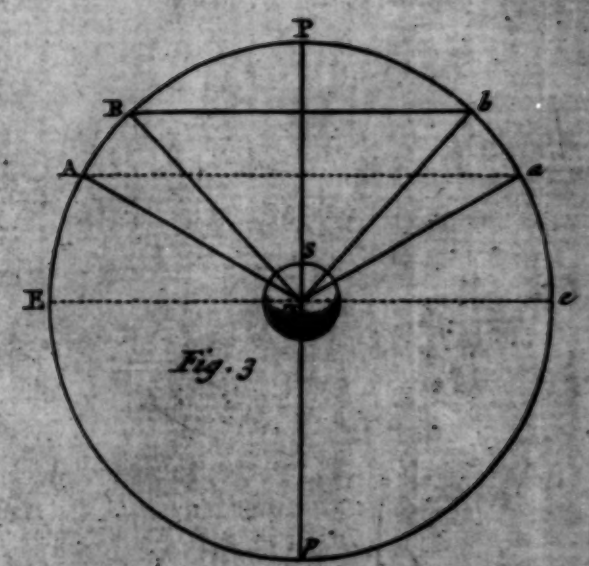
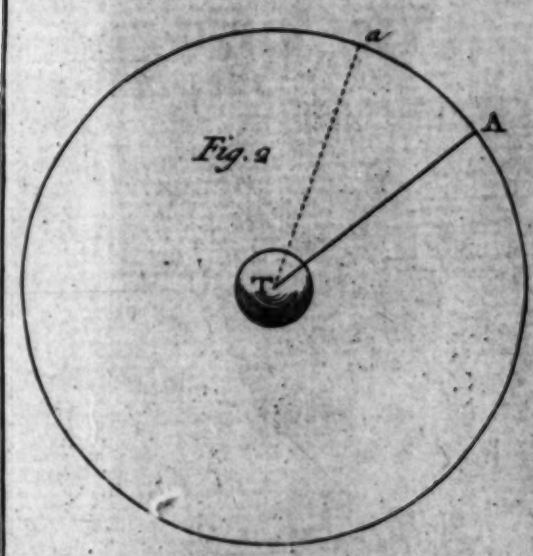
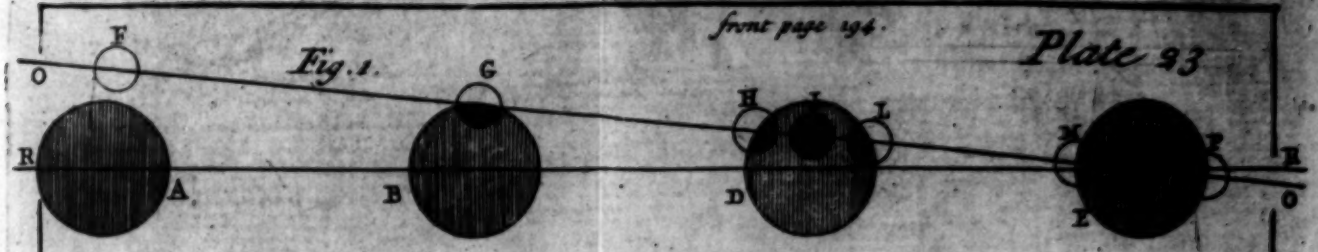
1148 We always speak of it when we mention Day
 in opposition to Night. In determining the Length
 of artificial Days, we shall not attend to the Equation
 of Time.

1149 The Crepusculum always comes before the Sun's
 1150 Rise, and follows its setting; this is that dim Light
 which we commonly call Morning and Evening Twi-
 light.

1151 The Twilight is produc'd by the Atmosphere, which
 201 is enlighten'd by the Sun's Rays, and whose Particles
 reflect the Light every way; from whence some
 Rays come to us, tho' the Sun be depressed 18 De-
 grees below the Horizon.

1152 In the parallel Sphere, that is, to all those that
 1138 dwell under the Æquator, * the Days and Nights
 1139 are equal to one another all the Year round*, and are
 1144 of 12 Hours*.

1153 In the oblique Sphere the Days are longer or shorter,
 according to the different distance of the Sun from the
 Equator





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*Equator towards either Pole**; for the Sun recedes ^{*1131}
from the Equator towards the Poles 23 Deg. ^{*1132}
29 Min. * ^{*1087}
¹⁰⁸⁸

The Sun is in the Equator about the 11th of March ¹¹⁵⁴
and the 12th of September, and then the Day is
equal to the Night, which happens all over the Earth, ** ^{*1194}
except just at the Poles.

DEFINITION XX.

Those Points of the Ecliptic, in which it is cut by ¹¹⁵⁵
the Equator, are called the Equinoctial Points, ** ^{*1085}
because the Sun is in those Points when the above
mention'd Equality of Day and Night happens.

DEFINITION XXI.

Those Points of the Ecliptic, in which the Tropics
touch that Circle, are called the Solstitial Points; ** ^{*1089}
because for a few Days, when the Sun comes to
those Points, and goes beyond them, it does not
sensibly change its Declination, and the Length
of the Days not sensibly vary.

Under the Poles, if there be any Inhabitants there, ¹¹⁵⁶
they can only once in a Year see the rising and
the setting Sun, and *only one Day with one Night*
make up their whole Year; the Sun continues above
the Horizon all the while it goes thro' one half of
the Ecliptic*, the rest of the Time it is hid under ^{*1085}
the Horizon; but yet *their Day is lengthen'd upon* ¹¹²³
*account of the Refraction**; and the *Twilights last* ¹¹⁵⁷
very long, for they last as long as the Declination ^{*1113}
of the Sun towards the hidden Pole does not
exceed 18 Degrees.* ^{*1151}

At the Arctic Pole, in the first six Signs, from ¹¹⁵⁸
Aries to Libra, the Sun is above the Horizon;
therefore at that Pole *the Day exceeds the Night 9*
natural Days, besides the Diminution of the Night ** ^{*1000}
on account of the Refraction.* ^{*1157}

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These general things which relate to the different Positions of the Horizon being explained, some more particular things are to be examin'd.

1159 *The whole Surface of the Earth is divided into five Zones, the first is contain'd between the two Tropics*

1160 *T T t t (Plate XXIV. Fig. 2.) and called the torrid Zone. There are two temperate Zones, and two frigid Zones; the northern temperate Zone is terminated by the Tropic of Cancer T T and the*

1161 *arctic polar Circle A A; the southern temperate Zone is contain'd between t t, the Tropic of Ca-*

1162 *pricorn, and the polar Circle a a; the frigid Zones are circumscrib'd by the polar Circles, and the Poles are in the Centers of them.*

1163 *In the torrid Zone, twice a Year, the Sun goes thro'*

1137 *the Zenith at Noon**; for the Elevation of the

1160 *Pole is less than 23 Deg. 29 Min.* and the di-*

1127 *stance of the Sun from the Equator towards the*

Pole, which is above the Horizon, is twice in a

1087 *Year equal to the height of the Pole**; for which

1088 *reason also in the Limits of that Zone, namely,*

1164 *under the Tropics, the Sun comes to the Zenith only*

1087 *once in a whole Year*.*

1089 *In the temperate and frigid Zones the least height*

1165 *of the Pole exceeds the greatest distance of the*

1087 *Sun from the Equator*, and therefore to their*

1162 *Inhabitants the Sun never goes thro' the Zenith*;*

1162 *yet the same Day the Sun rises to a greater height, the*

1166 *less the height of the Pole is, because thereby the*

1137 *Inclination of the Circles of the diurnal Motion*

with the Horizon is less.

1167 *In the torrid Zone, and in the temperate Zones,*

1129 *every natural Day the Sun rises and sets, for the di-*

1137 *stance of the Sun from the Pole always exceeds*

1168 *the height of the Pole*;* yet every where, but un-

1087 *der the Equator*, the artificial Days are unequal to*

1160 *one another*;* which Inequality is so much the

1161 *greater, the less the Place is distant from a frigid*

1152 *Zone*.*

1132 *But*

1134

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But in the polar Circles, just where the temperate Zones are separated from the frigid ones, the height of the Pole is equal to the distance of the Sun from the Pole when it is in the neighbouring Tropic * ; and therefore in that Case, that is, once a Year, the Sun in its diurnal Motion performs one entire Revolution without going down under the Horizon. 1169 1089 1091

But every where in a frozen Zone the height of the Pole is greater than the least distance of the Sun from the Pole * ; therefore during some Revolutions of the Earth, the Sun is at a distance from the Pole which is less than the Pole's height, and during all that time it does not set, nor so much as touch the Horizon * ; but where the distance from the Pole, as the Sun recedes from it, does exceed the height of the Pole or Latitude of the Place *, the Sun rises or sets every natural Day * ; then in its Motion towards the opposite Pole, it stays in the same manner below the Horizon, as was said of the Motion above the Horizon *. 1170 1089 1162 1135 1127 1129 1171 1136

These Times, in which the Sun makes entire Revolutions above the Horizon and below it, in its diurnal Motion, are so much the greater ; that is, the longest Day and Night last the longest, the less the Place in the frigid Zone is distant from the Pole, till at last, at the Pole itself, they take up the Time of the whole Year. 1172

From the same Causes, namely, the Obliquity of the Ecliptic in respect of the Equator, by which are occasion'd all the things which relate to the Inequality of Days, which is different in different Places, we also deduce the difference of Seasons which succeed one another every Year: I shall speak of them first in respect to the frigid and temperate Zones, and then in respect to the torrid Zone.

The Rays of the Sun communicate Heat to the Air, not only when they come directly from the Sun, but when they are reflected irregularly from Bodies or the Surface of the Earth*.

• 582

This Effect is so much greater as the Rays strike the less obliquely against the Surface of the Earth, and that upon a double Account. 1. If you resolve the Motion of the Light into two Motions*, one of which is parallel, and the other perpendicular to the Surface of the Earth, the Light acts upon Bodies only by this last Motion, which diminishes as the Obliquity increases. 2. There are more Rays acting at one time upon the same Part of the Surface of the Earth, the more directly they come upon it.

• 192

1173 Hence we deduce, that *the Causes of Heat increase when the Days increase*, by the Sun coming towards the Pole, which is above the Horizon, because the Sun does daily ascend to a greater height; so that to the diminish'd Obliquity is added the longer Continuance of the Sun above the Horizon, both which concur to the increasing of the Heat; the Nights also are diminish'd as the Days increase, and the Height that is produc'd by Day has less Time to decrease in.

In the northern Zones, as follows from this, the Cause of the Heat is the greatest of all when the Sun comes to the Tropic of Cancer*; yet the

• 1090

1174 *Heat is not always the greatest where the Cause of Heat is the greatest*, for the Heat increases as long as that which is acquir'd by Day is not wholly destroy'd by Night; for tho' the daily Augmentations be diminish'd, as long as there is an Aug-

- 1175

mentation the Heat increases. The most intense Cold is not upon the shortest Day, in which the Obliquity of the Sun's Rays is the greatest, and the Absence of the Sun the longest; but the Cold increases as long as the Diminution of Heat does

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does last; concerning which one may reason in the same manner as concerning the Increase of Heat.

The Year is divided into four Seasons, the hottest is called the Summer, the coldest the Winter; the temperate Season that follows the Winter, Spring; and the Autumn comes in between Summer and Winter. 1176

In the northern Regions, in the beginning of Spring the Sun appears to be in the beginning of Aries; in the beginning of Summer the Sun comes to the Tropic of Cancer; when the Sun enters Libra the Autumn begins; in the beginning of Winter the Sun performs its diurnal Motion in the Tropic of Capricorn; all which may be easily deduc'd from what has been explain'd.* 1177 1174 1175

In the southern Regions the Summer happens in the Time of the Winter above-mention'd, and they have their Spring whilst the former have their Autumn; and so of the other Seasons. 1178

The general Causes upon which the Division above-mention'd depends, are often disturb'd by Causes relating to particular Places, especially in the torrid Zone, of which we said we must treat separately; in most Places of this Zone there are only two Seasons observ'd (viz.) Summer and Winter, which are chiefly distinguish'd by dry and wet Weather. 1179

When the Sun comes to the Zenith of any Place there are almost continual Rains, upon which account the Heat is diminish'd, which Time is referred to, or called Winter; as the Sun recedes the Rains diminish, the Heat is increas'd, and that Time is referr'd to Summer. 1180 1181

In the middle of the torrid Zone there are two Summers, and as many Winters, because the Sun comes up twice to the Zenith.* 1182 1163

Towards the Sides of the Zone, tho' the Sun comes twice to the Zenith, yet since there is but a small Time between its coming to it the first and second time, both the Winters are confounded into one; wherefore only two Seasons in a Year are observ'd there.

C H A P. IX.

Concerning the Phænomena arising from the Motion of the Axis of the Earth.

• 952 **W**E have said that the Axis of the Earth is carried by a parallel Motion*; we have not consider'd a small Motion, whereby it is really moved, of which we shall now speak.

The Axis of the Earth keeping the Inclination of
 1183 66 Deg. 31 Min. *to the Plane of the Ecliptic, re-*
volves in antecedentia; that is, is successively car-
 1184 ried towards all Parts; and its Extremities (*viz.*)
the Poles of the World, describe Circles round the
Poles of the Ecliptic, from East to West; and this
Revolution is perform'd in the Time of about 25000
Years; which Period is called the great Year.

Because the Earth is look'd upon as immoveable by its Inhabitants, this Motion is referr'd to the heavenly Bodies, as has been said of the other Motions; therefore whilst the Poles of the World are mov'd about the Poles of the Ecliptic in *antecedentia*, and pass successively thro' all the Points, that are 23 Deg. 29 Minutes distant from these Poles, these Points themselves, or rather the fixed Stars that are in them, come towards the Poles of the World successively, and seem to be carried in *consequentia*, and to describe Circles which are
 really

really describ'd by the Poles of the World about the Poles of the Ecliptic, which being placed in Centers, alone are at rest; for together with the Stars above-mention'd, the rest of the Stars (because they keep the same Situation in respect to one another) * do also appear to be mov'd.

* 923

Therefore *the whole Sphere of the fixed Stars seems to move, in consequentia, about an Axis passing thro' the Poles of the Ecliptic, and each Star apparently describes a Circle parallel to the Ecliptic; by which Motion the Latitude of the Stars is not chang'd.*

1185

The Plane of the Equator makes a right Angle with the Axis of the Earth, therefore by the aforesaid Motion of its Axis, the Section of the Plane of the Equator, with the Plane of the Ecliptic, is mov'd round; wherefore *the first Points of Aries and Libra* *, which are always opposite, *move thro' the whole Ecliptic Line in the Space of about 25000 in antecedentia*; yet they are look'd upon as immoveable by the Inhabitants of the Earth, who imagine that the fixed Stars themselves are mov'd in *consequentia* *.

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CHAP. X.

Concerning the fixed Stars.

WE have said that the fixed Stars are lucid Bodies, removed so far off, that their Distances can be compar'd with no Distances in the planetary System; for *Astronomers have not been able, by their nicest Observations, to observe the Poles of the World carried out of their Place in the annual*

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annual Motion of the Earth, altho' they describe Circles in the Heavens which are equal to the
 • 952 *Earth's Orbit.* *

DEFINITION I.

1188 This Motion of the Pole is called the *annual Parallax*.

That the fixed Stars are at an immense distance, is also prov'd by Observations with the help of
 1189 Telescopes; if any fixed Star, even the lucid and conspicuous, be beheld with a Telescope, thro' which the Diameter of the Sun would appear equal to the Diameter of the Earth's annual Orbit, it will appear to be a lucid Point, without any sensible Magnitude; for all the fixed Stars appear less when they are seen thro' Telescopes, than they do to the naked Eye; for it is only their twinkling which makes them appear to have any sensible Magnitude.

1190 That the Stars may be distinguish'd, they are re-
 ferr'd to various Figures, which are imagin'd in the Heavens, and are called *Constellations*.

1191 Twelve Constellations are imagin'd to be in the Zodiac, which are called the Signs of the Zodiac; they receive their Names from the Animals or Things

1192 which they represent: *Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpius, Sagittarius, Capricornus, Aquarius, Pisces*. These Signs have given their Names to 12 Parts of the *Ecliptic*, of
 • 999 which we have spoken before*.

In the Time of *Hipparchus*, the Sections of the *Ecliptic* and *Equator* were between the Constellations of *Pisces* and *Aries*, and *Virgo* and *Libra*; and the Constellations gave their Names to those Parts of the *Ecliptic* which passed thro' each
 1193 Constellation; and the Parts of the *Ecliptic*, supposing the beginning of *Aries* and *Libra* in the Interse-
 ctions

*Sions of the Equator and the Ecliptic, have kept the Names which they had at that time, tho' these Intersections be carry'd from their old Places**; whence^{*1186} the Sun is said to be in *Taurus* when it moves among the Stars of the Constellation *Aries*.

The Zodiac separates the North Part of the Heavens from the South Part.

In the northern Region are the following Constella-¹¹⁹⁴ tions: the lesser Bear, the greater Bear, the Dragon, Cepheus, the Hounds, Bootes, the northern Crown, Hercules, the Harp, the Swan, the Lizard, Cassiopeia, Camelopardus, Perseus, Andromeda, the Triangle, the lesser Triangle, the Fly, Auriga, Pegasus, or the Flying-horse, Equuleus, the Dolphin, the Fox, the Goose, the Arrow, the Eagle, Antinous, Sobiesky's Shield, Serpentarius, the Serpent, Mount Mænalus, Berenice's Hair, the lesser Lion, the Lynx.

In the southern Region of the Heavens are the fol-¹¹⁹⁵ lowing Constellations, many of which are invisible to us; * namely, the Whale, the River Eridanus, the^{*1131} Hare, Orion, the great Dog, Rhinoceros, the lesser Dog, the Ship Argo, Hydra, the Sextant of Urania, the Cup, the Crow, the Centaur, the Wolf, the Altar, the southern Crown, the southern Fish, the Phoenix, the Crane, the Indian, the Peacock, the Bird of Paradise, the southern Triangle, the Cross, the Fly, the Chameleon, King Charles's Oak, the Flying fish, the Toucan, or American Goose, Hydrus, or the Water-Serpent, Xipbias, or the Sword-fish.

DEFINITION II.

The Stars which are between the Constellations are¹¹⁹⁶ called unform'd Stars.

The Stars are not equally lucid, and they are re-¹¹⁹⁷ ferr'd by Astronomers to six Classes; the most lucid are

are called Stars of the first Magnitude, others are said to be Stars of the second Magnitude, others of the third, &c. to the sixth Magnitude.

1198 Some are not referr'd even to this last Class, and are called nebulous Stars.

1199 There is also a certain Zone or Belt observ'd in the Heavens, which is not every where of the same Breadth, and goes round the whole Heavens, and in some Places is separated, so as to become dou-

1200 ble; from its Colour it is called *the milky Way*; it is plain from Observations, by the help of the Telescope, that this Way is an *Assemblage of innumerable fixed Stars, which cannot be seen by the naked Eye*, either because they are less than the other Stars, or more distant.

1201 Towards the *Antarctic Pole* there are two Nubeculæ, of the same Colour as the *milky Way*, which are also *Heaps of small Stars*, and cannot be seen without a Telescope; besides the Stars which are observ'd in these Nubeculæ, and in the *milky Way*,

1202 to whatsoever Part of the Heavens you point the Telescope, you may discover small Stars in a great Number, which are not visible to the naked Eye; very often an Heap of Stars appears to the naked Eye to be but one Star.

1203 Among the Stars, some are visible and invisible by Fits, and observe regular Periods; others are successively sometimes more lucid, sometimes of a duller Light, and to be seen only by the help of a Telescope, and that at certain Times.

1204 Yet they are not equally bright at every Period; sometimes Stars have appear'd suddenly exceeding the brightest in Light, which afterwards successively decreasing, have vanish'd in a short time, and still remain invisible.

1205 Besides the Stars, we observe in the Heavens several whitish Spots, which are in some measure lucid, and

and invisible to the naked Eye; their Light is re-
ferr'd to the Stars which are in them, or they
are look'd upon as nebulous Stars.

What these Spots are, cannot be determin'd;
perhaps they are a Congeries of Stars, which have
the same relation to the telescopic Stars, as those
which form the *milky Way* have to those which
are seen to the naked Eye.

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BOOK



BOOK IV.

PART II.

The PHYSICAL CAUSES of the Celestial Motions.

CHAP. XI.

Concerning universal Gravity.



Having explained the Motions of the heavenly Bodies, and the Phænomena arising from them, we must now examine by what Laws these Motions are performed.

We have before laid down the Laws according
 • 124 to which the Motions of Bodies are directed *;
 125 if we add one to these, we shall see the whole
 126 Contrivance by which that vast Machine, the
 Planetary System, is governed.

1206 The Law to be added to the rest is this: *All*
Bodies are mutually heavy (or gravitate) towards
 1207 *each other; this Gravity is proportional to the Quan-*
 1208 *tity of Matter; at unequal Distances it is inversely as*
the

the Square of the Distance ; that is, all Bodies mutually attract, or tend towards each other, with the Force which belongs to each Particle of Matter acting upon each Particle ; and the Force with which a Body acts upon others, is compounded of the joint Forces of all the Particles of which the Body consists ; so this Force increases in the same Proportion as the Quantity of Matter, and is unchangeable in every Particle ; it is always the same at the same distance ; but the distance increasing, the Force decreases as the Square of the distance increases.

We call this Force Gravity, when we consider a Body which of itself tends towards another ; because this Force is called by this Name near the Earth's Surface *.

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But when we consider a Body towards which another tends, we call this Force Attraction ; we mean the same Effect by these Names, and nothing but the Effect ; for since all Gravity is reciprocal *, it is the same to say, all Bodies gravitate mutually towards one another, as that Bodies mutually attract one another, or mutually tend towards each other.

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We look upon this Effect as a Law of Nature *, because it is constant, and its Cause is unknown to us, and cannot be deduc'd from Laws that are known, as we shall shew by and by. Now that there is such a Gravity, is to be prov'd from Phænomena.

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All the primary Planets are kept in their Orbits by Forces, which tend towards the Center of the Sun * ; therefore there is a Force by which the Planets are carried towards the Sun, and whereby the Sun tends reciprocally towards each of them * ; *that is, the Sun and Planets gravitate mutually towards each other.*

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1212 After the same manner it is plain, that the *Satellites of Jupiter and Saturn gravitate towards each other*; as also that the *Satellites of Saturn gravitate towards their primary Planet, and that towards them* *.

1213 The *Moon and the Earth also gravitate towards each other* *.

1214 All the *secondary Planets gravitate towards the Sun*, for they are all carried by a regular Motion about their primary Planets, as if the primary Planets were at rest; whence it is plain, that they are carried about by the common Motion with the primary Planets; that is, that the same Force by which the primary Planets are every Moment carried towards the Sun, acts upon the *secondary ones*, and that they are carried towards the Sun with the same Celerity as the primary Planets; even the Irregularities of the secondary Planets, which are so small as only to be sensible with respect to the Moon, confirm this Gravity of the secondary Planets towards the Sun; for we shall shew hereafter, that all the Irregularities are caused by the Change of the Moon's Gravity towards the Sun at a different distance; and because the Lines, in which the Earth and Moon tend towards the Sun, are not altogether parallel.

1216 From the Gravity of the secondary Planets towards the Sun, it follows that the *Sun gravitates towards them* *.

1217 In respect of the Gravitation of the primary Planets towards one another, Astronomers have observ'd, that *Saturn changes its Way when it is nearest to Jupiter*, which is far the greatest of all the Planets; so that it is plain from immediate Observations, that *Jupiter and Saturn gravitate towards each other*.

Jupiter also in this Case, as *Flamsteed* has observ'd, disturbs the Motion of the Satellites of *Saturn*, attracting them a little to itself; which proves that these Satellites gravitate towards *Jupiter*, and *Jupiter* towards them. From all which Considerations * compar'd together, it follows, that the 17 Bodies, of which the Planetary System is made up, mutually gravitate towards each other, altho' no immediate Observations can be made concerning the Gravitation of each particular one towards the rest *.

The second Part of the Law is *, that Gravity is proportional to the Quantity of Matter; that is, that all the Particles of Matter gravitate towards each other; and therefore that the Law of Gravity is universal, and that every Body acts upon all other Bodies; which is deduc'd from Phænomena.

The Forces of Gravity are as the Qualities of Motion which they generate *, and these Quantities, in unequal Bodies that are equally swift, are to one another as the Quantities of Matter; therefore since unequal Bodies, at the same distance from the attracting Body, move equally swift by Gravity *, it is evident that the Forces of Gravity are proportional to the Quantity of Matter; we find the same in all Bodies near the Earth's Surface, which have a Gravitation towards the Earth, proportional to their Quantity of Matter *; but the mutual Gravity of all these Bodies towards one another is not sensible, because it is exceeding small in respect of their Gravity towards the Earth, and therefore cannot disturb their Motion arising from their Gravity towards the Earth *, at least so as to make any sensible Change in the Direction of their Motions.

We shall presently shew, by another Method, that this universal Gravity of all the Particles of Matter, whereby they act upon one another, may be prov'd from Phænomena*.

The third Part of the Law which we examine is, that Gravity decreases when the Distance increases, and is inversly as the Square of the Distances, which also follows from Phænomena.

Bodies upon which Gravity acts according to their Quantity of Matter, as in our System, are mov'd with an equal Celerity in the same Circumstances, as we said before; so that it is no matter whether the Bodies are greater or less, and they are mov'd as if they were equal; but in this Case, if the Force towards a Point decreases in an inverse Ratio of the Square of the distance from that Point, and the Bodies move at various Distances from it, and are kept in Circles by that Force, the Squares of the periodical Times will be to one another as the Cubes of the Distances*, which is demonstrated by Geometers to obtain (in respect of the mean Distances) in elliptic Lines, whose Forces are directed to their Foci; but this is the Case in Bodies which revolve about the *Sun*, *Saturn*, and *Jupiter**; whence it follows, that the Force of Gravity receding from the Centers of these Bodies, decreases in an inverse Ratio of the Squares of the Distances.

By this Reasoning, supposing Gravity proportional to the Quantity of Matter, we demonstrate that it decreases in an inverse Ratio of the Square of the distance; and by the same Reasoning, supposing the Diminution of Gravity to be in this Proportion, it follows that Gravity is proportional to the Quantity of Matter; as is very evident.

But we prove by another Argument, that the Diminution of Gravity, so often mention'd, is in

an inverse Ratio of the Square of the Distance; so that there can remain no Doubt concerning the two Laws of Gravity, which we now treat of.

The Planets are mov'd in Orbits at rest *, and are kept in them by Forces, which are directed to an eccentric Point *; but it is plain that this would not obtain, if the central Force did not increase in an inverse Ratio of the Square of the Distance *.

It follows from the same Reasoning, that receding from the Center of the Earth, Gravity decreases, according to the same Law; for the Moon is retain'd in its Orbit by a Force which tends towards the Center of the Earth, that is, to an eccentric Point *; and tho' the Line of the *Apsides* is not carried by a parallel Motion, its Agitation is so small, if we consider every Revolution, that it may be look'd upon here as quiescent; for if we compute the Force which keeps the Moon in its Orbit so agitated, we shall find the Diminution of the Force of Gravity, in respect of the Moon, to differ very little from an inverse Ratio of the Square of the Distance; and we shall shew hereafter that this difference depends upon the Action of the Sun.

And no Doubt will remain concerning this Diminution, if we consider, that the Moon is kept in its Orbit by that very Force wherewith Bodies are carried towards the Earth, near the Earth's Surface; which is diminish'd, according to the Law of Diminution so often mention'd. The mean distance of the Moon is $60\frac{1}{2}$ Semidiameters of the Earth; we have before shewn that a Diameter of the Earth contains 3400669 *Rbynland* Perches *; whence knowing the periodical Time, we easily discover that the Moon in one Minute of Time goes thro' $16425\frac{1}{2}$ *Rbynland* Perches of her Orbit; this Arc is not the hundredth Part of one

Degree, and may be look'd upon as its Subtense, therefore the Diameter of its Orbit is to this Arc as the Arc itself is to its versed Sine, which is discover'd to be of 15,736 *Rbynland* Feet, and it is the Space which the Moon and Earth would go thro' in one Minute, coming to one another by their mutual Attraction. The Celerity with which a Body comes to another by the Force of Gravity, depends upon the Force with which it is attracted by that other, all whose Particles of Matter attract it; therefore the Celerities of the Moon and Earth, as they come towards each other, are inverfly as the Quantities of Matter in them, which is also deduc'd from the equal

• 126 Quantity of Motion that is in each Body*; there-
 65 fore by this Proportion we discover how much of the aforesaid Space (15,736 Feet) is gone thro' by the Moon; as the Quantity of Matter in both Bodies is to the Quantity of Matter in the Earth, so is the Space gone thro' by both Bodies in their mutual Access towards each other to the Way gone thro' by the Moon only. The Quantities of Matter in the Moon and in the Earth, as we shall shew hereafter, are to one another as 1 to 39, 37; and 40, 37 is to 39, 37 as 15,736 to 15, 344, the Space gone thro' by the Moon; which therefore would be gone thro' in one Minute by any Body, which at the Moon's distance should be impell'd by Gravity towards the Earth; this Force increasing in an inverse Ratio of the Square of the distance from the Center, the Space gone thro' in the same time at the distance of a Semi-diameter of the Earth, that is, on its Surface, will be $60' \times 60'' \times 15,344$ (*viz.*) 56158 Feet; but because in every Motion equably accelerated, as here (for we consider the Force as remov'd from the Earth's Center to the distance of its Surface) the Squares of the times are as the Spaces gone thro'

thro' in the Fall *, by dividing the Number by * 131
 60×60 , that is, 3600, we have the Space gone
 thro' by a Body in one Second near the Earth's
 Surface, by the Force with which the Moon is
 kept in its Orbit, which is discover'd to be 15,6.
Rbynland Feet.

Now if we examine the Gravity which we dai-
 ly find in all Bodies near the Earth's Surface *, it * 72
 is plain from what has been said concerning the
 Motion of Pendulums *, and from Experiments * 157
 made upon Pendulums, that a Body in falling 158
 goes thro' 15,6. *Rbynland Feet* in one Second of
 Time, and therefore falls with the Force by
 which the Moon is kept in its Orbit.

In this Computation we have neglected to con-
 sider the Action of the Sun, because it is small,
 and sometimes increases, sometimes diminishes
 the Gravity of the Moon towards the Earth.

We have consider'd the Centers of the Bodies
 in examining the Law of the Diminution of Gra-
 vity, altho' Gravity belongs to all the Particles
 of Bodies; because it is plain by mathematical
 Demonstration, that *the Action of a spherical Body*
(in which, in every Part, the Particles, that are 1226
equally distant from the Center, are homogeneous, and
which is made up of Particles, towards which there is
a Gravity that decreases, receding from each of them
in an inverse Ratio of the Square of the distance) is di-
rected towards the Center of the Body; and receding
from it, is diminish'd in the same inverse Ratio of the
Square of the distance; so that such a Body acts as
 if all the Matter of which it consists was collected
 in its Center; whence we deduce the following
 Conclusions.

That on the Surfaces of Bodies, in which the Mat- 1227
 ter is homogeneous at equal Distances from the Center;
 the Gravity is directly as the Quantity of Matter, and
 inversely as the Square of the Diameter *; for in * 1207

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1208 these Bodies the Distances from the Center are as the Diameters.

1228 That on the Surfaces of Bodies that are spherical, homogeneous, and equal, the Gravities are as the Densities of the Bodies; for the Distances from the Center are equal, in which Case the Forces of Gravity

•1207 are as the Quantities of Matter*, which in equal

• 228 Bodies are as their Densities*.

79 That on the Surfaces of the Bodies that are spherical, unequal, homogeneous, and equally dense, the Gra-

1229 vities are inversely as the Squares of the Diameters*,

•1208 because the Distances from the Center are in the

•1207 Ratio of those Diameters*, the Gravities also are directly as the Cubes of their Diameters; for the Quantities of Matter in Spheres are in that Ratio; and the Ratio compounded of that direct Ratio of the Cubes of the Diameters, and the inverse Ratio of their Squares, in the direct Ratio of the Diameters themselves.

1230 Therefore if both the Densities and the Diameters differ, the Gravities on the Surface will be in a Ratio

•1228 compounded of the Densities* and the Diameters*;

•1229 therefore dividing the Gravity on the Surface by

1231 the Diameter, you will have the Density; which consequently is in a direct Ratio of the Gravity on the Surface, and an inverse Ratio of the Diameter.

1232 If a Body be plac'd in a Sphere that is homogeneous, hollow, and every where of the same thickness, where-soever it be placed it has no Gravity, the opposite Gravities mutually destroying one another pre-

1233 cisely; whence it follows, that in an homogeneous Sphere a Body coming towards the Center, gravitates towards the Center only from the Action of the Sphere, whose Semidiameter is the distance of the Body from the Center; which Gravity decreases in coming towards the Center, in the Ratio of

•1229 the distance from the Center*; for all Matter, which is at a greater distance from the Center, forms an hollow

hollow Sphere, in which the several Actions on a Body destroy each other*.

*1232

We have said, that the Gravity which we have hitherto explain'd, is to be taken for a Law of Nature, because we don't know the Cause of it, and because it depends upon no Cause that is known to us; which will evidently appear, if we attend to what follows.

(Viz.) That Gravity requires the Presence of the attracting Body; so the Satellites of Jupiter, ex. gr. gravitate towards Jupiter, wheresoever it be*.

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That the distance remaining the same, the Celerity with which Bodies are carried by Gravity, depends upon the Quantity of Matter in the attracting Body*; and that the Celerity is not chang'd, let the Mass of the gravitating Body be what it will*.

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*1207

Besides that, if Gravity depends upon any known Law of Motion, it ought to be referr'd to a Stroke from an extraneous Body; and because Gravity is continual, a continual Stroke would be required.

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If there be such a Sort of Matter continually striking against Bodies, it must of necessity be fluid and very subtile, so as to penetrate all Bodies; for Bodies that are any how shut up in others are heavy.

Now let a Mathematician consider, whether a Fluid so subtile as freely to penetrate the Pores of all Bodies, and so rare as not sensibly to hinder the Motion of Bodies (for in a Place void of Air the Motion of a Pendulum will be continu'd very long) can impel vast Bodies towards one another with so much Force; let him explain how this Force increases in a Ratio of the Mass of the Body towards which another is carried*.

*1235

Lastly, let him shew, what seems most difficult to me, how all Bodies, in any Situation whatsoever (if the distance, and the Body towards which the Gravitation is, remain the same) are carried

*1236 with the same Velocity *; that is, how a Liquid, which can only act on the Surfaces either of the Bodies themselves, or their internal Particles, to which it is not hinder'd from coming by the Interposition of other Particles, can communicate such a Quantity of Motion to Bodies, which in all Bodies exactly follows the Proportion of the Quantity of Matter in them, and which in this Chapter we have prov'd to obtain every where in Gravity, and which we have demonstrated by a direct Experiment, in respect of the Gravity near the Earth's Surface *.

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1238 Yet we don't say that *Gravity does not depend upon any Stroke*, but that it does not follow from that Stroke, *according to any Laws known to us*; and we confess that we are entirely ignorant of the Cause of Gravity.

CH A P. XII.

Of the celestial Matter; where a Vacuum is proved.

HAVING explained the Laws whereby the whole planetary System is govern'd, several things must be first laid down before we proceed to the physical Explication of the System. We must begin, by saying something of the celestial Matter, that is, of the Medium in which the Bodies that make up the System are mov'd; which would be done in few Words, if all Philosophers agreed that there is a *Vacuum*.

We

We have before prov'd that a *Vacuum* is possible*, now we are to demonstrate that there is really one; *from only considering Motion we can deduce a Vacuum*, and this is a very common and usual way of proving it; to see the Force of which Argument, we must consider, that indeed all Motions are not impossible without a *Vacuum*, but most of those which are daily observ'd, which might be fully evinc'd by a longer Discussion; but it seems to me to be so evident from the following Consideration, that it would be useless to add much more.

The Figure of the least Particles is unchangeable, for the Particle whose Figure may be changed, consists of smaller Particles, which are moved in respect to each other; and therefore if it has a changeable Figure, it is not one of the least Parts.

But if the Figure of these Particles be unchangeable, and a Body can move between them, without such a Separation of the Particles as to leave a void Space, this will depend on the Figure of the Particles, and the Relation which they have to one another, which a Mathematician will not deny; therefore if keeping things in this State (as to their Figure and Relation) the Particles are increased, even in that Case Bodies may be mov'd without a *Vacuum*.

Now supposing the smallest Particles increas'd to the bigness of a cubic Foot, whatever be their Figure and Relation with the other Parts, which we suppose increas'd in the same Proportion as the first; let any one consider, whether Bodies of any bigness can be carried between those Parts in right Lines, and in all sorts of Curves, and yet never separate the Particles, so as to leave Vacuities between them?

We

We cannot conceive how the most subtile Parts are made, and therefore often attribute to them such Properties as do not follow from their Figure; but these Errors will be corrected by imagining the Particles increased.

1240 *We also prove a Vacuum by an Argument taken from Resistance.*

- 12 We have said that Matter is inactive*; some dispute about the Word, but no Man denies the thing; whence it follows, that a Body cannot move thro' a Fluid without undergoing a Resistance*, and consequently a Retardation*. The
- 319
- 330 Resistance arising from the *Inertia* of the Matter (which Resistance alone is here consider'd) depends upon the Quantity of Matter to be remov'd out of its place, which is the same whether the
- Parts of a Fluid be greater or less, if the Celerity of the Body remains the same; whence it follows, that in determining what relates to the Resistance, we must have no regard to the Subtlety of the Fluid, as long as it cannot go thro' the Pores of Bodies; for if we come to such a Fineness of Parts, that a Fluid shall partly penetrate a Body, it will less resist the Body.

Now let us suppose any Ball or spheric Body to be mov'd along in a Medium of the same Density as itself, and so close, that the Parts of the Medium cannot pass thro' the Pores of the Body, this Body will be retarded every Moment; so that its Velocity at last will be reduc'd to half (as may be prov'd by a mathematical Demonstration) before the Body has gone thro' twice the Length of its Diameter.

In order to apply this Proposition to a Motion in a very subtile Fluid, which freely penetrates the Pores of all Bodies, and fills all Places, we must conceive a spherical Body without any Pores at all; and that such a Body may be supposed, by

by imagining all the Particles of Matter closely join'd, no Body will deny.

That the Resistance of such a Body in any Fluid does not depend upon the Bigness of the Parts of the Fluid, and is the same, whether the Parts of the Fluid be equal, or any how unequal, is evident.

If every thing be full of Matter, this Body can only move thro' a Fluid of the same Density as itself, for it must run against all the Matter which is in those Places thro' which it passes; and in them the Matter is without Interstices, as it is in the Body; therefore it will lose half its Velocity before it has run thro' the Length of twice its Diameter.

Now let us suppose the Body to be increas'd, the Quantity of Matter remaining the same, and the Body continuing homogeneous; that is, let there be Pores in the Body, thro' which the most subtile Particles of Matter may pass very freely, and let these Pores be equally dispersed all over the Body; if the Body thus chang'd be mov'd, the very subtile Fluid, of which we speak, will not run against the whole Surface, but only those Parts of the Surface which are between the Pores; which Parts being taken together, because we suppose the Body homogeneous, are equal to the Surface of the Body in the first Supposition, when we conceiv'd it to be without Pores; for the Body being increas'd, the Surface has not been changed, but only dilated by the Interposition of Pores; so that in both Cases the Body will undergo the same Resistance from the Impulse upon the Surface, and the Resistance on the dilated Body is greater, from the Fluid running against the internal Parts of the Body; wherefore this Body will sooner lose half its Motion in the second than in the first Case, that is, before it has
run

run thro' the Length of two Diameters of the first supposed Bigness; and therefore it loses a greater Quantity of Motion in going thro' two Diameters of the Bigness supposed in the second Case.

But this is contrary to Experience; for a homogeneous Ball of Gold, or Lead, &c. loses a much less Quantity of Motion than what we have mention'd in Water or Air; whence it follows, that the Supposition that all Things are full of Matter, is false; therefore there is a *Vacuum*.

- 1241 *That there is a Vacuum, does also agree with the Phænomena relating to Gravity*; by which it follows, that it is proportional to the Quantity of Matter; if all was full of Matter, Gravity would act equally every way, and the Forces which are directed towards opposite Parts, would destroy one another; and therefore no sensible Gravity would be observ'd, which is contrary to Experience.

These being premised, we must return to the celestial Matter.

- 1242 The Motions of the heavenly Bodies do not depend upon the Motion of the celestial Matter, if
 • 1238 there be such a Matter*; *whereby is overthrown the Opinion of those which say that the heavenly Bodies are carried along by the common Motion of the Matter which fills our System*. This Opinion is also overthrown by the Motion of the Comets; if there was a Medium in the System which carried about the Planets in its Motion, and also the Comets, it would at least sensibly disturb these last in their Motions, whilst they come almost directly towards the Sun, or directly recede from it, or are carried in *antecedentia*, that is, in a Motion contrary to the Motion of that Matter; now as this Motion is not disturb'd, but follows the Way which depends upon Gravity, as it is observ'd,
 it

it is plain, that if there be any celestial Matter, and that it is in motion, it does not exert a sensible Action on the Bodies of the planetary System, which is also deduc'd from the small Resistance of such a Medium; for by comparing the most ancient Observations with the modern, it does not appear that the Planets are sensibly retarded in their Motions: Yet in Air the Resistance is sensible, wherefore the Density of the Medium, in which the Planets should move, must be almost immensely less; therefore *the planetary System is not filled, unless it be by such a subtile Medium.* 1243

But we may from the Divisibility of Matter deduce, that a Quantity of Matter, how small soever it be, may be dispersed all over the planetary System, leaving but very small Interstices*. * 29

C H A P. XIII.

Concerning the Motion of the Earth.

BESIDES the Question that has been handled in the foregoing Chapter, there is also another to be examin'd, before we proceed to the Explanation of the whole System.

That no Doubt may be made concerning the System, which has been explained in the first Chapter of this Book, we must here prove the Motion of the Earth, concerning which it is no Wonder that many have doubted; for the celestial Motions cannot be determin'd by us, but by Observations made by Observers on the Earth,
and

and the same Phænomena appear, whether the Bodies themselves be moved, or the Spectator be moved * ; so that it is not to be prov'd by immediate Observations, whether the Motion of the Earth is to be referr'd to the heavenly Bodies or not.

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1244 *That the Earth is carried about the Sun, is deduced from the Analogy of the Motions, and from an Examen of the Laws of Nature.*

As to what relates to the Analogy of the Motions, it is to be observ'd, that Satellites revolve about *Jupiter* and *Saturn*, which are less than the central Body ; that the Moon revolves about the Earth, than which it is less : lastly, that the Sun has revolving about it less Bodies than itself, as *Mercury*, *Venus*, *Mars*, *Jupiter*, and *Saturn* ; now if the Earth revolves with the rest, then every

1245 *where in our System the lesser Bodies move about the greater : Now there would be an Exception in this Rule, in respect of the Sun, if that vast Body was*

• 975 *to go round so small a Body as the Earth *.*

About the Sun *Jupiter* and *Saturn*, about which several Bodies revolve, *those move the slowest which*
1246 *are most distant from the central Body, and according to this Rule, that the Squares of the periodical Times follow the Ratio of the Cubes of the*
• 974 *Distances * ; which Rule may be applied to the Earth, if it be carried about the Sun with the rest of the Planets, as appears, if its periodical Time (namely, the Time in which the Sun appears to perform an entire Revolution) and its Distance from the Sun be compared with the Distances and periodical Times of the rest of the Planets.*

Now this Rule has only one Exception ; if the Sun be mov'd about, the Earth is at rest ; in this Case *Mercury*, *Venus*, *Mars*, *Jupiter* and *Saturn*, are subject to this Rule in their Motions, as also the
five

five Satellites of *Saturn*, and the four Planets that accompany *Jupiter*, only the Moon and the Sun would move above the Earth in a Proportion quite different, and then the Celerity of the Sun would not only be greater than is required by this Law, but its Velocity would at least be 26 times greater than that of the Moon, tho' it be remov'd to a vast distance from the Earth in respect to the Moon's distance; and therefore in this respect the Analogy of the celestial Motions would be disturb'd.

To these Arguments I shall add others, whereby it will clearly appear, that the Motion of the Earth is a necessary Consequence of the Laws of Nature, which are deduc'd from Phænomena.

All Bodies gravitate towards one another*, 1247
therefore the Sun and Earth do; but the Motion *1206
whereby these Bodies tend towards one another, is deduc'd from direct Observations; which soever of these Bodies moves about the other, describes Area's by Lines drawn to the Center of it, proportional to the Times, which is evident from astronomical Observations; therefore the Body mov'd is retain'd in a Curve, by a Force which is directed towards the Center of the other*. Now as Re-action is always equal to * 226
Action*, unless the Laws of Nature, which ob- * 126
tain constantly every where, be wholly overturn'd, these two Bodies tend towards one another with equal Motions, that is, with Celerities that are inversely as their Masses*, which is * 65
also immediately deduc'd from the Law of Gravity*. *1235

The Quantity of Matter in the Earth is next to nothing, in comparison to the Quantity of Matter in the Sun, as we shall shew in the following Chapter; wherefore the *Sun must move*
very

very slowly whilst the Earth comes towards it very swiftly.

Whence it follows, that the Earth is carried round the Sun, lest it should fall upon the Sun by that very violent Motion whereby it is retained in its Orbit.

This Motion of the Earth is also deduc'd from the same Principles another way.

Two Bodies that are carried towards one another by any Force, will at last concur, or continually recede from one another, unless each of them be so mov'd as to have a centrifugal Force, equal to the Force whereby it is carried towards the other Body; but as the Bodies which gravitate towards one another, tend towards each other with equal Forces *, or what is the same *, * 65 with Celerities that are inversly as the Quantities of Matter *, these Bodies cannot persevere in their Motions about one another, unless both of them be so mov'd as to have equal centrifugal Forces, which does not happen, unless they both revolve in equal times about their common Center of Gravity; that is, if this Proposition be applied to the Sun and Earth, unless they both move about a Point, whose distance from the Center of the Sun is to its distance from the Center of the Earth, as the Quantity of Matter in the Earth is to the Quantity of Matter in the Sun, they cannot persevere in their Motions about one another *: This Point, or Center of Gravity, must of consequence be very near to the Sun's Center. Now since which soever of these Bodies moves, it perseveres in its Motion about the other, it follows that both of them are affected by the Motions above-mention'd, and that the Sun is moved but

but a little, whilst the Earth describes a very great Orbit; whence it follows, that the Motion of the Earth cannot be denied by any one who reasons from the Laws of Motion, that are deduc'd from Phænomena.

Having prov'd *the annual Motion of the Earth*, 1249 and brought back the Earth among the Planets, there remains but little difficulty in relation to the Motion of it *about its Axis*, for no Body that believes the annual Motion doubts of this; a great many who allow of the Motion about the Axis, deny the annual Motion; therefore it will be enough to observe by the bye, that all the Planets, concerning which any Observation could be made in respect of this Motion, do move about their Axes; and that the Earth has such a Motion, *the uniform diurnal Motion in Bodies, at any Distances, does plainly enough shew*; to which we must add, that the Celerity of the fixed Stars going thro' one whole Revolution in less than 24 Hours, can hardly be more probable than it is conceivable.

This Motion also is disagreeable to the Nature of all *the heavenly Bodies*; for if they are carried 1250 round, they must every Day, with an equable Motion, describe Circles that have the Earth for their Center; that is, they must, by Lines drawn to the Center of the Earth, sweep thro' Area's proportionable to the times, and be retain'd in their Orbits by Forces which are directed towards the Center of the Earth*, and by which (by reason that Action and Re-action* are equal) *the Earth* must also be continually attracted towards those Bodies, so that it *must necessarily be agitated by a very violent Motion*; whence it appears that the diurnal Motion must not be referr'd to the heavenly Bodies, but to the Rotation of the Earth about its Axis.

1251. Those who obstinately affirm that the Earth is at rest, object, that Bodies upon the Surface of the Earth must (on account of their centrifugal Force) recede from the Earth along a Tangent to a Circle parallel to the Equator*: We answer;
- * 217 That the Bodies, in the Places where they are, are carried round with the same Motion as the Surface of the Earth; and therefore, that in respect of the Points of the Surface, they endeavour
- * 223 to recede in Lines perpendicular to the Axis*; but also that Bodies by Gravity tend to the Center of the Earth*; and therefore by a Motion compounded of both these the Body is continually moved, or endeavours to move*; but because
- * 190 198 the first Motion is extremely small in respect of the other, a heavy Body is turn'd but very little out of its Direction towards the Center, and the Gravity is a little diminish'd, so much the more as the Place is more distant from the Pole; which agrees with Experience. We shall hereafter shew, when we come to speak of the Figure of the Earth, that the above-mention'd Direction of heavy Bodies is every where directed perpendicularly to the Surface of the Earth. A Body which is thrown upwards is acted upon, not only by the Motion wherewith it is thrown up, but it is also carried by the Motion that is impress'd to the Person or Machine that impels the Body; that is, it is carried by the Motion which is common with the Surface of the Earth, and therefore the
- * 212 Body moves in the same Line (the Line being
- * 211 carried on with the Surface of the Earth) as it would do if the Earth was at rest.

C H A P. XIV.

Concerning the Density of the Planets.

BEfore we proceed to the physical Explanation of the System, we must determine the Quantities of Matter in some Bodies, and their Diversities; which being known, the Effects of the Laws by which these Bodies are govern'd will more easily appear.

The Quantities of Matter in different Bodies are to one another as the Gravities at the same distance from these Bodies *, which Gravities are to one another inverſly as the Squares of the periodical Times of the Bodies revolving about those different Bodies at the same distance *; by multiplying the Quantities which are in this Ratio by the same Quantity (*viz.*) by the Cube of this distance, the Ratio of these Quantities will not be chang'd, which are therefore to one another as the Quotients of the Divisions of the above-mention'd Cube by the Squares of the periodical Times aforesaid; but the Quotient of such a Division is found for any Body by dividing the Cube of the other distance, let it be what it will, by the Square of the periodical Time of the Body revolving at that distance, for such Quotients are equal to one another for all Bodies that revolve about the same Body at any Distances, as follows from the Equality of the Ratio between the Cubes of the Distances and the Squares of the periodical Times at those Distances *; from which we deduce, that the Quantities of Matter in any Bodies in our System, are to one another directly as the Cubes of the Distances at which other Bodies revolve about these,

these, and inversely as the Squares of the periodical Times of these revolving Bodies.

These things are demonstrated by setting aside the Agitation of the central Body, whose Quantity of Matter is enquired after.

By reason of the Sun's Magnitude in respect of *Venus*, *ex gr.* which alone we consider of the Planets, the Sun is scarce mov'd by the Action of
 1235 that Planet; and *Venus* may be consider'd as moving about a quiescent Body.

The Satellites of *Jupiter* and *Saturn* are indeed carried by the common Motion along with the primary Planets, but by reason of the Magnitude of the primary Planets, they are carried about them as about Bodies that are at rest.

But the Moon acts sensibly enough upon the Earth, and moves it; wherefore before we can compute the Motion of the Moon by the help of
 1252 the aforesaid Rule, in order to compare the
 1253 Quantity of Matter in the Earth with the Quantities of Matter in the Sun, *Jupiter*, and *Saturn*, we must determine *the distance at which the Moon would move about the Earth*, if it was at rest (that is, *not carried about by the Action of the Moon*) in the same periodical Time in which it now performs its Revolution; here also we don't take notice of the Motion that is common to the Earth and Moon, by which they are both carried about the Sun.

The Moon perseveres in its Motion about the Earth, therefore the Earth and Moon are mov'd about a common Center of Gravity, as follows from what has been demonstrated concerning the
 1248 Earth and the Sun; and the Moon (with that Force with which it tends towards the Earth) revolves in an Orbit, whose Semidiameter is the distance of the Moon from the aforesaid common
 1251 Center of Gravity of the Moon and the Earth.

Let L be this distance of the Moon from the common Center of Gravity, T the distance of the Earth from the same Center, $L + T$ therefore is the distance of the Moon from the Earth, and is 60 half Semidiameters of the Earth; for here we consider the mean distance. Let D be the distance which we would have, at which the Moon, by its Gravity towards the Earth, would move about the Earth, if it was at rest, in the same Time in which it is now mov'd about the common Center of Gravity at the distance L .

By reason of this Equality of the periodical Times, the Force whereby the Moon would be kept in its Orbit at the distance D , is to the Force whereby it is kept in its Orbit at the distance L , as D to L *.

But the Force whereby the Moon would tend to the Earth, and be kept in its Orbit at the distance D , is to the Force whereby it is now kept in its Orbit at the distance $L + T$, as $L + T$ to D *. Therefore,

$D, L :: L + T, D^2$ consequently $D^2 = L \times \frac{L + T}{D}$, and $D^2 \times \frac{L}{L + T} = L \times \frac{L}{L + T}$: whence we deduce the following Proportion:

$L + T, D^2 :: L + T, L$. Therefore $L + T, D :: L + T$, is to the first of two mean Proportionals between $L + T$ and L .

$L + T$ is to L , as the Quantity of the Matter in the Earth and Moon taken together, to the Quantities of Matter in the Earth alone *; which Quantities of Matter, as we shall shew hereafter, are to one another as 40,37. to 39,37. and the first of two mean Proportionals to these Numbers

is 40,035; therefore 40,37. is to 40,035. as $60\frac{1}{2}$ to the distance required, which is found to be 60 Semidiameters of the Earth.

Concerning this Operation, it is to be noted, that the distance D cannot be discover'd, unless the Ratio between the Mass of the Moon and the Earth be known; which cannot be determined, unless the Ratio between the Density of the Sun and the Earth be found; to discover which, it is necessary that the distance D be known; wherefore D is discover'd at first by Trials, and is exactly determin'd by Approximation; but it is certain that this is 60 Semidiameters of the Earth; because this being suppos'd, it is found that the Ratio between the Quantities of Matter of the Moon and the Earth is as 1 to 39,37. as we shall see hereafter; by making use of which Proportion, this distance is discover'd to be 60 Semidiameters, as we have shewn.

These things being premis'd, we proceed to the Computation.

The distance of *Venus* from the Center of the Sun is 723, and its periodical Time is 5393 Hours*.

The fourth Satellite of *Jupiter* is distant from the Center of *Jupiter* 12,507. such Parts of which *Venus* is distant from the Sun 723; the periodical Time of this Satellite is 402 Hours, 5 Minutes*.

The fourth Satellite of *Saturn* is distant from the Center of *Saturn* 9,292. of the same Parts; and its periodical Time is 382 Hours, 41 Min.*

Lastly, the distance of the Moon is 60 Semidiameters of the Earth from its Center, and 2,909. of the aforesaid Parts; its mean periodical Time is 655 Hours, 43 Minutes.

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If you divide the Cubes of these Distances respectively by the Squares of the periodical Times, you will have in the Quotients, Numbers which are to one another as the Quantities of Matter in the aforefaid central Bodies*; which Quotients are to one another as the following Numbers.

Quantities of Matter	In the Sun.	In Jupiter.	In Saturn.	In the Earth.
	10000.	9,248.	4,223.	0,0044.

We have also the Proportion of the Diameters of these Bodies from astronomical Observations, as follows.

Diameters	Of the Sun.	Of Jupiter.	Of Saturn.	Of the Earth.
	10000.	1077.	889.	104.

If the Quantities of Matter abovesaid be divided by the Squares of the Diameters, the Quotients will be to one another as the Weights on the Surfaces of the aforefaid Bodies*; and these Quotients are as the following Numbers.

Gravities on the Surfaces	Of the Sun.	Of Jupiter.	Of Saturn.	Of the Earth.
	10000.	797,15.	534,337.	407,832.

If you divide these Numbers by the Diameters, you will have the Proportion of the Densities of these Bodies*.

The Quotients found by these Divisions are as the following Numbers.

Densities	Of the Sun.	Of Jupiter.	Of Saturn.	Of the Earth.
	10000.	7404.	6011.	39214.

We shall determine the Density of the Moon in the last Chapter.

It is not probable that the aforesaid Bodies are homogeneous; we shall shew in relation to the Earth, in the 17th Chapter, that it is denser towards the Center than towards the Surface; from whence it follows, that the Densities cannot be exactly determin'd; wherefore we only determine the mean Densities, that is, which the Bodies would have, if the same Bodies, keeping the same Quantity of Matter and Bulk which they now have, should become homogeneous.

The above-mention'd Proportion* between the Densities, in respect of all the Bodies, and the rest of the Computations in respect of the Sun, Jupiter, and Saturn, are free from any sensible Error; when they are compared with the Earth there may be some Error, which must be corrected by Observations to be made hereafter; for we suppose the distance of the Moon (which is 60 Semidiameters of the Earth) to be 2,909 such Parts, of which Venus is distant from the Sun 723, that is, of which the Earth is distant from the Sun 1000*; which distance of the Moon is discover'd by supposing the horizontal Parallax of the Sun 10", which cannot be look'd upon as absolutely true, altho' it be deduc'd from the most exact Observations that have hitherto been made of the Parallax of Mars when it is nearest of all to the Earth, which is too small to leave us without Suspicion of some Mistake*.

But the Error, in not determining truly the Proportion between the Semidiameter of the Earth and the distance from the Sun, does not change the determin'd Density of the Earth, as is deduc'd from Computations made about it.

For it follows from these, that the Densities of Bodies are to one another in a Ratio compounded of the direct Ratio of the Cubes of the Distances

ces of the Bodies carried about, and the inverse Ratio of the Squares of the periodical Times of these revolving Bodies * ; as also of the inverse Ratio of the Cubes of the Diameters of the central Bodies, whose Denfities are required * ; the Ratio, compounded of these, is compounded of the direct Ratio of a Fraction, whose Numerator is the Cube of the distance of the revolving Body, and whose Denominator is the Cube of the Diameter of the central Body, and the inverse Ratio of the Square of the periodical Time of the Body carried about ; but you have such a Fraction, if you know the Ratio between the Diameter of the central Body and the distance of the revolving Body from that Center, altho' this distance can be compar'd with no other ; but this Ratio is given in respect of the Earth and Moon, as well as in respect of the other Bodies ; wherefore also the Ratio of the Density of the Earth to the Denfities of the other Bodies is exactly discover'd.

C H A P. XV.

The physical Explanation of the whole planetary System.

IN the first Part of this Book we have shewn what are the Motions of the Bodies in the planetary System ; now we must explain how these Motions follow from the Laws of Nature * ; that is, how these Bodies, being once put in motion, persevere in those Motions which we observe.

Let us conceive the *Sun* and *Mercury* to be left to themselves, and they will come together * ; but if they be projected, they may revolve about a common

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common Center of Gravity in equal Times, and
 1248 describe immoveable elliptic Lines, and perse-
 1298 vere in that Motion; for it is plain, by mathe-
 241 matical Demonstration in that Case, that the Bo-
 1291 dies will describe Ellipses about the common Cen-
 1291 ter of Gravity, similar to that which the one of
 them could describe with the same Forces about
 the other, if it was at rest; this Center, on ac-
 * 235 count of the Sun's Magnitude*, is very little di-
 1260 stant from the Center of the Sun itself.

Let us conceive besides, *Venus* to be projected
 at a greater distance from the Sun, it will a little
 disturb the Motion of *Mercury*, which also, by its
 Action upon *Venus*, will turn it a little out of the
 way, and both will draw the Sun, sometimes the
 same way, and sometimes different ways; but
 we find all these Irregularities are insensible, if we
 consider the Magnitude of the Sun, and therefore
 that these three Bodies tend towards a Point that
 is between them near the Sun, which therefore is
 very little distant from the common Center of
 Gravity of them all.

If the Earth, *Mars*, and the other Planets be
 successively projected at different Distances from
 1264 the Sun, the same Reasoning will hold good;
 whence it follows, that *all the Planets are revolved*
about the common Center of Gravity of all the Bodies
which compose the System, which is but little distant
 from the Sun, and that the Planets do not sensi-
 1265 bly disturb one another in their Motions; where-
 fore *they all describe the same Line singly, which they*
 1291 *would describe about the Sun, if every one of them was*
 1291 *alone with the Sun in the planetary System*, that is,
 1291 immoveable Ellipses; for it is plain that these
 1208 will be describ'd by the Force of Gravity; and
 241 it is prov'd by mathematical Demonstration, that
 1291 no other immoveable eccentric Lines can be de-
 1291 scrib'd by a central Force acting equally at equal
 Distances.

It

It will also more plainly appear, that all the Planets tend to a Point near the Sun, if we consider that the Quantity of Matter in the Sun is 1000 times and more greater than the Quantity of Matter in *Jupiter*, which is far the greatest of all the Planets*.

*1255

When all the Planets move, tho' they move the Sun but little, yet they do move it, and draw it differently, according to their different Situation in respect of one another; whence there arises a small Motion in the Sun, which always depends upon the Motion already acquired, and the Change which happens in it from the Action above-mention'd, which varies every Moment.

1266

It is owing to this Agitation of the Sun, that the Planets disturb one another less in their elliptic Motions round the Sun, than if the Sun was at rest in the middle of the System. If *Jupiter*, ex gr. was equally distant from *Mercury* and the Sun, it would attract both those Bodies to itself with an equal Celerity*; whence the Situation, in respect of the Sun, is less chang'd than if the Sun was not agitated by this Motion, and *Mercury* only was attracted by *Jupiter*. According to the various Distances of *Mercury* and the Sun from *Jupiter*, the one or the other is more attracted, and there is always a less Change in their respective Situation when both are carried the same way, than if (the Sun being at rest) *Mercury* only should be carried towards *Jupiter*.

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*1235

This Reasoning may be applied to all the Actions of the Planets that are more distant from the Sun upon those that are less distant. As to what relates to the Action of those that are nearer upon those that are farther from the Sun, according to the different Situation they draw a Planet to the Sun, or drive it from the Sun, and in considering one whole respective Revolution, that is,

the

the Motion from one Conjunction to another, the Disturbance is less than if the Sun was immovable.

1268 The Magnitude of the Sun, compar'd with the rest of the Bodies of our System, is the reason (as appears by what has been already demonstrated) that the Planets disturb one another but little; but since the Magnitude is not infinite, these mutual Actions must not be wholly overlook'd.

We have said, that it appears by astronomical Observations, that *Jupiter* alters the Way of *Saturn* when it is nearest to it*; why this Disturbance is more sensible than the rest, is deduc'd from the Law of Gravity.

1269 The Actions of *Jupiter* upon *Saturn* when it is nearest to it, and of the Sun upon the same Planet, by which it is kept in its Orbit, are to one another directly as the Quantities of Matter in

*1207 *Jupiter* and the Sun* (*viz.*) as 9,248. to 10000*,

*1255 and inversly as the Squares of the Distances of *Jupiter* and the Sun from *Saturn*, that is, directly

as 81 to 16; for the Distances of *Saturn* and *Jupiter* from the Sun are almost as 9 to 5; wherefore when *Jupiter* is nearest to *Saturn*, the Distances of *Saturn* from *Jupiter* and the Sun are as 4 to 9. The Ratio compounded of the two aforesaid Ratio's is as 749 to 160000, or as 1 to 214. This

1270 Action of *Jupiter* conspires with the Gravity of *Saturn* towards the Sun, and therefore increases $\frac{1}{214}$ Part; whence it is no Wonder that the Disturbance is sensible.

We don't here consider the Force by which *Jupiter* attracts the Sun, for the Orbit of *Saturn* is not chang'd by it; and what we had to explain was, why Astronomers observe *Saturn* to be turned out of the way; yet by the Action of *Jupiter* upon the Sun, the Sun is brought nigher to *Saturn*,

turn, and the respective Situation of these Bodies is more disturb'd than is discover'd by astronomical Observations; the Force with which *Jupiter* in the aforesaid Position attracts the Sun, and with which therefore the Sun is attracted towards *Saturn*, is to the Force with which *Jupiter* attracts *Saturn* as 16 to 25^{*}; that is, as 479 to 749; ^{*1208} which Number expresses the Force with which *Saturn* tends towards *Jupiter*, when the Gravity of *Saturn* towards the Sun is express'd by 160000. If we collect into one Sum the Forces of *Jupiter*, by which it attracts *Saturn* and the Sun, the Force by which, from the Interposition of *Jupiter*, these Bodies tend towards each other, will be to the Gravity of *Saturn* towards the Sun as 1228 to 160000; but this Gravity is to the Gravity of the Sun towards *Saturn* as 160000 to 67,5^{*}; ^{*1207} wherefore *the mutual Access or Approach of the Sun* ¹²⁵⁵ *and Saturn, is to the Increase of this Approach by the* ¹²⁷¹ *Action of Jupiter interposed, as 160067 to 1228, or as 130 to 1.*

This Disturbance is remarkable, and far the greatest of any that happens in the Motion of any of the primary Planets; this also obtains only in this one Case of the Conjunction; for when *Jupiter* recedes from *Saturn*, the Disturbance of the Motion of *Saturn* in a short time becomes insensible.

In the same Position of *Jupiter*, when it is nearest to *Saturn*, the Force of *Saturn*, altho' it be the greatest of all in this Case, does not so sensibly alter the Way of *Jupiter* about the Sun; the Action of *Saturn* attracting *Jupiter*, is to its Action by which it attracts the Sun, as 81 to 16^{*}; there- ^{*1208} fore it attracts *Jupiter* with greater Celerity; and since they are both attracted the same way, the difference of these Forces is the Force with which
(from

(from the Action of *Saturn*) *Jupiter* and the Sun are separated from each other*, which is therefore to the Gravity of the Sun towards *Saturn* as 65 to 16; but this Gravity of the Sun towards *Saturn* is to the Gravity of *Jupiter* towards the Sun as 4,223. to 10000*, and as 25 to 81*; that is, as 106 to 810000, or as 16 to 122756; therefore the disturbing Force of *Saturn* is to the Gravity of *Jupiter* towards the Sun as 65 to 122756, or as 1 to 1888; therefore by the greatest Action of *Saturn*, the Gravity of *Jupiter* towards the Sun is diminish'd only by $\frac{1}{1888}$ Part, which Disturbance is insensible.

The other mutual Disturbances of the Planets are much less, as will appear by determining that, which is the greatest of them all (*viz.*) that of *Mars* by *Jupiter*, which is discover'd by the same sort of Computation as the foregoing.

The distance of *Jupiter* from *Mars* and the Sun, when *Mars* is between the Sun and *Jupiter* in the same Line, are as 7 to 10*; wherefore the Forces with which *Jupiter* attracts these Bodies, are as 100 to 49*; the difference of which Forces is to the Gravity of the Sun towards *Jupiter* as 51 to 49. This Gravity of the Sun towards *Jupiter*, is to the Gravity of *Mars* towards the Sun, as 9,248 to 1000*, and as 9 to 100*; that is, as 83 to 1000000, or as 49 to 590443; and the disturbing Force of *Jupiter*, is to the Gravity of *Mars* towards the Sun, as 51 to 590443, or as 1 to 11577; wherefore the Gravity of *Mars* towards the Sun is diminish'd only $\frac{1}{11577}$ Part by the Action of *Jupiter* when nearest to it.

Altho'

Altho' these Disturbances, arising from the Actions of the Planets upon each other, be very small, and altho' those which happen in a different Position of the Planets, do in some measure compensate each other, yet the Proportion in which the Force which keeps the Planets in their Orbits decreases, is a little chang'd by these Actions; so that it does not decrease exactly in an inverse Ratio of the Square of the Distance; therefore altho' the Orbits are at rest as to Sense, after a great many Revolutions, a small Change is observ'd in their Situation*.

From all this it follows, that if we suppose the Planets at first once projected at the Distances from the Sun, at which they are mov'd, they will, by the Laws already explain'd, persevere in those Motions; and the Eccentricity of the Orbits depends upon the Celerity and Direction of the first Projection; but these Motions may be preserv'd very long by reason of the small Resistance of the celestial Matter.

It is also plain, why, by Lines drawn to the Center of the Sun, they describe Area's proportionable to the Times; namely, because all other Gravities in the System are very small in respect to the Gravity towards the Sun*; therefore by this Gravity alone it is that they are retain'd in their Orbits; whence follows this Proportion of the Area's*. And also the Motion in elliptic Lines, which are carried on very slowly, follows from the Law of Gravity; and these Lines would also be immoveable, if the Planets gravitated only towards the Sun*; but this slow Motion of the Orbits is deduc'd from the Action of the Planets upon one another*. Now in respect to the Proportion which is observ'd between the Cubes of the Distances and the Squares of the periodical Times, it is also deduc'd from the

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• 239 the Law of Gravity * ; so that if we add to these
 1208 what we have said of the Deflection of *Saturn**,
 • 1217 nothing will remain to be explain'd in respect to
 1270 the primary Planets.

1276 That the Motion of Comets depends upon the Law of Gravity, is also deduc'd from Observations; and in respect of them, as has been said concerning the Planets, the Sun's Gravity prevails, and by that Gravity they deflect from a rectilinear

• 980 Course * ; but that the Curvature of their Way
 226 depends upon the same Gravity, follows from this; that a Body, by that Gravity, will describe an Ellipse, or a Parabola, or an Hyperbola * ; which Lines it appears that those Comets
 • 241 have described, whose Trajectories have been determined.
 1208

1277 The Satellites of Jupiter and Saturn are moved
 • 944 by the same Laws about their Primaries, as the Pri-
 966 maries are moved about the Sun * ; wherefore the
 974 Explication of those Motions * may be also re-
 • 1275 ferr'd to them; for in these three Cases, smaller Bodies are revolving at different Distances about a much greater Body; namely, Satellites about Jupiter and Saturn, and primary Planets about the Sun.

1278 Whilst secondary Planets are moved about a primary one, it is evident that they may all be moved with one common Motion, whereby the respective Motions with which they are mov'd in respect of each other, will not be disturb'd, because a Body may at the same time be mov'd by different Impressions * ; the Motion that a primary Planet has in common with its Satellites, is the Motion of a primary Planet about the Sun.

1279 Yet the Motion of the secondary Planets is disturb'd by the Action of the Sun, towards which they are carried, sometimes faster, sometimes slower, according to the different Position of the Primary; and they

they also often concur towards the Sun's Center in different Directions. These Irregularities, which are very small, cannot be observ'd in the Satellites of *Jupiter* and *Saturn*, tho' they be really like those which are observ'd in the Motion of the Moon; the least Deviation of this last is very sensible to us. But that the Irregularities of the Moon exactly follow from the Theory of Gravity, will appear in the next Chapter.

C H A P. XVI.

The physical Explication of the Moon's Motion.

IT is certain that the Moon and Earth having 1280
 once a projectile Motion given them, they
 can perceive in their Motion about their common
 Center of Gravity*, if they be carried any way *1263
 by a common Impression directed in parallel
 Lines, as was said of the Satellites of *Jupiter* and
*Saturn**, this Motion will not disturb the Motion *1278
 about the common Center of Gravity, which will
 follow that Direction only, because in respect of
 the two Bodies it is at rest; but the Bodies are
 carried by a Motion compounded of that Impres-
 sion, and of the Motion about the common Cen-
 ter of Gravity*, that is, they are whirl'd about *190
 that Center as it is carried along, as before its
 Motion when it was at rest. If every Moment
 new Impressions common to both the Bodies act
 upon them, the Way of the Center of Gravity
 may be chang'd every Moment; which Change
 will be like that which the Bodies themselves
 would undergo if they had no respective Motion.

Hence we deduce, that if whilst the Moon and Earth are whirl'd round their common Center of Gravity they be both projected, the Way of the Center of Gravity, by the Action of the Sun acting upon both Bodies, is the same as a Body projected in the same manner would describe about the Sun.

1281 Whence it follows, *that the Moon disturbs the Motion of the Earth, and that the common Center of Gravity of those Bodies describes that Orbit about the Sun, which we have hitherto said that the Earth describ'd*; because we look'd on the Action of the

1282 Moon; but *the Earth describes an irregular Curve.*

1283 *Fig. 4. Plate XXIV.]* Let the Sun be at S, and the common Center of Gravity of the Moon Q, and the Earth at M, at the time of the Full-moon, be at F; after one whole Lunation, that is, the next Full-moon, let that Center be at A; and let F D A be the Orbit which we call that of the Earth, but in which it is the Center of Gravity above-mention'd that does really move.

If this Lunation be divided into four equal Parts, after the first, the Center of Gravity will be at E, the Moon at P, and the Earth at L; after the second, Part of the Time at New-moon the Center of Gravity will be at D, the Moon at R, and the Earth at I; in the following Quadrature the Center of Gravity will be at B, the Moon at O, and the Earth at H. Lastly, at Full-moon the Center of Gravity being at A, the Moon will be at N, the Earth at G; all which follows from the Revolution of the Earth and Moon about their common Center of Gravity, whilst it is carried in the Orbit about the Sun.

Therefore we see the Earth moves in the Curve M L I H G, which is twice inflected in each Lunation; which Curve also does not return into it-

self,

self, because the Inflections in the several Revolutions about the Sun do not coincide; for 12 Lunations, and a third Part of another, are perform'd every Year.

This Irregularity of the Motion of the Earth, which is deduc'd from the Laws of Nature, is too small to become sensible in astronomical Observations; wherefore we may without any Error say, that the Center of the Earth itself describes the Orbit FDA; for MF, or DI, the greatest distance of the Earth from that Orbit, is about the 40th Part of the distance IR, which distance itself is not the 300th Part of the distance FS.

In explaining what relates to the Moon, we also neglect the Consideration of the Motion of the Earth about the common Center of Gravity above-mention'd, but we suppose it to revolve at the distance of 60 Semidiameters from the Center of the Earth; because, as we have before demonstrated, such is the distance at which in its periodical Time it could revolve about the Earth at rest, or be carried along in an Orbit in which it should not be disturb'd by the Moon's Action; by this Method the Moon's Irregularities will be much more easily discover'd, for they are the same, as is evident, whether the Moon moves about the common Center of Gravity of the Moon and Earth, or about the Center of the Earth itself.*

Plate XXV. Fig. 1.] Let S be the Sun, T the Earth, and the Orbit of the Moon ALB; and last of all, let the Moon be at A in the Quadrature; it tends towards the Sun in the Direction AS, in the same manner and with the same force that the Earth is carried towards S along TS, because the Distances AS and TS are equal; this Force may be represented by TS or AS, where- by the Moon endeavours to descend along AS,

and is resolv'd into two Forces, by drawing the Parallelogram $A DST$; so that the Moon will endeavour to move in the Directions AD and
 • 192 AT by Forces represented by those Lines*.

By the Force which acts along AD , the Moon is carried with the same Celerity and the same Way of the Earth, by reason of the equal and parallel Lines TS and AD ; wherefore by this Motion the relation between the Moon and Earth is not chang'd, but the Force along AT conspires with the Gravity of the Moon towards the Earth; and this Gravity is increased by the Action of the Sun when the Moon is in the Quadratures; and the Augmentation or Addition is to the Gravity of the Earth towards the Sun, as AT , the Moon's distance from the Earth, is to TS , the Earth's distance from the Sun.

1287 TS , the Earth's distance from the Sun, remaining the same, the above-mention'd Addition of Gravity increases, and diminishes in the Ratio of the Line AT , the distance of the Moon from the Earth.

But this distance of the Moon from the Earth AT , if it remains the same, and TS be increased, then AT will be less in respect of AS ; therefore tho' there should be no Change in the Force, whereby the Earth and Moon fall towards the Sun, the Addition will be less, and so much less as TS is greater, that is, it will be inverfly as TS ; but the Force of Gravity does not remain the same when TS is increas'd, but is diminish'd; wherefore also in that respect the above-mention'd Addition is diminish'd, and in the same Ratio with that Force of Gravity, therefore it is the inverse Ratio of the Square of the distance TS *; if this Diminution be added to that above-mention'd, we see that the Addition of which we speak,

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speaks, follows the inverse Ratio of the Cube of the distance of the Earth from the Sun. 1289

The distance of the Earth from the Sun remaining the same, the Gravity of the Moon towards the Earth decreases more slowly in the Quadratures, than according to the inverse Ratio of the Square of the distance from the Center of the Earth; for if the Addition in that Case should follow the inverse Ratio of the Square of the distance, as the Gravity from the Action of the Earth does *, this Ratio would not be disturb'd; but the Addition increases when the Gravity itself is diminish'd; wherefore the Addition, when the distance is increas'd, is always greater than is required, and consequently the Diminution the less. 1290

This Addition is determin'd in the mean Distances of the Moon from the Earth, and of the Earth from the Sun. Let A T and T S be these mean Distances, the Addition required is to the Gravity of the Earth towards the Sun as A T to T S; the Gravity of the Earth towards the Sun, is to the Gravity of the Moon towards the Earth (because these Bodies are retain'd by these Gravities in their Orbits) directly as T S to T A, and inversely as the Squares of the periodical Times of the Earth about the Sun, and of the Moon about the Earth; therefore the Addition required is to the Gravity of the Moon towards the Earth, in a Ratio compounded of these Ratio's; that is, the above-mention'd inverse Ratio of the periodical Times of the Earth and Moon, the other Ratio's destroying one another; these Times are given, and their Squares are inversely as 1 to 178,773. 1291

Now if the Moon be at L [Plate XXV. Fig. 1.] in which Situation the Sun attracts the Moon and Earth in the same Line, but not equally, the Moon it draws with a greater force, because it is less distant from it; the difference of those Forces 1292

is the Force by which the Moon is drawn back from the Earth, and by which the Gravity of the Moon towards the Earth is diminish'd.

The Forces whereby the Moon at L , and the Earth at T , tend towards the Sun, are to one another as the Squares of the Lines ST and SL^* ; and the difference of the Forces, that is, the disturbing Force, is to the Force by which the Earth descends toward the Sun, as the difference of those Squares to the Square of the Line LS , that is, nearly as double LT to LS or TS , for these Lines do but very little differ from one another; and the difference of the Squares, whose Roots differ but little, is keeping the Proportion double that which is between the Roots.

If therefore TS , as before, represents the Force whereby the Earth descends towards the Sun, Ll will represent the disturbing Force and diminishing Gravity, when AT represents the disturbing Force in the Quadratures*. Let the Moon be at l , it is again (together with the Earth) attracted by the Sun in the same Line; but the Earth, because less distant, moves more swiftly towards the Sun*; so that there is a Force which separates the Earth from the Moon, namely, the difference of the Forces which attract the Moon and the Earth; which Force always acts contrary to the Gravity of the Moon towards the Earth, and diminishes it, in the same manner as has been demonstrated from the greater Gravity of the Moon towards the Sun, supposing it at L . At l also the separating Force scarce differs from the separating Force at L ; for this Force, as we have shewn, is proportional to the difference of the Squares of the Lines TS and LS ; and that (as appears by such another Demonstration) proportional to the difference of the Squares of the Lines LS and TS ; which difference, by reason that Ll

is very small in respect of T S, scarce differ among themselves; so that the Force which diminishes the Moon's Gravity at *l*, is also represented by *L l*.

Yet the disturbing Force is something greater at the 1295
Conjunction *L* than at the Opposition *l*; for supposing the Differences between the Roots to be equal, the Squares, keeping the Proportion, will differ so much the more the less they are; and so keeping the Proportion, the Forces differ more at *L* and *T* than at *T* and *l*, which also are less*. *1208

From this we conclude, that the Force which 1296
diminishes the Gravity of the Moon in the Syzygies, is double that which increases it in the Quadratures; namely, as *L l* to *A T*; wherefore in the Syzygies the Gravity of the Moon from the Action of the Sun is diminish'd by a Part, which is to the whole Gravity as 1 to 89,36; for in the Quadratures the Addition of Gravity is to the Gravity as 1 to 178,73*. *1291

In the Syzygies the disturbing Force follows the 1297
same Proportion with half added to it, that is, with the disturbing Force in the Quadratures*; *1296
it is therefore directly as the distance of the Moon from the Earth*, and inversely as the Cube of the distance of the Earth from the Sun*. *1288
*1289

At the Syzygies the Gravity of the Moon towards 1298
the Earth receding from its Center, is more diminished than according to the inverse Ratio of the Square of the distance from that Center; for it would be diminish'd in that Ratio, if the Force to be taken away follow'd the same Ratio; but on the contrary, as it increases when the distance becomes greater*, the Diminution is always greater than *1297
in that Ratio.

1299 *Plate XXV. Fig. 1.]* Lastly, let the Moon be at F, in any intermediate Place between the Syzygy and Quadrature, it is drawn towards the Sun along FS, by which, since it is less distant than the Earth T, it is attracted with more Force than the Earth; let the Force with which the Moon tends to the Sun be to the Force with which the Earth is carried towards it as FM to TS, which also before has been made use of to express the same Gravity of the Earth; draw the Parallelogram FHMI, whose Diagonal is FM, and whose Side FH is parallel and equal to the Line TS; the Motion of the Moon towards the Sun is resolv'd into two Motions, one along FH, the other along FI, and these Lines denote the Forces whereby the Moon endeavours to move
 • 192 along them*; the Motion along FH is common to the Moon and the Earth, which with an equal Force, and in a Line parallel to it, does also tend to the Sun; so that by this Motion of the Moon, the Situation of it, in respect of the Earth, will not be chang'd, and the disturbing Force will be only the Motion along FI.

By reason of the immense distance of the Sun; the Part MS of the Line MF is small in respect of the whole, and the Angle FST, where it is the greatest, as AST, is hardly more than the sixth Part of a Degree; whence it follows, that the Lines MI and SN are very near one another, and that the Points I and N are scarce sensibly distant, and may, without any sensible Error, be confounded together; which Error, notwithstanding how little soever it need be regarded in consideration of one whole Revolution, is compensated by a contrary Error when the Moon is at E; therefore the disturbing Force is express'd by FN.

It

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B E E B

Fig. 1.

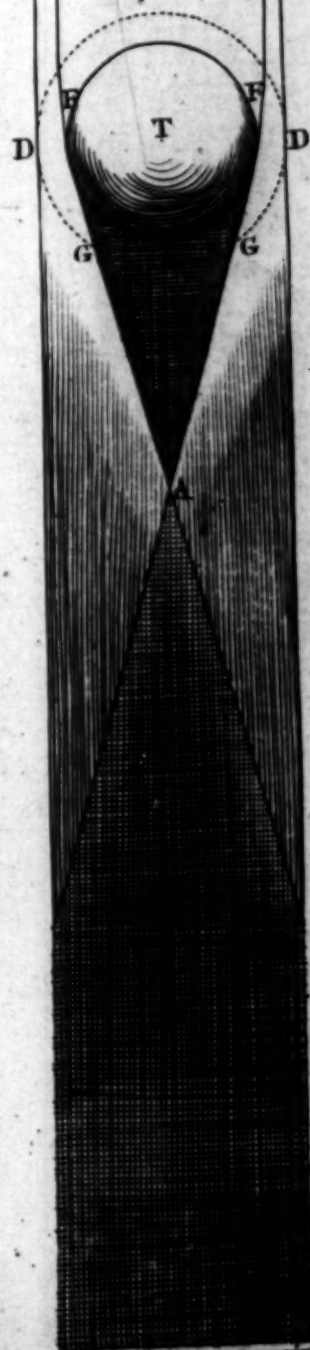


Fig. 2.

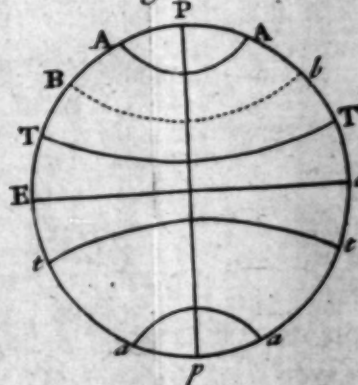


Fig. 3.

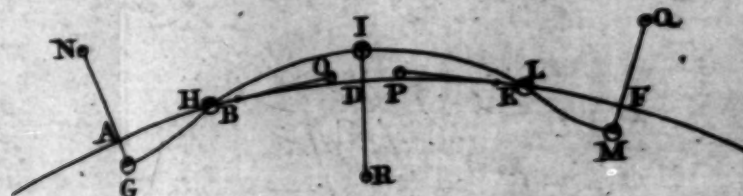
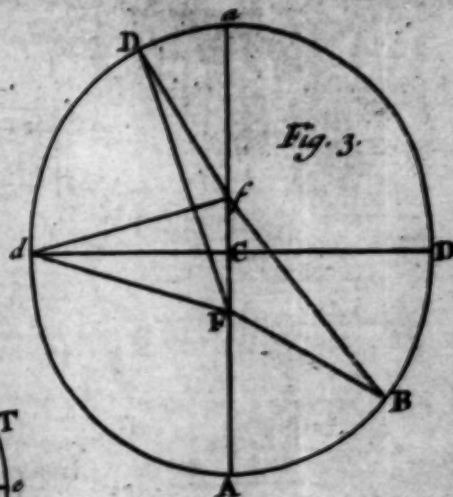
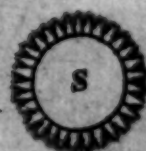


Fig. 4.





It is to be observ'd, when only the Part E F of the Line E S is consider'd, that it is to be look'd upon as parallel to the Line L l, because of the small Angle which these Lines make; from the Point N draw N Q perpendicular to the Line F T, continu'd if need be, in which the Moon gravitates towards the Earth, and let the rectangular Parallelogram E P N Q be drawn; let us conceive the Force along F N resolv'd into two others, acting in the Directions F Q and F P, and represented by these Lines*; by the Force along F Q the Force of Gravity is diminish'd in the Case represented by this Figure, but it is increased when the Point Q falls between F and T; but by the Force along F P the Moon in its Orbit is drawn towards the next Syzygy L, and the Motion of the Moon is accelerated or retarded, according as this Force conspires with, or acts contrary to the Moon's Motion.

Near a Syzygy the Gravity of the Moon is diminish'd, and the Line F Q, which follows the Proportion of this Diminution, grows less, receding from the Syzygy, till it vanishes at the distance of about 54 Deg. 44 Min. from it; at a greater distance of the Moon from the Syzygy Q falls in between F and T, and the Gravity of the Moon towards the Earth is increas'd by the Sun's Action; the Force along F P vanishes in the Syzygy L; receding from it, it increases quite to the Octant, which is the middle Point between the Syzygy and the Quadrature; and then it diminishes again, till it vanishes quite at B.

Between B and l, or l and A, the disturbing Motions are determin'd in the same manner as in A L B, the opposite inferior Part of the Orbit; at E and F the Diminution of Gravity is equal, and in that Position it is drawn in the Orbit with

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an equal Force towards the Syzygy l , with that with which at F it is impell'd towards the Syzygy L .

1303 Hence it follows, *that in the Motion of the Moon from the Syzygy to the Quadrature, between L and B , as also between l and A , the Gravity of the Moon towards the Earth is continually increas'd, and the*

1304 *Moon is continually retarded in its Motion; but in the Motion from the Quadrature to the Syzygy, between B and l , as also between A and L , every Moment the Moon's Gravity is diminish'd, and its Motion in its Orbit is accelerated.*

You may determine the Forces upon which these Effects depend, by comparing them with the known Force whereby Gravity is increas'd in the

*1291 *Quadratures**, and which is represented by the Moon's distance from the Center of the Earth.

1305 The Lines MI , HF , ST , are equal by Construction; therefore when the Points I and N are confounded, MN is equal to ST , and MS is equal to NT ; the Lines MF and ST represent the Forces whereby the Moon at F and the Earth at T are carried towards the Sun S , therefore they are as the Square of the Line TS to the

*1208 *Square of the Line FS **; wherefore as FG is the difference of those Lines, FM and TS differ

*1293 *from one another double the Line GF **, and adding GF to the Line FM , the difference between GM and TS , that is, MS will be triple the Line FG ; and therefore this is also the Quantity of the Line NT ; now as FE is double

*1300 *FG **, therefore NT will be to FE as Three to Two.

Let FT be continu'd, if need be, and from E draw EV perpendicular to it, the Triangles EVF and NQT , which are rectangular, will be similar, by reason of the alternate Angles

*1300 *VFE and QTN **; therefore NT is to FE ,
that

that is, 3 is to 2 as NQ , equal to FP , is to EV , which therefore is proportional to 2 third Parts of the Force which is express'd by FP ; but EV is the Sine of the Angle ETV at the Center, which is double the Angle EFV at the Circumference, equal to the Angle FTL , which is the distance of the Moon from the Syzygy; therefore *as the Radius TA or TE is to a Sine* 1306 *and an half of double the distance of the Moon from the Syzygy, namely, FP , so the Addition of Gravity in the Quadratures (which is express'd by the Radius TA) is to the Force which accelerates or retards the Moon in its Orbit.*

The Computation of this Diminution of Gravity, and of its Increase at a less distance from the Quadratures, is deduc'd from the same Principles.

This Diminution is represented by the Line FQ , which is equal to QT , minus the Radius; but from the Consideration of the Triangles above-mention'd, VF taken once and an half is equal to QT ; therefore VT and an half, with half the Radius added to it, expresses the requir'd Diminution of Gravity; and *the Radius is to the* 1307 *Sum or Difference of once and a half the Co-sine of double the distance of the Moon from the Syzygy and half the Radius; as the Addition of Gravity in the Quadratures, to the Diminution or Increase of Gravity in that Situation of the Moon, concerning which the Computation is made.*

We make use of the difference of the Co-sine from half the Radius, when the Angle, whose the Co-sine is, is greater than a right Angle, because in that Case we make use of the Co-sine of the Complement of the Angle to two right Angles; when in this same Case the Co-sine and a half, which we make use of, is greater than half the Radius,

Radius, the Quantity found is to be added ; that is, increases the Gravity, which obtains every where between the Quadrature, and 35 Deg. 16 Min. from it.

1308 *These Forces, whatever is the Figure of the Moon's Orbit, are exactly determin'd ; for they are compared with the Addition of Gravity in the Quadratures, supposing the Moon in the Quadrature to be at the same distance from the Earth, at which it really is in the Place which is consider'd ;*
 1291 but this Addition is discover'd in every Case.

1288 Tho' it be foreign to the Purpose of this Work
 1289 to give a Computation of the Moon's Motion, I thought it necessary to explain in a few Words what is the Method whereby to discover the Forces that govern the Moon ; because the more exactly we know the Forces, the more easily we shall conceive their general Effect.

Now to examine the Moon's Motion, we must singly consider its several Irregularities ; which to do without Confusion, we must remove several Irregularities, and conceive the Moon as moving in a Circle about the Earth, in which Curve it is plain that it can be retain'd by Gravity* ; this
 • 241 Motion is disturb'd by the Action of the Sun,
 1308 *and the Orbit is more convex in the Quadratures than in the Syzygies.* The Convexity of a Curve, which a Body describes by a central Force, is so much the greater, as the central Force does more strongly every Moment turn the Body out of the way ; it is also the greater the more slowly the Body moves, because the central Force acting the longer, has a greater Effect in inflecting the way of the Body. From contrary Causes the Convexity of the Curve is diminish'd ; both concur in increasing the Convexity of the Orbit in the Quadratures*, and diminishing it in the Syzygies*.
 •1303
 •1304

From

From this it follows, that the circular Figure 1310 of the Moon's Orbit is chang'd into an Oval, whose greater Axis goes thro' the Quadratures, so that the more convex Parts are in the Quadratures; wherefore *the Moon is less distant from the Earth at the Syzygies, and more at the Quadratures*; and it is no Wonder that the Moon comes towards the Earth when its Gravity is diminish'd, because the Access is not the immediate Effect of this Diminution, but of the Inflexion of the Orbit towards the Quadratures.

The Motion of the Moon, taking away the Action of the Sun, is not in a Circle, but in an Ellipse, one of whose Foci coincides with the Center of the Earth*; for the Orbit of the Moon * 967 is eccentric, and it is retain'd in it by the Force 241 1280 of Gravity.

Therefore what has been demonstrated cannot be exactly applied to the Moon's Motion; for as the Forces, which generate the Deviations explain'd, do really act upon the Moon, the Ellipse, which the Moon would describe if the Sun was taken away, is chang'd, and cæteris paribus, 1311 *the Propositions of N^o 1309, 1310, may be applied to the Moon's Motion.*

In the Quadratures and Syzygies the disturbing Force acts in the same Line as the Force of Gravity towards the Earth*; therefore the Force * 1286 which continually acts upon the Moon, and retains it in its Orbit, is directed towards the Center of the Earth; and *the Moon describes Area's, by Lines drawn to the Center of the Earth, proportional to the Times* *. 1292 1294 * 215

Plate XXV. Fig. 1.] In other Points of the Orbit, 1313 as F, besides the Force which acts in the Line F T, there is also another, whose Direction is perpen-

- *1301 perpendicular to FT^* , which is here represented by FP ; the Direction of the Force compounded of both, is directed sometimes sidewise to the Line FT , and does not tend to the Center of the Earth^{*}; wherefore *the Area's, by Lines drawn to the Center of the Earth, are not exactly proportional to the Times**. In the Octants FP is the greatest of all; and the Force, which is represented by that Line, is to the Gravity of the Moon towards the Earth, in that Point, in the mean Distances of the Sun and Moon, as 1 to 119,49^{*}; wherefore the Direction of the Force, compounded of the Actions of the Sun and Earth upon the Moon, makes an Angle of above half a Degree with the Line FT .

The Motion of the Moon is subject to several other Irregularities, so that it describes a Curve wholly irregular; which Astronomers, in order to subject it to the most exact Computations that can be made, do *reduce to an Ellipse, which they conceive to be agitated by various Motions, and also to be changeable, lest the Moon should go out of it.*

- In respect to central Forces, we have observ'd, that a Body does not describe an Ellipse, if the central Force, by which it is retain'd in its Orbit, decreases in any other Ratio than the inverse Ratio of the Square of the distance, but that the Curve may be often reduc'd to a moveable Ellipse^{*}; concerning which it is to be observ'd, that *the Ellipse in that Motion turns about one of its Foci, and the Motion of the Ellipse is directed the same way as the Motion of the Body in it, when the central Force decreases faster than in the inverse Ratio of the Square of the distance; but if the central Force decreases slower as you recede from the Center, the Ellipse is carried the contrary way; as these things may be demonstrated mathematically.*

Hence

Hence it follows, the Orbit of the Moon cannot be referr'd to an elliptic Orbit, unless you suppose it agitated by four Motions every Revolution; that is, unless the Line of the *Apsides*, which goes thro' the Center of the Earth, goes forwards twice, and backwards twice.

*The Apsides of the Moon go forward when the Moon is in the Syzygies**, or rather whilst the Moon moves between the Points, which are 54 Deg. 44 Min. distant from them*. *In the Quadratures*, and between the Points, distant from them 35 Deg. 16 Min. *the Apsides go backwards*, that is, move in *antecedentia**.

The Forces, upon which *the Progress* and *Regress* of the *Apsides* depend, are the Forces which disturb the Motion of the Moon, which have been before explain'd; therefore since the disturbing Force in the Syzygies is double the disturbing Force in the Quadratures*, the Progress, considering one entire Revolution of the Moon, exceeds the Regress, cæteris paribus.

In a Circle whose Center is in the Center of the Forces, the Diminution of the Force in receding from the Center produces no Effect, because in such a Line the Body does not recede from the Center; therefore the Effect of this Diminution is so much the greater, as the Curve describ'd by the Body differs more from such a Circle.

In an elliptic Orbit, one of whose Foci coincides with the Center of the Forces, the Curvature in the *Apsides* differs most of all from such a Circle, and the Effect of the Diminution of the Force, in receding from the Center of the Forces, is the greatest of all; if this Orbit is but a little eccentric at the Ends of the lesser Axis, the Ellipse differs very little from the Circle above-mention'd, and the Effect of the Diminution is the least of all.

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1322 The *Progress* and *Regress* of the *Apsides* depend upon the Proportion according to which the Force of Gravity decreases receding from the
 1315 Center of the Earth, it is therefore the Effect of
 1316 the Diminution of the central Force.

1323 Plate XXV. Fig. 2.] This Motion of the *Apsides*, which we have explain'd, undergoes several Changes; the *Apsides* go forward fastest of all in a Revolution of the Moon, supposing the Line of the *Apsides* in the *Quadratures**; and in that very Case
 1317 they go back the slowest of all in the same Revolution;
 1322 because, by reason of the same Eccentricity of the
 *1318 Moon, the *Quadratures* are but very little di-
 1321 stant from the Ends of the lesser Axis of the Orbit.
 1322

1324 Supposing the Line of the *Apsides* to be in the *Quadratures*, the *Apsides* are carried in consequentia the
 *1317 least of all in the *Syzygies**, but they return the swift-
 1322 est in the *Quadratures**; and in this Case, in one
 *1318 entire Revolution of the Moon, the *Regress* exceeds the
 *1322
 1320 *Progress*.

1325 Whilst the Earth is carried along in its Orbit, the Line of the *Apsides* does successively go thro' all Situations in respect of the Sun; wherefore, considering a great many Revolutions of the Moon
 *1319 taken together, the *Apsides* go forwards*; and it is plain from Observations, that in the Space of about 8 Years the Line of the *Apsides* performs one entire Revolution.

We have also said that the Eccentricity of the Orbit is not constant.

1326 The Eccentricity of a Body is increased, if the central Force, the Diminution being continu'd, decreases faster than before, whilst the Body is carried from the lower to the upper Apis; for then it is every Moment less attracted, than if the Force did not decrease; and therefore it recedes the more. The Eccentricity of the Orbit is also thereby increas'd, in the same Case, in the

the Motion from the upper to the lower Apfis, because in this Case coming towards the Center, the Force increases so much the faster as the Body descends more towards the Center; so that in each Case the difference between the greatest and least distance from the Center of the Forces may become greater, and the Eccentricity be thereby increas'd. By the same Reasoning it appears that *the Eccentricity is diminish'd when the central Force is increas'd in the motion of the Body from the lower to the upper Apfis, and likewise when that Force is diminish'd in the motion from the upper Apfis to the lower; that is, when it decreases more slowly than before in the receding from the Center.*

Applying this to the Moon's motion, it appears, that *the Eccentricity of the Orbit, every Revolution, undergoes various Changes; that it is the greatest of all when the Line of the Apfides is in the Syzygies, because the Forces in the Apfides, being compared, do decrease faster than in an inverse Ratio of the Square of the distance*, whence this Addition follows*, which prevails in this Position*; but the Orbit is the least Eccentric of all when the Line of the Apfides is in the Quadratures, the Diminution of the Eccentricity prevailing*.*

We have said that the Moon moves in a Plane inclin'd to the Plane of the Ecliptic; that the Line of the Nodes is carried round in *antecedentia**; and that the Inclination of the Orbit is not constant*; these Effects are also deduc'd from the Action of the Sun upon the Moon.

By reason of the small Inclination of the Moon's Orbit, the Forces which we have hitherto considered acting in the Plane of the Ecliptic, not regarding the Inclination of the Orbit, may (without any sensible Error) be referr'd to the Plane of the Orbit, and the Moon in it is subject to the

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1330 Motions before explain'd ; but *there is a Force which removes the Moon from the Plane of the Or-*

*1314 *bit**, so that we must conceive that Plane to be agitated, otherwise the Moon would go out of the Orbit.

1331 *Plate XXV. Fig. 1.]* Let the Moon be at F ; if we attend to what has been said above concern-

1299 ing the Actions of the Sun, it is plain that the Plane of the Parallelogram F H M I goes thro' the Line T S, which joins the Centers of the Earth and Sun, and therefore it is in the Plane of the Ecliptic ; so that the Point N, to which is directed the Force F N, disturbing on account of the Action of the Sun, is in that Plane.

1332 *Plate XXV. Fig. 4.]* Let the same Force be represented by F I ; at F let F R be rais'd perpendicular to the Plane of the Orbit, and imagine the Parallelogram F R I i, whose Side F i is in the Plane of the Orbit, and whose Diagonal is F I, the disturbing Force along F I is resolv'd into two in the Directions F R and F i, which these
192 Lines represent, and of which this last acts in the Plane of the Orbit ; so that we must refer to this what relates to the disturbing Force, of which we have treated in N^o 1299 ; for the Lines F i and F I scarce differ, and the Plane of the Parallelogram F R I i is perpendicular to the Plane of the Moon's Orbit.

1333 The Line F R must be determin'd, which represents the force that acts perpendicular to the Plane of the Orbit, and removes the Moon from that Plane ; now the relation of the Line F R or I i to the Radius E T, is the Ratio of the disturbing force which is spoken of here, to the Increase
1286 of Gravity in the Quadratures.

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In the Case of this Figure, in which the Line of Nodes Nn is in the Quadratures, we find out FR ; because IT (which is NT of *Fig. 1.*) is given*, and because IT is to Ii or to FR as the Radius to the Sine of the Inclination of the Orbit. *1325

But in every Case the Force must be determin'd which drives the Moon out of the Plane; let us therefore suppose the Line of Nodes carried to the Situation Mm , whereby, every thing else remaining as before, Ii is chang'd; to Mm continu'd, if need be, let iX and IX be drawn perpendicular, which make an Angle equal to the Inclination of the Plane of the Orbit. 1335

The Ratio between ET and Ii , that is, the Ratio between the Addition of Gravity in the Quadratures, and the Force, which we seek, which removes the Moon out of its Orbit, is compounded of the Ratios of the Line ET to TI , of the Line TI to IX , and lastly, of the Line IX to Ii ; the first is the Ratio between the Radius and 3 times the Sine of the distance of the Moon from the Quadrature*; the second is the Ratio of the Radius to the Sine of the Angle ITX , that is, of the distance of the Node from the Syzygy; lastly, the third is the Ratio of the Radius to the Sine of the Inclination of the Orbit; and the Ratio compounded of these, is the Ratio of the Cube of the Radius to 3 times the Product of the Sines of the distances of the Moon from the Quadrature, and of the Node from the Syzygy, as also of the Inclination of the Plane. To this Force is also to be referr'd N° 1308. *1305

This Force vanishes in the Quadratures, because the Point I coincides with the Point T , which is the Center of the Earth, and the Line Ii vanishes; the Lines FI and Fi concurring in the Plane of the Orbit, which also follows from the Computation above-mention'd*; the Sine of the distance *1136

of the Moon from the Quadrature vanishing, and consequently the whole Product which is multiplied by that Sine.

1338 That same Product vanishes also, and with it *the Force* which it represents, when the Sine of the distance of the Node from the Syzygy *vanishes*, that is, *supposing the Line of the Nodes in the Syzygies*. It is also deduc'd from this, that the Line of Nodes Nn (*Plate XXV. Fig. 5.*) continu'd, goes thro' the Sun; wherefore the Sun is in the Plane of the Orbit itself, and therefore cannot draw the Moon but in that Plane.

1339 *The Force* also, which we examine, is increased as the Moon advances towards the Syzygy, and as the

1336 Node recedes from it.

1340 *Plate XXV. Fig. 6.*] Let Pp be the Plane of the Ecliptic, PA the Orbit of the Moon; when the Moon is come to A , that is, is receded a little from the Node, it is remov'd out of the Plane of the Orbit, and in the second Moment it is not carried along AB (the Continuation of the Orbit PA) but along Ab , because it comes towards the Plane of the Ecliptic along Bb , therefore it is moved as if it came from a more distant Node p ; whence it appears that *the Nodes go backward whilst the Moon moves in its Orbit*, as long as it recedes from the Node; the Nodes also go back whilst the Moon is going to the opposite Node, because as the Moon is continually driven out of its Orbit towards the Plane of the Ecliptic, it is continually directed to a Point less distant, and comes sooner to the Node, than if, not being agitated by such a Motion, it had continu'd in motion with the same Celerity.

1342 Considering one entire Revolution of the Moon, cæteris paribus, the Nodes move in antecedentia, swift-

1339 est of all when the Moon is in the Syzygies, then slower and slower, till they are at rest, when the Moon

1337 is in the Quadratures. Whilst

Whilst the Earth is carried round the Sun (even when we do not attend to the above-mention'd Motion of the Nodes) the Line of Nodes does successively acquire all possible Situations in respect of the Sun, and every Year goes twice thro' the Syzygies, and twice thro' the Quadratures. 1343

If now we consider several Revolutions of the Moon, the Nodes in one whole Revolution go back very fast, the Nodes being in the Quadratures*; then slower, till they come to rest, when the Line of Nodes is in the Syzygies*. 1344 1339 1338

By the same Force with which the Nodes are mov'd, the Inclination of the Orbit is also chang'd; it is increased as the Moon recedes from the Node, and diminish'd as it comes to the Node. 1345

Plate XXV. Fig. 6.] For the Angle $b p L$ is less than the Angle $A P L$, and for the same Cause it is continually diminish'd, and the Inclination becomes greater; but when the Moon is come to the greatest distance from the Plane of the Ecliptic, and is going towards the opposite Node, the Direction of the Moon is continually inflected towards the Plane of the Ecliptic, and less inclin'd to it than if it continu'd in motion in its Orbit; let $N p n$ be the Plane of the Ecliptic, the Curve $N n$ the Orbit of the Moon, by the force whereby the Moon is continually remov'd out of it, the Way of the Moon is chang'd, and it goes in the Curve $N p$, which is more inclin'd to $N p n$ at N than at p ; so that we must conceive the Inclination of the Plane of the Orbit to be twice changed* whilst the Moon moves from one Node to the other, therefore this happens four times; in each Revolution of the Moon it is twice diminish'd, and twice again increased. 1346 1314 1347

Plate XXV. Fig. 4.] Supposing the Nodes $N n$ to be in the Quadratures, the Forces which in one Revolution increase the Inclination and diminish it 1348

are equal to one another, for by reason of the equal distance of each Node from the Syzygies, the Forces that change the Inclination at ND and nE are equal to the Forces in the correspondent Points in the Arcs Dn and EN*; by the former the Inclination is increased, by the latter it is diminish'd*; the Diminution of the Angle of Inclination, on account of the first, is restor'd by the Action of the second; and here it is *not chang'd*. In the Motion above-mention'd* of the Line of Nodes in respect to the Sun, which depends upon the parallel Situation of this Line, the Node N is carried to the Syzygy E. When (*for Ex.*) the Line of Nodes is come to the Situation Mm, the Moon in its Recess from the Nodes goes thro' the Quadratures Nn, in which the Force which changes the Inclination vanishes*, and near which it is the least of all*; but in coming towards the Nodes, the Moon is every where distant from the Quadratures, and a greater force acts upon it*; therefore *considering one entire Revolution*, the Increase of the Angle of Inclination exceeds its Diminution*; that is, that Angle is increased, or which is the same, *the Inclination is diminish'd*; which obtains every where in the Motion of the Nodes from the Quadratures to the Syzygies.

1350 *When the Nodes are come to the Syzygies, the Inclination of the Plane of the Orbit is the least of all*;
 1351 *for in the Motion of the Nodes from the Syzygies to the Quadratures, the Plane of the Orbit is continually more and more inclin'd*; for in that Case, as the Moon goes to the Node, it passes thro' the Quadratures; in its Recess from them the Moon is distant from the Quadratures, and in one whole Revolution of the Moon, the Force which increases the Inclination*, exceeds that which diminishes it*, therefore *the Inclination is increased*; and it is the

the greatest of all when the Nodes are in the Quadratures, where the Diminution of the Angle made by the Plane of the Orbit with the Plane of the Ecliptic is terminated.*

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All the Errors in the Moon's Motion, that we have explain'd, are something greater in the Conjunction than in the Opposition.*

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All the disturbing Forces are determin'd by discovering their relation with the Addition of Gravity in the Quadratures, for which reason they all undergo the same Changes as that Addition does, that is, they are inversly as the Cube of the distance of the Sun from the Earth*, which, when it remains the same, they are as the distance of the Moon from the Earth*. Considering all the disturbing Forces together, the Diminution of Gravity prevails*, which follows immediately from the Progress of the Apfides; for it appears from thence, that considering several Revolutions together, the Effect of the Diminution of Gravity exceeds the Effect of the Increase of it*.*

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Therefore the Motion of the Moon being consider'd in general, the Gravity of the Moon towards the Earth is diminish'd coming near the Sun; therefore when it is less attracted by the Earth, it recedes more from it than it would recede if there was no such Diminution of Gravity; therefore in that Case the Moon's distance is increas'd, as also the periodical Time*; and that Time is the greatest, as also the distance of the Moon (cæteris paribus) the greatest when the Earth is in the Perihelion*, because then it is least distant from the Sun.*

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CHAP. XVII.

Concerning the Figures of the Planets.

IF we consider the Figures of the Planets, we shall find that they have such Figures which

follow from these very Laws by which the System is govern'd, which is very agreeable to that admirable Order which we observe every where, that no Forces act upon the Planets to destroy
 1358 them; that is, that *the Figure of a Planet, whether it be a primary or secondary Planet, is such as it would acquire if it wholly consisted of fluid Matter*; which agrees with the Phænomena.

1359 Whence it follows that *all the primary and secondary Planets are spherical*, for they consist of a Matter whose Particles gravitate towards one another*; from which mutual Attraction a spherical
 • 1206 Figure is generated, in the same manner as a Drop
 1297 becomes round from another sort of Attraction
 • 34 of the Parts*.

1360 *This spherical Figure of the Planets is not chang'd from their motion round the Sun, or from the motion of the secondary Planets about their primary ones, because all the Particles are carried by the same motion; but this Figure undergoes some Change by the motion round the Axis, and so much the greater as this motion is swifter.*

1361 *Plate XXV. Fig. 7.] Let PP be the Axis of a Planet, Ee the Diameter of the Equator, perpendicular to the Axis; let there be a Canal PCE fill'd with a Liquid; this Fluid will descend by its Gravity in both Legs towards C, and will not be at rest till the Pressure in both Legs be equal; if the Planet be at rest, the height of the Fluid in
 • 1359 both Legs will be equal*; but if the Planet be mov'd about its Axis Pp, all the Liquid in the Leg CE will endeavour to recede from the Center by its centrifugal Force*, which Force acts
 • 217 contrary to Gravity*, and therefore diminishes
 • 223 the Gravity; so that there is no *Equilibrium* till CE exceeds CP: Now if the Canal be taken away, the lateral Pressure of the Fluid, of which the Planet consists, does not change the Gravity
 towards*

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towards C, nor the difference between the heights of the Columns CE CP*; therefore the Planet • 280 is every where higher in the Equator than in the Poles, and *acquires, by its motion round its Axis, the* 1362 *Figure of a Spheroid depress'd in its Poles*; for the Elevation is continually diminish'd as you go towards the Pole, because the centrifugal Force is diminish'd by reason of the Diminution of the distance from the Axis*. • 232

If what has been demonstrated be compar'd with the Phænomena, it will appear why all the Bodies in our System are spherical*; but that this • 924 Figure is not exact, but a little chang'd by their motion round their Axis*, tho' this cannot be • 1362 observ'd in most of them, may be deduc'd from Observations made upon *Jupiter* and the Earth. *Astronomers have observ'd that the Axis of Jupiter is* 1363 *shorter than its equatorial Diameter*; altho' this Planet be the greatest of all the Planets, it is moved the swiftest about its Axis*, and therefore this • 962 difference may be observ'd.

The Elevation of the Earth at the Equator is de- 1364 *termin'd by us*, altho' perhaps to the Inhabitants of the other Planets, if there are any, it may not be more sensible than the Elevations on *Mars* and *Venus* are to us, which are so small that we cannot perceive them.

Suppose the Earth to be fluid, it will acquire 1365 the aforesaid spheroidical Figure*; if the Parts • 1362 cohere towards the Center, the Position of the other Parts will not be chang'd thereby, nor will it be chang'd if in some Places the Parts cohere together quite to the Surface; so that the Surface of the Sea must necessarily acquire a spheroidical Figure depress'd at the Poles. But since the Shores are every where but a little elevated above the Surface of the Sea, it is certain that the Continent acquires the same Figure.

Now

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Now to measure this Elevation, that is, how much the Diameter of the Equator of the Earth is longer than the Axis, we must consider its motion round its Axis in the Space of 23 Hours,
 • 960 56 Min. 4 Sec.* and supposing the Earth homogeneous, the Computation will be made in the following manner.

1366 The Periphery of the Earth is 128,202,185 *Rbynland* Feet, therefore in one Second of Time a Point of the Equator goes thro' 1488 Feet; the versed Sine of which Arc is 0,054, a Space which could be gone thro' by a Body in such a Time by the centrifugal Force.

By Gravity a Body in one Second, as we have shewn before, falls thro' 15,607 *Rbynland* Feet; but these Experiments were made at the distance of 48 Deg. from the Equator *Ee* (*Plate XXV. Fig. 7.*) at the Point *A*; the centrifugal Force at *E* is to the centrifugal Force at *A* as *CE* to *CA*,

• 232 for these Lines are very little different at *AB**; let this centrifugal Force be *Ab*, having drawn the Perpendicular *ba* to *CA* continu'd, let the Force thro' *Ab* be resolv'd into two other Forces

• 192 directed along *Aa* and *ab**, the Gravity is diminish'd only by the former, and *Ab* is to the Force diminishing it as *CA* to *AB*, by reason of the similar rectangular Triangles *Ab a* and *ABC*, which have their opposite vertical Angles equal at *A*; therefore the centrifugal Force at the Equator, with which a Body in one Second goes thro' 0,054, is to the Force which diminishes the Gravity at *A*, in a duplicate Proportion of the Radius *AC* to *AB*, which is the Cosine of the Latitude *AE* of 48 Degrees; so that from this diminishing Force the Body in one Second goes thro' 0,0243; wherefore if the Earth was at rest, in falling it would not go through 15,607 Feet, but 15,632; with which Gravity
 a Body

a Body falls under the Poles, because these Points are not mov'd; at the Equator, by the centrifugal Force, a Body goes thro' 0,054, and falls as much in the same Time from the Height of 15,578 Feet; whence it appears that the Gravity under the Poles is to the Gravity under the Equator as 289 to 288.

If *Fig. 7.* represents the Figure of the Earth, the Weight of a Column of Liquid CE will be to the Weight of a Column of Liquid CA, the Earth being at rest, as 289 to 288; for otherwise, the Earth moving, there will not be an *Equilibrium*, because $\frac{1}{15}$ of the Column CE is sustain'd by the centrifugal Force; for the centrifugal Force decreases as you come towards the Center, in the Ratio of the distance*, in which Ratio also the Gravity decreases*; so that in all the Points of the Column the same Part of the Weight is sustain'd as towards the Surface. * 232
* 1233

Whence we deduce, that *the height CP at the Pole is to the height EC at the Equator as 229 to 230*; for supposing this Ratio between the Axis and the equatorial Diameter, if a Computation be made of the Gravities in the Places P and E, the Earth being at rest, they are found to be to one another as 1121,71. to 1120,71; which Ratio obtains every where in correspondent Points, that is, which are distant from the Center as CP to PE, because in both Legs the Gravity decreases in proportion to the distance from the Center*; you have the Weight by multiplying the Quantity of Matter by the Gravity, for the Weight increases in a Ratio of both; by multiplying 1121,71. by 229, and 1120,71. by 230, the Products are to one another as 288 to 289, which is the Ratio of the Weights before discover'd; the mean Diameter of the Earth is 3,400,669 Perches*, 1367
* 1233

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• 976 ches*, therefore the Axis P P is 3,393,261, and the equatorial Diameter E e is 3,408,078 Perches, which exceeds the Axis by 14,817 Perches (*viz.*)

1368 $\frac{1}{15}$, and the Equator is more elevated by 7408,5.

1369 In this Computation, as we have said, we have look'd upon the Earth as homogeneous; but if it be more dense towards the Center, the Matter which is added to it may be look'd upon as a separate Body, from whose Center the Points P and E are unequally distant, and towards which therefore the Bodies P and E have a different Gravi-

• 1226 ty*; and the difference is so much the greater as these differences are greater; and it will be also so much the greater in respect of the whole Gravity, as the Quantity of Matter which is added, or which is the same, as the Density is greater towards the Center.

It is plain that the Forces of Gravity at the Poles and the Equator differ from one another more than $\frac{1}{15}$ Part, by comparing together Experiments made at several Distances from the Equator by the help of Pendulums, by which the Forces of Gravity may be compar'd together, as

• 164 we have shewn*, and which difference is truly
165 nearly double that which is found by Computa-
1370 tion; whence it follows, that the Elevation of the Equator is nearly double that which we have deter-
• 1368 min'd to be 7408,5 Perches*.

Now if we consider the spheroidical Figure of
1371 the Earth, we shall see that heavy Bodies do not tend directly to the Earth's Center, unless at the Poles and the Equator, but every where perpendicularly to the Surface of the Spheroid; for a Liquid will not be at rest unless its upper Surface forms a right
• 272 Angle with the Direction of heavy Bodies*; and the Figure of a Spheroid is form'd by the Surface of a quiescent Fluid. We also deduce this Direction

tion of heavy Bodies from the centrifugal Force. 1372

(Plate XXV. Fig. 7.) the Body A by its Gravity tends towards C, and is carried by its centrifugal Force along A b; this Force at the Point A is to the Gravity along A C as 1 to 430,8. having formed a Parallelogram with the Sides A c and A b, supposing these to one another as 430,8. to 1, the Diagonal will shew the Direction of heavy Bodies*, forming a small Angle with the Line A C. • 190

The Force along A b increases as you go towards the Equator, whereby this Angle is increas'd, but is diminish'd by the Increase of the Angle C A b; so that in the Equator, where the centrifugal Force is greatest, the Direction of heavy Bodies coincides with E C; at the Pole it coincides with P C, because there is no centrifugal Force there.

In this spheroidical Figure *the Latitude of the Place is determin'd by an Angle, as ACE, which is made with the Equator by a Line drawn from the Place of the Center*: Dividing this whole Arc P A E by this Method into 90 Parts, that is, into Degrees, it will easily appear, that going towards 1373
the Poles the Degrees are increased on the Surface; but this difference is so very small, that in measuring Degrees that are not very distant, it cannot be discover'd; because the Error, arising from the Make and Use of the Instruments, exceeds this difference; whence Degrees measur'd at the South and North of *France*, as also in *England*, differ little from one another, and the middle one is the least of all; wherefore *nothing can* 1374
be concluded concerning the Earth's Figure from these Measures. 1375

C H A P. XVIII.

The physical Explanation of the Motion of the Axis of the Earth.

•¹³⁴¹ **T**HAT the Nodes of the Moon go back,
 •¹³⁴⁷ that is, are mov'd in *antecedentia**, and that
 the Inclination of its Orbit is liable to change*,
 we have already demonstrated; let us conceive
 several Moons to be at the same distance, revolving
 in equal Times about the Earth, in a Plane
 inclin'd to the Plane of the Ecliptic, it is plain
 they will all be agitated by the same Motions;
 let us conceive the Number of the Moons to be
 increas'd, so as to touch one another, and form
 a Ring, whose Parts cohere; whilst one part of
 the Ring is attracted, to increase the Inclination,
 •¹³⁷⁶ the other part is agitated by a contrary motion,
 •¹³⁴⁵ to diminish its Inclination*; the greater Force
 •¹³⁷⁷ in this Case prevails, that is, in the motion of the
 Line of Nodes, from the Quadratures towards the
 Syzygies, the Inclination of the Ring is diminish'd in
 •¹³⁴⁹ each of its Revolutions*, and it is the least of all when
 •¹³⁵⁰ the Line of Nodes is in the Syzygies; on the contra-
 ry, its Inclination is increas'd when the Line of Nodes
 is carried from the Syzygies towards the Quadra-
 •¹³⁵¹ tures*, and it is the greatest of all when the Line of
 •¹³⁷⁸ the Nodes is in these last*; the Line of the Nodes is
 •¹³⁵² continually carried in *antecedentia*, unless when it is
 •¹³⁴¹ at rest in the Syzygies*.
 •¹³⁴⁴

379 If the Quantity of Matter in the Ring be dimi-
 nish'd, its Motions will not be chang'd, because they
 depend upon Gravity, which acts equally upon
 •¹⁰²⁷ every Particle of Matter*.

If

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If the Diameter of the Ring be diminish'd, these 1380
Motions are diminish'd in a Ratio of this Dimi-
nution*, but none of them wholly vanish; and 1354
it is agitated by the same Motions.

Let us conceive the Earth to be spherical; and 1381
in the Plane of the Equator, which makes an An-
gle of 23 Deg. 29 Min. with the Plane of the
Ecliptic, let there be a Ring, revolving in the
same Time as the Earth; let it be diminish'd so
as to touch the Earth, and cohere with it; by
this the aforesaid motion of the Ring will not be
destroy'd; for since the Earth is kept in a deter-
minate Situation by no Force, it yields to the
Impressions of the Ring, whose Agitations are
yet diminish'd, the Matter to be mov'd being in-
creas'd, and the moving Power remaining the
same.

And this is truly the Case; for the Figure of
the Earth is spherical, encompass'd with a Ring
at the Equator, whereby the Earth is more ele-
vated towards the Equator*, the Line of Nodes 1370
of which Ring is the Section of the Planes of the
Equator and Ecliptic; whence we deduce the
following Conclusions.

In the Equinoxes the Inclination of the Equator 1382
is least of all*, and therefore the Inclination of the 1377
Axis is the greatest, for it makes a right Angle
with the Plane of the Equator*; the Inclination 1075
of the Equator is increas'd, that is, the Inclination
of the Axis is diminish'd till the Sun comes to the Sol- 1383
stices, where this Inclination of the Axis is least of
all, and that of the Equator the greatest*; there- 1377
fore twice in a Year the Inclination of the Axis of the
Earth is diminish'd, and twice increas'd; and the 1384
Section of the Plane of the Equator with the Plane of 1385
the Ecliptic, which is at rest in the Equinoxes, the
rest of the Time is mov'd in antecedentia*. 1378

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- 1386 The Plane of the Equator is inclin'd to the Plane of the Moon's Orbit, for it makes a small
 • 969 Angle with the Plane of the Ecliptic*, and therefore the Moon acts in the same manner upon the Ring as the Sun; and altho' the Moon be less, yet because it is much less distant than the Sun, it exerts a greater Action upon the Ring; wherefore
 1387 also *the Inclination of the Axis of the Earth to the*
 • 1384 *Plane of the Moon's Orbit** (and consequently to the Plane of the Ecliptic) is twice chang'd in every Revolution, and twice restor'd by the Action of the Moon; and the Section of the Plane of the Equator
 • 1385 with the Plane of the Moon's Orbit* is carried in *antecedentia*; from which Motion necessarily follows, that the Section of the Plane of the Equator with the Plane of the Ecliptic changes its Place.
 1388 *The Changes of the Inclination of the Axis are too small to be observ'd, but the change of Place of the*
 1389 *Line of the Equinoxes, and the motion of the Axis* which follows from it, being always carried the same way, at last become sensible; and from these
 • 1185 follow the Phænomena before explain'd*.
 1186

C H A P. XIX.

Concerning the Tides.

THAT we may explain the Tides from the Principles already laid down, we must consider, that the Earth, as also all Bodies near it,
 • 1206 gravitate towards the Moon*; therefore the Particles of Water in the Earth's Surface which tend towards the Center of the Earth (for here we neglect the Consideration of N^o 1371.) are carried with it towards the Moon. Since also the solid Mass of the Earth is carried towards the Moon, according to the Laws, which would obtain,

obtain, if all the Matter of which it consists was collected in its Center*, *what has been demonstrated in Chap. 16. of the Action of the Sun upon the Moon* falling towards the Earth, whilst with the Earth it goes towards the Sun, *may be applied to the Action of the Moon upon the Particles of Water in the Earth's Surface*, which do not cohere with the Mass of the Earth, but tend towards its Center, and continually with its Mass fall towards the Moon; by which Force, as we have shewn*, the Earth is kept in its Orbit, about the common Center of Gravity of the Earth and the Moon.

Plate XXV. Fig. 1.] Let S be the Moon, A L B L / the Surface of the Earth, whose Mass tends towards the Moon, as if it was all collected at T; by the Action of the Moon, the Particles of Water A and B acquire a greater Gravity towards T*; on the contrary, the Particles at L / lose of their Gravity*; whence we deduce, that if the whole Earth was cover'd with Water there would not be an *Equilibrium*, unless this Water was more elevated in the Points L and /, than in a whole Circle 90 Degrees distant from these Points, and therefore passing thro' the Points A and B; therefore *by the Action of the Moon the Water acquires a spheroidical Figure, formed by the Revolution of an Oval about its greater Axis, which being continu'd, goes thro' the Moon.*

Let us suppose the Moon in the Equator, all the Sections of the Earth, which are parallel to the Equator, as they are also parallel to the Axis of the Spheroid*, are oval, whose greater Axes pass thro' the Meridian of the Moon; whence it follows, that *the Earth being at rest in any Circle of Latitude, the Water is more elevated in the Meridian in which the Moon is, and in the opposite Meridian, than in the intermediate Places.*

DEFINITION.

1394 The lunar Day is the Time spent between the Moon's going from the Meridian and coming to it again; this Day is divided into 24 lunar Hours, it is 50 Minutes longer than the natural Day.

From the Motion of the Earth round its Axis every lunar Day, every Place passes thro' the Meridian of the Moon and the opposite Meridian, that is, twice passes thro' that Place where the Water is raised by the Action of the Moon, and twice thro' that Place where the Water is dispersed by the same Action*; and so in a lunar Day the Sea is twice elevated and twice depressed in any assign'd Place.

1396 By the Motion of the Earth round its Axis, the elevated Water continually recedes from the Meridian of the Moon; yet by the Action of the Moon the Axis of the Spheroid passes thro' the Moon*; therefore the Water is continually agitated, that the Elevation, which (on account of the Motion of the Earth) is remov'd, may be brought under the Moon; therefore the Water continually flows from A and B (Plate XXV. Fig. 1.) towards L and l, whilst by the Motion of the Earth the Elevation is carried from L towards B, and from l towards A; that is, between L and B, as also between l and A, there are two contrary Motions, by which the Water is accumulated; so that the greatest Elevations are between these Points (viz.) not directly under the Moon, but on one side of that Point, and likewise aside of the opposite Point; that is, in any Place the Water most elevated two or three Hours after the Moon has passed the Meridian of the Place, or the opposite Meridian.

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*The Elevation towards the Moon a little exceeds the opposite one**; the Ascent of the Water is diminish'd as you go towards the Poles, because there is no Agitation of the Water there. 1398 1390 1295

What has been demonstrated in relation to the Moon, may be applied to the Sun; therefore from the Action of the Sun every natural Day, the Sea is twice elevated and twice depressed*; this Agitation is much less, on account of the immense distance of the Sun, than that which depends upon the Moon, yet it is subject to the same Laws. 1399 1400 1401 1395 1402

The Motions which depend upon the Actions of the Moon and Sun, are not distinguish'd, but confounded; and from the Action of the Sun the lunar Tide is only chang'd; which Change varies every Day, by reason of the Inequality between the natural and lunar Day*. 1403 1404 1394

In the Syzygies the Elevation from the Actions of both Luminaries concur, and the Sea is more elevated; the Sea ascends less in the Quadratures, for where the Water is elevated by the Action of the Moon, it is dispersed by the Action of the Sun, and so on the contrary; therefore whilst the Moon passes from the Syzygy to the Quadrature, the daily Elevations are continually diminish'd; on the contrary, they are increas'd when the Moon moves the Quadrature to the Syzygy: At a New-moon also, cæteris paribus, the Elevations are greater; and those that follow one another the same Day, are more different than at a Full-moon*. 1405 1406 1407 1398

The greatest and least Elevations are not observ'd till the second or third Day after the New or Full-moon, because the Motion acquir'd is not presently destroy'd from the Attrition, and other Causes, by which acquir'd Motion the Ascent of the Water is increas'd; altho' the Action by which the Sea is rais'd be diminish'd, somewhat like 1402 1408

like to what we have demonstrated elsewhere concerning Heat*.

1409 If now we consider the Luminaries receding from the Plane of the Equator, we shall perceive that *the Agitation* is diminish'd, and *becomes less, according as the Declination of the Luminaries becomes greater*; which plainly appears, if we conceive them to be in the Poles, for then the Axis of the spheroidical Figure coincides with the Action of the Earth; and all the Sections that are parallel to the Equator, are perpendicular to the Axis of the Spheroid, and therefore circular; so that the Water in every Circle of Latitude will have every where the same Elevation; and so in the Motion of the Earth, the height of the Sea is not chang'd in particular Places. If the Luminaries recede from the Poles, it is easy to find that the Agitation will be more and more increas'd, till it be the greatest of all, the Spheroid revolving about a Line perpendicular to the Axis, the Axis of the Spheroid being supposed in the Plane of the Equator.

1410 Hence it is plain, why in the Syzygies, near the Equinoxes, the Tides are observ'd to be the greatest, both Luminaries being in or near the Equator.

1411 *The Actions of the Moon and Sun are greater, the less those Bodies are distant from the Earth**; but 1390 when the Distance of the Sun is less, and it is in the South Signs, often both the greatest equinoctial Tides are observ'd in that Situation of the Sun; that is, before the vernal, and after the autumnal Equinox, which yet does not happen every Year, because some Variation may arise from the Situation of the Moon's Orbit, and the distance of the Syzygy from the Equinox.

Plate XXV. Fig. 8.] In Places distant from the Equator, as the Luminaries recede from the Equator, the Elevations that happen the same Day are unequal. Let PP be the Axis of the Earth, EE the Equator, Ll a Circle of Latitude, AB the Axis of the spheroidical Figure which the Water forms; when a Place in the Circle Ll is given at L or l, it is given in the same Meridian with the Axis of the Spheroid, and the Water is most elevated in both Cases; yet at L it is more elevated than at l, for Cl exceeds Cl; which Lines measure the Heights of the Waters, that is, the Distances from the Center; these Lines would be equal if AL and Bl (which are the Distances from the Axis of the Spheroid) were equal; but Cl is less, because Bl exceeds AL, which arises from the Inclination of the Axis of the Spheroid to the Equator.

As long as the Moon is on the same side of the Equator in any Place, that is, towards the Line CA continu'd, the Elevation of the Water is observ'd to be greatest every Day after the Moon has passed the Meridian of the Place, for there is the greatest Elevation when the Place is come to L; but if the Equator separates, or is between the Moon and the Place of which we speak, that is, if the Moon be towards the Line CB continu'd, the Water again at L will come to the greatest height, and every Day the greatest Elevation of the Sea will be after the Moon has passed thro' the opposite Meridian.

All things which have been hitherto explain'd, would exactly obtain, if the whole Surface of the Earth was cover'd with Sea; but since the Sea is not every where, some Changes arise from thence, not indeed in the open Sea, because the Ocean is extended enough to be subject to the

Motions we have spoken of; but *the Situation of*
 1415 *the Shores, the Streights, and many other things, de-*
pending upon the particular Situation of the Places,
disturb these general Rules; yet it is plain from the
 most general Observations, that the Tide follows
 the Laws which we have explain'd. What re-
 mains is, to determine the Forces with which the
 Sun and Moon act upon the Sea, that it may ap-
 pear that they are able to produce the Effects
 which we have mention'd; and that the Actions
 of those Bodies upon Pendulums and other Bodies
 are insensible.

- 1416 The Increase or Addition to the Gravity of the
 Moon in the Quadratures from the Action of the
 Sun, is to the Gravity of the Moon towards the
 1291 Earth as 1 to 178,73; in which Computation
 we have suppos'd the mean distance of the Moon
 from the Center of the Earth to be 60 Semidia-
 1285 meters of the Earth; therefore the Gravity of
 the Moon is to the Gravity of the Earth's Surface
 *1208 as 1 to $60 \times 60 = 3600$ *; therefore the above-
 mention'd Increase is to the Gravity on the
 Earth's Surface as 1 to 643428; in which Com-
 putation there is an Error to be corrected.

- This Computation would be exact, if the In-
 crease of which we speak, was to the Force with
 which the Earth descends towards the Sun, as the
 distance of the Moon (which is 60 Semidiameters
 of the Earth) to the distance of the Earth from
 1287 the Sun; but it is as the true mean distance of
 the Moon, which is $60\frac{1}{2}$ Semidiameters of the
 Earth to the distance of the Earth from the Sun;
 wherefore the Increase that we have just deter-
 min'd ought to be $\frac{1}{11}$ Part increased, and will
 be to the force of Gravity on the Earth's Surface
 as $1\frac{1}{11}$ to 643428, or as 1 to 638110,4.

This Increase of the Gravity of the Moon in
 the Quadratures from the Action of the Sun, is
 to

to the Increase of the Gravity of the Water on the Earth's Surface, in Places which are 90 Deg. distant from the Sun (from the same Action of the Sun) as $60\frac{1}{2}$ to 1^{*}; therefore this Increase of ¹²⁸⁸ the Gravity is to the Gravity of the Water as 1 to 38605679. The Diminution of the Gravity under the Sun, and in the opposite Place, is double this Increase^{*}; therefore it is to the Gravity as 1 ¹²⁹⁶ to 1930839; and the whole Change in the Gravity ¹⁴¹⁷ arising from the Action of the Sun, is to the Gravity itself as 1 to 12868560.

In order to compare the Action of the Moon ¹⁴¹⁸ with the Action of the Sun, we must make Experiments in Places, in which, by reason of the Narrowness, the Sea is sensibly raised. Near Bristol, at the Autumn and Spring, at which Times the Agitation of the Sea is greatest^{*}, the Water ¹⁴¹⁰ ascends in the Syzygies about 45 Feet, more or less; in the Quadratures about 25 Feet, more or less; which Numbers are to one another as 9 to 4.

The Determination of the Forces, which we would find, if the greatest and least Elevations were exactly at the Time of the Syzygies, would be very easy, which we have shewn before not to happen so^{*}. ¹⁴⁰⁸

The distance of the Moon from the Syzygy or the Quadrature is not always the same in the greatest or least Elevation, for this distance varies, because the Moon is sometimes more, and sometimes less distant from the Meridian, when it goes thro' the Syzygy or Quadrature; the mean distance of the Moon from the Syzygy or Quadrature, to which the aforesaid Observations ought to be referr'd, is about 18 Deg. 30 Min. so that the whole Action of the Sun neither conspires with the Action of the Moon in the Syzygies, nor acts contrary to it in the Quadratures; also

in such a Case, if at the Syzygy both the Luminaries be in the Equator, in the said distance from the Quadrature, the Declination of the Moon is 22 Deg. 13 Min. more or less, whereby the force of the Moon to move the Sea is diminish'd*; besides, *cæteris paribus*, the distance of the Moon from the Earth at the Syzygies is less than at the Quadratures*, whence also the Action of the Moon is diminish'd at the Quadratures*; by attending to all which things we may discover, that the mean Force of the Sun to move the Sea, is to the mean Force of the Moon to move the same, as 1 to 4,4815; but the Force of the Sun is to the Force of Gravity as 1 to 12868560*; wherefore the Force of the Moon is to the same Force of Gravity as 1 to 2871485; from whence it follows, that these Forces of the Moon and Sun are too small to be sensible in Pendulums, and other Experiments; but it is easily prov'd that they are capable of agitating the Sea.

By diminishing the Gravity $\frac{1}{12}$ Part, the Sea is raised to the height of 88902 Rbynland Feet*, for each Perch contains 12 Feet; whence, by the Rule of Proportion*, we find, that the Action of the Sun changes the height of the Sea two Feet, and that the Action of the Moon changes it 8,95*; and that from the join'd Action of both, the mean Agitation is of about 11 Feet, which agrees pretty well with Observations; for in the open Ocean, as the Sea is more or less open, the Water is raised to the height of 6, 9, 12, or 15 Feet; in which Elevations also there is a difference arising from the Depth of the Waters; but those Elevations, which far exceed these, happen where the Sea violently enters into Streights or Gulfs, where the Force is not broken till the Water rises higher.

C H A P. XX.

Of the Moon's Density and Figure.

THE Forces of the Sun and Moon for giving motion to the Sea, are to one another in a Ratio compounded of the Ratio of the Quantities of Matter in these Bodies* (for all the Particles of Matter act) and the inverse Ratio of the Cubes of the Distances of the Sun and Moon from the Earth*.

1425

*1207

*1390

1402

1354

* 288

79

The Quantities of Matter are in a Ratio compounded of the Ratio of the Bulks, that is, of the Cubes of the Diameters and the Ratio of the Densities* ; wherefore the Forces above-mention'd are directly as the Densities and the Cubes of the Diameters, and inversely as the Cubes of the Distances.

The apparent Diameters of Bodies, that is, the Angles under which they are seen, increase as the Diameters themselves, and diminish as the Distances ; that is, they are directly as the Diameters, and inversely as the Distances ; therefore the Ratio compounded of the Ratio's of the Cubes of the apparent Diameters of the Sun and Moon, and of the Ratio of the Densities, will be the Ratio of the Forces whereby those Bodies act upon the Sea ; therefore the Densities of those Bodies are directly as the Forces whereby they move the Sea, and inversely as the Cubes of their apparent Diameters ; and dividing the Forces by the Cubes of those Diameters, you have the Ratio of the Densities.

1426

8281

1351

The

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The Force of the Sun is to the Force of the
 1419 Moon as 1 to 4,4815 ; the mean apparent Dia-
 meter of the Sun is 32 Min. 12 Sec. and the mean
 apparent Diameter of the Moon is 31 Min. 16½
 Sec. that is, they are to one another as 3864 to
 1427 3753 ; therefore *the Density of the Sun is to the*
Moon's Density as 10000 to 48911 ; which Den-
 sity of the Moon may be compar'd with the Den-
 sities of *Jupiter, Saturn, and the Earth** ; and the
 *1260 Moon is denser than the Earth.

The Quantities of Matter in two Bodies are to
 one another in a Ratio compounded of the Denfi-
 * 79 ties and Bulks* ; that is, if the Body be a Sphere,
 in a Ratio compounded of the Denfities and the
 Cubes of the Diameters.

The Denfities of the Moon and Earth are to one
 1428 another as 48911 to 39214* , the Diameters as 11
 *1427 to 40,2. therefore the Quantities of Matter in
 *1260 those Bodies are as 1 to 99,13. Tho' the Denfi-
 ties be discover'd, if you suppose the Bodies to be
 homogeneous, yet the Quantities of Matter will
 be rightly defin'd tho' the Bodies are not homo-
 geneous ; for we determine the Density which
 that Body would have, if the Matter of which
 the Body really consists was equally diffus'd all
 over it.

The Gravities on the Surfaces of the Earth and
 1429 Moon are determin'd by multiplying the Denfities
 1230 by the Diameters , that is, they are to one ano-
 ther as 2,93 to 1, or as 407,8 to 139,2 ; which
 Number also does express the relation of Gra-
 vity on the Surface of the Moon with the Gravity
 1258 on the Surfaces of the Sun, Jupiter, and Saturn .

The common Center of Gravity of the Moon and
 1430 Earth, about which both Bodies are mov'd, is
 determin'd ; for its distance from the Center of
 the Earth, is to the distance between the Centers
 of

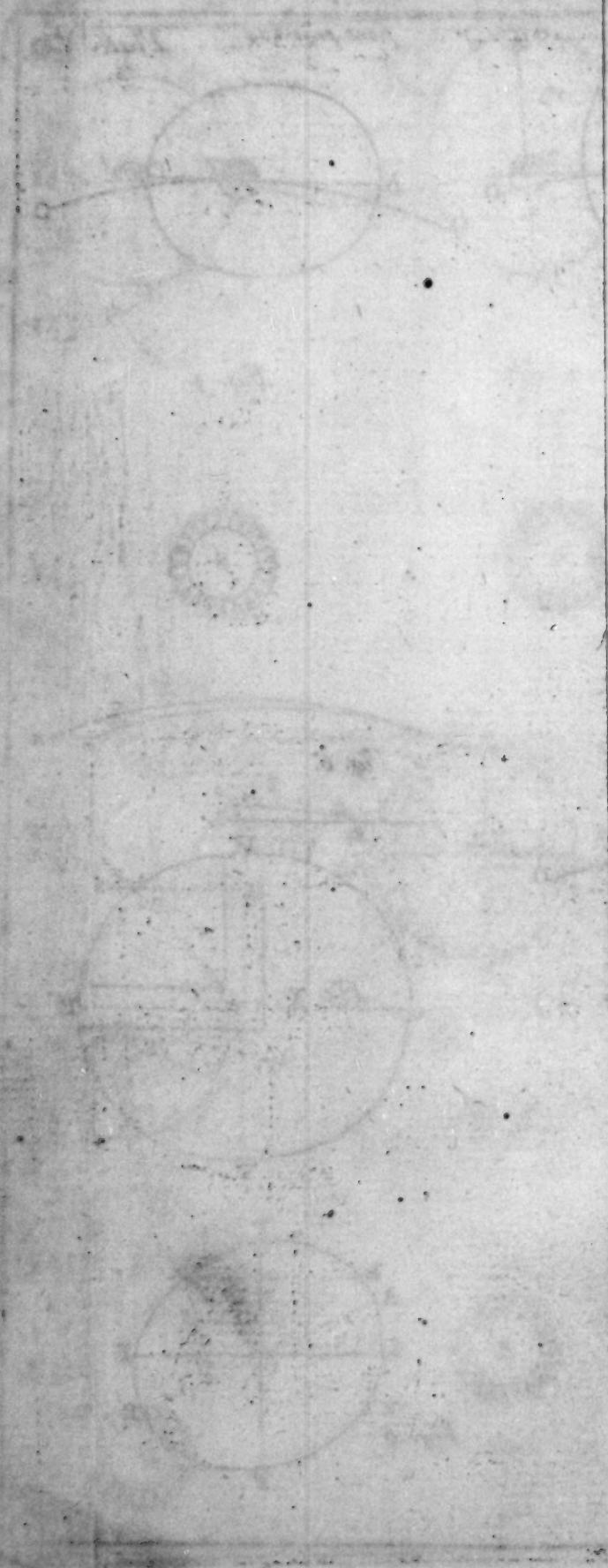
of both Bodies, as the Quantity of Matter in the Moon to the Quantity of Matter in both Bodies*; * 234
therefore 40,13 is to 1 as the distance of the Moon * 235
from the Earth, is to the requir'd distance of the
Center of Gravity *from the Center of the Earth*,
which is found to be of 5126950 Perches, as is
deduc'd from the known Diameter of the Earth* * 976
and the distance of the Moon.

To determine the Figure of the Moon, we must 1431
examine what Figure it would have if it was
fluid*. If we consider the Moon alone at rest, it *1358
would be spherical*; if we consider the Action*1359
of the Earth upon the Moon, the Moon would
acquire the Figure of a Spheroid, whose Axis
would go thro' the Earth*. The Force of the *1392
Earth for changing the Figure of the Moon, is
to the Force of the Moon upon the Earth as
39,13 to 1*, and as the Diameter of the Moon *1428
to the Earth's Diameter*, which are to one ano- *1207
ther as 11 to 40,2; and it is a Ratio compound- *1390
ed of these 10,7 to 1. This Force of the Moon *1354
is to the Gravity upon the Earth's Surface as 1 to
2871485*; which Gravity on the Earth's Sur- *1420
face is to the Gravity on the Surface of the Moon
as 407,8 to 139,2*, or as 2871485 to 980028; *1429
wherefore *the Action of the Earth for changing the
Moon's Figure is to the Gravity upon the Moon's Sur-
face as 10,7 to 980028, or as 1 to 91524*; the 1432
Gravity being chang'd on the Earth's Surface by
~~111111~~ Part, the Water is rais'd 8,95 Feet*; *1420
and therefore if Gravity was to be chang'd ~~111~~ 1422
Part, the Elevation would be of 280,7 Feet, as
it is found by the Rule of Three. If, keeping
this Diminution of Gravity, we consider a less
Body, this Height must be diminish'd in propor-
tion to the Diameter; therefore from the Action
of

of the Earth the Elevation of the Moon is of 76,8 Feet; and if the Moon be homogeneous,
 1433 *there will not be an Equilibrium, unless the Axis of the Spheroid exceeds the Diameter, which is perpendicular to it, 153,60 Feet.*

1434 *The Elevation of the Moon from the Action of the Earth, may be discover'd by one single Proportion, by knowing the Elevation of the Sea from the Moon's Action; for these Elevations are in a duplicate inverse Ratio of the Gravities on the Surfaces of those Bodies.*

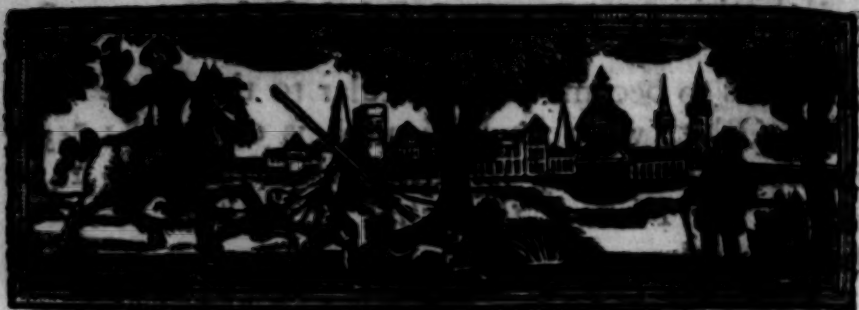
1435 *If, supposing this to be the Figure of the Moon, we conceive the Parts to cohere, there will not be an Equilibrium between the Parts of the Moon, unless the Axis of the Spheroid be directed towards the Earth; whence we see the reason why the Moon always turns the same Face towards the Earth, by which continual Agitation the Moon has at least acquired the Motion about its Axis, of which*
 1436 *we have before spoken**; which Motion must necessarily be perform'd in the same Time as the Moon performs one Revolution, for from the Action above-mention'd it must necessarily adapt itself to such a Celerity; for if the Celerity was greater, it would be continually retarded by the Force whereby the same Face is always directed towards the Earth; and if this Celerity was less, it would be continually accelerated; yet
 1437 *this Force is not great enough sensibly to disturb the Equibilty of the Motion acquired about the Axis every Revolution; therefore the Motion about the Axis is equable, though the Moon be moved in its Orbit by an unequal Motion*.* The Position also of the Moon's Axis cannot be so chang'd by the Force above-mention'd,



tion'd, as to become perpendicular to the Plane of the Orbit, when its Inclination is chang'd*; *1345
therefore the Axis of the Moon is sometimes in- 1438
clin'd to the Plane of the Orbit; as we have be-
fore shewn*. *1093

The End of the Fourth Book.





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